Pitch and Flight Path Control with Simultaneous Forward Gust Rejection

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Abstract: - The problem of rejecting atmospheric disturbances with simultaneous input-output decoupling of the pitch angle and the flight path angle of an aircraft, is studied. The problem is proven to be solvable for almost all flight conditions via static state controllers. All static controllers solving the problem are explicitly characterized. Stability requirements are fulfilled. The present results are illustrated via simulation.

KeyWords: -Aircraft control, linear systems, input-output decoupling, disturbance decoupling, stability, feedback.

Nomenclature

U , W: translation velocities at the x and z axis
U_g: gust velocity at the x axis
Q : pitch rate
Θ , Γ: pitch angle and flight path angle
m: aircraft mass; I_y: y-axis moment of inertia
X , Z: x and z axis aerodynamic propulsion and gravitational forces
M: y-axis external aerodynamic & propulsion moment
$u_a(t)$, $w(t)$: forward and vertical velocity increments
$\vartheta(t)$, $q(t)$, $\gamma(t)$: pitch angle, pitch rate and flight path angle increments
γ_0, U_0: nominal values of flight path angle and forward velocity
$\delta_\tau(t)$, $\delta_e(t)$: thrust and elevator deflection
g: gravitational acceleration
Z_u, X_u, Z_w, X_w, $Z_{\delta e}$, $Z_{\delta\tau}$: dimensional force stability derivatives
M_u, M_ϑ, $M_{\delta\tau}$, M_{δ_e}, M_w,M_q, $M_{\dot{w}}$: dimensional pitching moment derivatives
$u_g(t)$: gust component increment along stability axis

1 Introduction

The problem of rejecting atmospheric disturbances with simultaneously controlling independently the flight variables of a multimode aircraft is one of the central problems in flight path control. The design objective is to eliminate the coupling between the flight variables of the aircraft, and to completely reject the undesirable effects of unknown forward disturbances, thus allowing the pilot to perform precise manoeuvres by applying simple commands.

Pitch mode decoupling or flight path control have extensively been studied and solved in [1]-[7]. The problem of rejecting unknown disturbances is solved in [8]. Results regarding the combined problem of independent control and disturbance attenuation using a prediction of gust responses, is studied in [9].

In this paper, disturbance rejection (decoupling) with simultaneous input-output decoupling, is used in order to control the longitudinal motion of an aircraft flying in non uniform atmosphere. In particular, a static state feedback law is applied to independently control the flight path angle and the pitch angle of the aircraft while an unknown gust acceleration is completely rejected from the pitch and the flight path angle. The problem is proven to be solvable for almost all flight conditions. Explicit characterization of all controllers solving the problem, is derived in terms of the aerodynamics parameters of the aircraft as well as free parameters that can be used to satisfy pole assignment. In particular, if the pitch angle and the flight path angle can be controlled independently without being influenced by the disturbance, via a disturbance rejection with simultaneous decoupling control law, the conditions for stabilizability are established.

The goal of our study is to improve the aircraft effectiveness to follow the pilot's commands. Using the present control scheme the requirements of certain flight manoeuvres, such as pitch and flight path pointing etc., can easily be met inspite of the presence of the unknown forward gust. Using the free parameters of the controller a satisfactory closed loop performance is achieved in the sense of good flying qualities. All above results are illustrated for a jet fighter/bomber aircraft, using simulation.

2 Equation of longitudinal motion

The nonlinear equations describing the longitudinal motion of a an aircraft are as follows [10-11]:

$$X - mg\sin(\Theta) = m(\dot{U} + \dot{U}_g + QW)$$
$$Z + mg\cos(\Theta) = m(\dot{W} - QU)$$
$$M = I_y\dot{Q}\,,\quad \dot{\Theta} = Q\,,\quad \Theta - \Gamma = \tan^{-1}(W/U) \quad (2.1)$$

The respective linearized model can easily be derived using the small disturbance theory and expressing the aerodynamic forces and moments as functions of all motion variables (see f.e. [10-11]). Here we study the longitudinal motion of an advanced aircraft, in stability axis system, for straight symmetric flight with wings level. In case where the increment of the angle of attack, is sufficiently small, the approximate equation, $\vartheta - \gamma \cong w/U_0$, can be used to derive the following aircraft description [9]

$$\dot{x}(t) = Ax(t) + Bu(t) + D\dot{u}_g(t)\,,\ y(t) = Cx(t) \quad (2.2a)$$

with

$$x(t) = \begin{bmatrix} u_a(t) & \gamma(t) & q(t) & \vartheta(t) \end{bmatrix}^T$$
$$y(t) = \begin{bmatrix} \gamma(t) & \vartheta(t) \end{bmatrix}^T,\ u(t) = \begin{bmatrix} \delta_\tau(t) & \delta_e(t) \end{bmatrix}^T \quad (2.2b)$$
$$A = \begin{bmatrix} X_u & -X_w U_0 & 0 & X_w U_0 - g\cos\gamma_0 \\ -Z_u/U_0 & Z_w & -Z_q/U_0 & -Z_w + g\sin\gamma_0/U_0 \\ \tilde{M}_u & -\tilde{M}_w U_0 & \tilde{M}_q & \tilde{M}_\vartheta + U_0\tilde{M}_w \\ 0 & 0 & 1 & 0 \end{bmatrix}$$
$$B = \begin{bmatrix} X_{\delta\tau} & X_{\delta e} \\ -\frac{Z_{\delta\tau}}{U_0} & -\frac{Z_{\delta e}}{U_0} \\ \tilde{M}_{\delta\tau} & \tilde{M}_{\delta e} \\ 0 & 0 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, D = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (2.2c)$$

where

$$\tilde{M}_u = M_u + M_{\dot{w}}Z_u\,,\ \tilde{M}_w = M_w + M_{\dot{w}}Z_w$$
$$\tilde{M}_q = M_q + U_0 M_{\dot{w}}\,,\ \tilde{M}_\vartheta = gM_{\dot{w}}\sin(\gamma_0)$$
$$\tilde{M}_{\delta e} = M_{\delta e} + M_{\dot{w}}Z_{\delta e}\,,\ \tilde{M}_{\delta\tau} = M_{\delta\tau} + M_{\dot{w}}Z_{\delta\tau}$$

The unknown gust $\dot{u}_g$ is a wind acceleration gust due to spatial and temporal variation in the horizontal gust component. The point approximation for the above gust is used where in spacewise variations over aircraft size are ignored in frozen gust [9].

3 The feedback system

The objective of the present design scheme is to control independently the flight path angle γ and the pitch angle ϑ and to completely reject the influence of the forward acceleration gust $\dot{u}_g$ from γ and ϑ. The combined design scheme facilitates the aircraft placement and maintenance to desired orientation. To meet these requirements the input-output diagonal decoupling with simultaneous disturbance rejection technique, is proposed. To this end, a static state feedback law of the form

$$u(t) = Fx(t) + G\omega(t) \quad (3.1)$$

is applied to the system (2.2), where $\omega(t) = [\gamma_c(t)\ \vartheta_c(t)]^T$ is the external command vector with $\gamma_c(t)$ and $\vartheta_c(t)$ denoting the pilot commands for driving the performance variables γ and ϑ. The disturbance rejected and diagonally decoupled closed loop system, is of the following form

$$\mathcal{L}\begin{bmatrix} \gamma(t) \\ \vartheta(t) \end{bmatrix} = \begin{bmatrix} h_1(s) & 0 & 0 \\ 0 & h_2(s) & 0 \end{bmatrix} \mathcal{L}\begin{bmatrix} \gamma(t) \\ \vartheta(t) \\ \dot{u}_g(t) \end{bmatrix}$$

where the operator $\mathcal{L}$ denotes the Laplace transform. The most beneficial characteristic of the above disturbance rejected and diagonally decoupled structure is the ability to control each output variable by using only one external input independently from the disturbances and without influenced by any other output. Another benefit, resulting from the decoupled closed loop, is the ability to tune each output separately by arbitrary shifting the poles of $h_i(s)$.

4 Solvability Conditions

In this section it is investigated under which conditions (over the aerodynamic parameters) the proposed configuration results to disturbance rejected and diagonally decoupled closed-loop system.

Theorem 4.1. The necessary and sufficient condition for forward gust rejection with independent control of the pitch and the flight path angle of the aircraft (2.2), via the feedback (3.1), is

$$\tilde{M}_{\delta\tau}Z_{\delta e} \neq \tilde{M}_{\delta e}Z_{\delta\tau} \quad (4.1)$$

Proof: As proven in [12], disturbance rejection with simultaneous decoupling is solvable if and only if the $\det[C^*B] \neq 0$, and $C^*D = 0$, where

$$C^* = \begin{bmatrix} c_1 A^{d_1} \\ c_2 A^{d_2} \end{bmatrix}\ ;\ c_i : \text{the } i\text{-th row of } C$$
$$d_i = \begin{cases} j : c_i A^j B \neq 0 & j = 0, 1, \ldots, n-1 \\ n-1 \ \text{ if } \ c_i A^j B = 0 & \forall j \end{cases} \quad (4.2)$$

Since $c_1 B = \begin{bmatrix} \frac{-Z_{\delta\tau}}{U_0} & \frac{-Z_{\delta e}}{U_0} \end{bmatrix} \neq 0$ then $d_1 = 0$.Since $c_2 B = 0$ and $c_2 AB = \begin{bmatrix} \tilde{M}_{\delta\tau} & \tilde{M}_{\delta e} \end{bmatrix} \neq 0$ then $d_2 = 1$.Note that U_0 is always $\neq 0$. For the present case it holds that $C^* = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ and $C^*D = 0$.Thus, (4.1) is proven. ■

Remark 4.1. Note that, condition (4.1) involves only the stability derivatives $M_{\delta e}$, $Z_{\delta\tau}$, $M_{\delta\tau}$ and $Z_{\delta e}$ (since

$\tilde{M}_{\delta\tau} = M_{\delta\tau} + M_{\dot{w}} Z_{\delta\tau}$ and $\tilde{M}_{\delta e} = M_{\delta e} + M_{\dot{w}} Z_{\delta e}$) and it is true for almost all values of these derivatives [10-11].

5 Explicit Characterization of all Decoupling Controllers

According to [12] the general expressions of the feedback matrices F and G solving the problem, are given by the following expressions

$$G = \frac{1}{\tilde{M}_{\delta e} Z_{\delta\tau} - \tilde{M}_{\delta\tau} Z_{\delta e}} \begin{bmatrix} -\tilde{M}_{\delta e} U_0 (p_1)_0^{-1} & -Z_{\delta e} (p_2)_0^{-1} \\ \tilde{M}_{\delta\tau} U_0 (p_1)_0^{-1} & Z_{\delta\tau} (p_2)_0^{-1} \end{bmatrix} \quad (5.1)$$

$$F = \frac{1}{\tilde{M}_{\delta e} Z_{\delta\tau} - \tilde{M}_{\delta\tau} Z_{\delta e}} \begin{bmatrix} -\tilde{M}_{\delta e} U_0 & -Z_{\delta e} \\ \tilde{M}_{\delta\tau} U_0 & Z_{\delta\tau} \end{bmatrix} \times \begin{bmatrix} \frac{Z_u}{U_0} & (\lambda_1)_1 & \frac{Z_q}{U_0} & -\frac{g \sin\gamma_0}{U_0} + Z_w \\ \tilde{M}_u & \tilde{M}_w U_0 & (\lambda_2)_1 & (\lambda_2)_2 \end{bmatrix} \quad (5.2)$$

where $(\lambda_i)_j$, $(p_i)_0$ are arbitrary parameters. Relations (5.1 and 2) are explicit formulae yielding controllers that can easily be implemented by elementary operations upon the values of the stability derivatives of the aircraft, and the nominal values γ_0 and U_0.

6 Decoupled Closed Loop System

The degrees of freedom $(\lambda_i)_j$ in (5.1 and 2), can be used to shift the closed loop system poles. To this end, the general form of the transfer function matrix of the decoupled closed loop system is derived to be

$$C(sI - A - BF)^{-1} \left[\begin{array}{c|c} BG & D \end{array}\right] = \left[\begin{array}{cc|c} \frac{(p_1)_0^{-1}}{s - Z_w - (\lambda_1)_1} & 0 & 0 \\ 0 & \frac{(p_2)_0^{-1}}{s^2 - [\tilde{M}_q - (\lambda_2)_1]s - (\lambda_2)_2 - \tilde{M}_w U_0 - \tilde{M}_\vartheta} & 0 \end{array}\right] \quad (6.1)$$

The polynomial which is cancelled out in the general form of the closed loop transfer function, is

$$p_u(s) = s + \frac{\tilde{M}_{\delta e}(X_{\delta\tau} Z_u - X_u Z_{\delta\tau}) + \tilde{M}_{\delta\tau}(X_u Z_{\delta e} - Z_u X_{\delta e}) + \tilde{M}_u (X_{\delta\tau} Z_\tau - Z_{\delta e} X_{\delta\tau})}{\tilde{M}_{\delta e} Z_{\delta\tau} - \tilde{M}_{\delta\tau} Z_{\delta e}} \quad (6.2)$$

*Theorem 6.1.*Disturbance rejection with simultaneous independent control of the pitch angle and the flight path angle and stabilizability can be achieved if (4.1) is satisfied and $p_u(s)$ is Hurwitz. ■

7 Simulation Results

Consider the linear model of the a twin-engined jet fighter/bomber aircraft [11] flying at height 13700 m and with 2.15 Mach. For this model the aircraft parameters are [11]: $U_0 = 650$ m/sec, $Z_u = -0.001$, $Z_w = -0.494$, $Z_{\delta e} = -25.45$, $Z_{\delta\tau} = -0.00005$, $Z_q = -0.39$, $M_u = 0.07$, $M_q = -0.41$, $M_w = -0.07$, $M_\vartheta = 0$, $M_{\delta e} = -16.1$, $M_{\delta\tau} = -0.000003$, $M_{\dot{w}} = -0.001$, $X_u = 0.016$, $X_w = 0.004$, $X_{\delta e} = 0.62$, $X_{\delta\tau} = 0.00006$. The variables u, q, ϑ and γ, are in meters, rad/sec, rad and rad, respectively. The disturbance rejection with simultaneous decoupling conditions (4.1,2) are satisfied. According to (5.1, 2) the controller is

$$G = \begin{bmatrix} 1.43394 \times 10^7 (p_1)_0^{-1} & 34927.6 (p_2)_0^{-1} \\ -2.63158 (p_1)_0^{-1} & -0.0686201 (p_2)_0^{-1} \end{bmatrix} \quad (8.1)$$

$$F = -\begin{bmatrix} 1.43394 \times 10^7 & 34927.6 \\ -2.63158 & -0.0686201 \end{bmatrix} \times \begin{bmatrix} 1.53846 \times 10^{-6} & -(\lambda_1)_1 & 6 \times 10^{-4} & 0.494 \\ 0.0700009 & 45.1788 & -(\lambda_2)_1 & -(\lambda_2)_2 \end{bmatrix} \quad (8.2)$$

The resulting closed loop transfer function is

$$C(sI - A - BF)^{-1} \left[\begin{array}{c|c} BG & D \end{array}\right] = \left[\begin{array}{cc|c} \frac{(p_1)_0^{-1}}{s + 0.494 - (\lambda_1)_1} & 0 & 0 \\ 0 & \frac{(p_2)_0^{-1}}{s^2 - [-1.06 - (\lambda_2)_1]s - (\lambda_2)_2 + 45.1788} & 0 \end{array}\right] \quad (8.3)$$

and the cancelled out pole is $s = -0.129042$. Choosing $(\lambda_1)_1 = -19.506$, $(\lambda_2)_1 = -38.94$, $(\lambda_2)_2 = -354.821$, the poles of the closed loop system are assigned at -20. Choosing $(p_1)_0 = 0.05$ and $(p_2)_0 = 0.0025$, the responses of the state variables of the closed loop system for flight path pointing ($\gamma_c = 1^o (0.01748$ rad), $\vartheta_c = 0$) are illustrated in Figures 1-7. As is shown in these figures the performance of the state vector is quite satisfactory since the rising time of the flight path angle is also short while the pitch angle remains zero.

8 Conclusions

The pitch angle and the flight path angle of a multimode aircraft have been independently controlled and without influence from the forward gust, via static state feedback yielding disturbance rejection with simultaneous decoupling and stabilizability. The necessary and sufficient conditions are established. The set of all controllers and the respective general form of the closed-loop transfer function, have been derived. The conditions for the problem with simultaneous stabilizability have also been derived.

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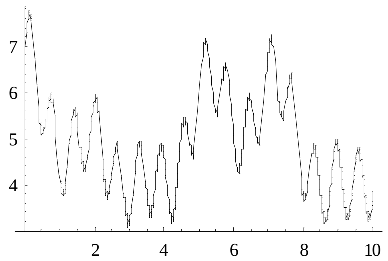

Figure 1: Unknown Gusts (m/sec^2)

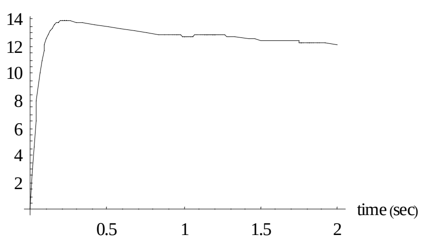

Figure 2: Forward Velocity increment (m/sec)

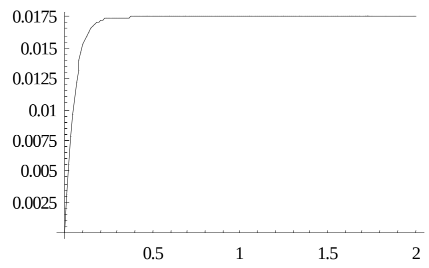

Figure 3: Flight path angle (rad)

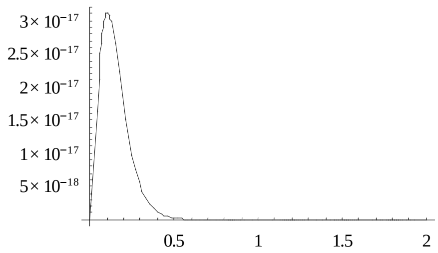

Figure 4: Pitch rate (rad/sec)

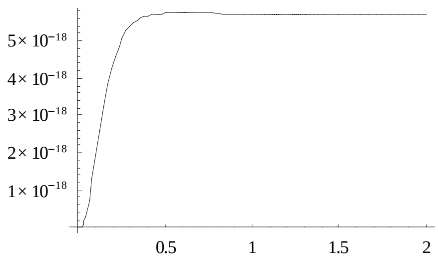

Figure 5: Pitch angle (rad)

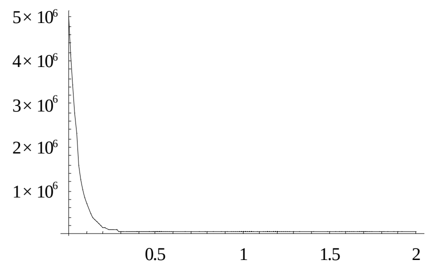

Figure 6: Thrust deflection (N)

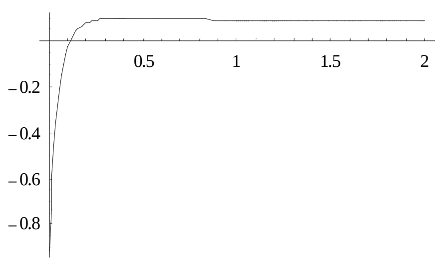

Figure 7: Elevator deflection (rad)