Robust Disturbance Decoupling with Simultaneous Exact Model Matching via Static Measurement Output Feedback

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Abstract:- The problem of robust exact model matching with simultaneous robust disturbance decoupling for linear systems with nonlinear uncertain structure (NLUS) and with measurable and non measurable disturbances is solved via an independent of the uncertainties static measurement output feedback law. The necessary and sufficient conditions for the problem to have a solution, are established. The general analytical expressions of the feedback matrices are derived. IMACS/IEEE CSCC'99 Proceedings, Pages:6111-6113

Key-Words: -Disturbance decoupling, model matching, robust control, linear systems, NLUS, feedback.

1 Introduction

Consider the linear system with (NLUS):

$$\dot{x}(t) = A(q)x(t) + B(q)u(t) + D_1(q)z_1(t) + D_2(q)z_2(t) \ ,$$
$$y_M(t) = M(q)x(t) \ , y(t) = C(q)x(t) \qquad (1.1)$$

where $x \in \Re^n$ denotes the state vector, $u \in \Re^m$ denotes the input vector, $y_M \in \Re^\mu$ denotes the measurement output vector, $y \in \Re^p$ denotes the performance output vector, $z_1 \in \Re^{\zeta_1}$ denotes the measurable disturbance vector and $z_2 \in \Re^{\zeta_2}$ denotes the nonmeasurable disturbance vector. The elements of the matrices $A(q)$, $B(q)$, $C(q)$, $D_1(q)$, $D_2(q)$ and $M(q)$ belong to $\wp(q)$ i.e. they are nonlinear function matrices depending upon the uncertainty vector $q = (q_1, \ldots, q_l) \in \mathcal{Q}$, where $\mathcal{Q}$ is the uncertainty domain and $\wp(q)$ is the set of nonlinear functions of q. The uncertain domain can be any set, while the values of the function of $\wp(q)$ are considered to be real. The uncertainties $q_1, \ldots, q_l$ do not depend upon the time. With regard to the nonlinear structure of $A(q)$, $B(q)$, $D_1(q)$, $D_2(q)$, $M(q)$ and $C(q)$ it is mentioned that, no limitations or specifications (continuity, boundness, smoothness,etc) are required.

The problem of robust disturbance rejection is a significant control design requirement having attracted considerable attention [1]-[4]. In [1] and [2] sufficient conditions for the problem to have a solution are derived. In [3] the necessary and sufficient conditions for the solution of the problem are derived for the case of left-invertible systems involving uncertain structure of the polynomial type. In [4] the robust disturbance rejection of left invertible systems with uncertain structure, of the general nonlinear type have extensively been solved. Robust elimination of the influence of disturbances to the system outputs is a desirable goal which however does not guarantee satisfactory control of the outputs. To this end the design requirement of having a desired closed loop input-output map is simultaneous requested. The specification of having an ideal closed loop transfer function is the well known exact model matching problem [5-7]. Robust exact model matching with simultaneous robust disturbance rejection via static state feedback has been solved in [8]. In the present paper the problem of robust disturbance rejection with simultaneous robust model matching (RDRRMM) for linear systems with NLUS and with measurable and nonmeasurable disturbances is extensively solved.

The contribution of the present paper consist in establishing the following aspects: The necessary and sufficient conditions for the problem to have a solution via an independent of q static measurement output feedback. The general analytical expressions of the independent of q feedback .

2 Main Result

The problem will be studied, via an independent of q static measurement output feedback law. In the frequency domain system (1.1) takes the form

$$sX(s) = A(q) + B(q)U(s) + D_1(q)Z_1(s) + D_2(q)Z_2(s)$$

$$Y_M(s)=M(q)X(s),\quad Y(s)=C(q)X(s) \tag{2.1}$$

To system (2.1) apply the regular static measurement output feedback law

$$U(s)=F_1Y_M(s)+F_2Z_1(s)+G\Omega(s) \tag{2.2}$$

where $\Omega(s)\in\mathbb{C}^m$ is the external input vector and G is assumed to be invertible in order to insure linear independence of the m external inputs. The RDRMM problem for systems with measurable and nonmeasurable disturbances is formulated as in the definition:

Definition 3.1. The RDRMM problem for linear systems with NLUS and with measurable and nonmeasurable disturbances of the form (2.1), via a feedback law of the form (2.2), consists in finding independent of q matrices F_1, F_2 and G such that

$$\begin{aligned} &C(q)\left[sI-A(q)-B(q)F_1M(q)\right]^{-1}\times\\ &\left[\,B(q)G\;\middle|\;D_1(q)+B(q)F_2\mid D_2(q)\,\right]=\\ &=\left[\,H_M(s)\;\middle|\;0_{p\times\zeta_1}\mid 0_{p\times\zeta_2}\,\right] \end{aligned}\tag{2.3}$$

where $H_M(s)$ is the transfer function of the ideal model. The ideal model is considered not to depend upon q. Clearly, in many practical applications, the presence of the uncertainties is undesirable. □

To solve the problem, the equation (2.3) is first reduced to the following linear equation

$$\begin{aligned} &C(q)\left[sI-A(q)\right]^{-1}\left[B(q)\mid D_1(q)\mid D_2(q)\right]=\\ &H_M(s)\Big[\Gamma-\Phi M(q)\left[sI-A(q)\right]^{-1}B(q)\Big|\\ &-\Phi M(q)\left[sI-A(q)\right]^{-1}D_1(q)-\Theta\Big|\\ &-\Phi M(q)\left[sI-A(q)\right]^{-1}D_2(q)\Big] \end{aligned}\tag{2.4}$$

where $$\Gamma=G^{-1},\ \Phi=G^{-1}F_1,\ \Theta=G^{-1}F_2 \tag{2.5}$$

According to (2.4 and 5), Γ must be invertible, i.e.

$$\det\Gamma\neq 0 \tag{2.6}$$

Expansion of both sides of (2.4) in negative power series of s yields

$$\begin{aligned} &C(q)\Delta(q)s^{-1}+C(q)A(q)\Delta(q)s^{-2}+\cdots=\\ &=\left\{H_{M,1}s^{-1}+H_{M,2}s^{-2}+\cdots\right\}\times\\ &\times\left\{J-\Phi M(q)\Delta(q)s^{-1}-\Phi M(q)A(q)\Delta(q)s^{-2}-\cdots\right\} \end{aligned}\tag{2.7}$$

where $\Delta(q)=\left[B(q)\mid D_1(q)\mid D_2(q)\right]$, $J=\left[\Gamma\;\middle|\;-\Theta\mid 0\,\right]$ and where $H_M(s)=\sum_{k=1}^{\infty}H_{M,k}s^{-k}$.

Equating coefficients of like powers of s in (2.7) derive an infinite set of algebraic equations. Since it suffices to keep only the $2n+1$ equations, the following nonhomogeneous system of equations is derived

$$\eta\Pi(q)=\xi(q) \tag{2.8}$$

where

$$\eta=\left[\gamma_1,\ldots,\gamma_m\mid\theta_1,\ldots,\theta_m\mid\phi_1,\ldots,\phi_m\,\right] \tag{2.9}$$

$$\begin{aligned} \xi(q)=&\Big[c_1(q)\left[A(q)\right]^0\Delta(q)\cdots c_p(q)\left[A(q)\right]^0\Delta(q)\Big|\cdots\\ &\cdots\Big|c_1(q)\left[A(q)\right]^{2n}\Delta(q)\cdots c_p(q)\left[A(q)\right]^{2n}\Delta(q)\Big] \end{aligned}\tag{2.10}$$

$$\Pi(q)=\left[\,\Pi_1\;\middle|\cdots\middle|\;\Pi_{2n+1}(q)\,\right] \tag{2.11}$$

where γ_i,ϕ_i,θ_i are the i-th rows of Γ,Φ,Θ; $\Pi_i(q)=$

$$\left[\begin{array}{c} J_m\otimes H_{M,i}^T\\ \hline -\sum_{k=0}^{i-2}M(q)[A(q)]^k\Delta(q)\otimes H_{M,i-k-1}^T\end{array}\right],\ i=2,..,2n+1$$

$\Pi_1=\left[\begin{array}{c} J_m\otimes H_{M,1}^T\\ \hline 0\end{array}\right]$ and where $c_i(q)\quad i=1,2,\ldots,p,$ is the i-th row of $C(q)$ and $J_m=\left[\begin{array}{cc|c} I_m & 0 & \\ 0 & I_{\zeta_1} & 0\end{array}\right]$.

The symbol $\otimes$ denotes the Kronecker product:

$$A\otimes B=\begin{bmatrix} b_{11}A & \ldots & b_{1q}A\\ \vdots & & \vdots\\ b_{r1}A & \ldots & b_{rq}A\end{bmatrix};\quad B=\{b_{ij}\}\in\Re^{r\times q}$$

Clearly, Γ, Θ and Φ are independent of q if and only if the rows γ_i, θ_i and ϕ_i, $(i=1,\ldots,m)$ are independent of q and consequently if and only if the vector η is independent of q. Till now, the problem is solvable, via independent of the uncertainties static measurement output feedback controllers, has been reduced to that of finding an appropriate vector η independent of q, satisfying (2.8) and (2.6).

Before establishing the necessary and sufficient conditions some definitions will be established. Let the operator $\mathrm{rank}_{\Re}[\cdot]$ denote the rank of an uncertain matrix on the field of real numbers, the operators $[\cdot]_{\mathbb{R}}^{\perp}$ denote an independent from q matrix which is orthogonal to the argument matrix, and the operator $\langle\,\cdot\setminus\cdot\,\rangle_{\mathbb{R}}$ denote the projection (in the field of real numbers) of an uncertain vector to the subspace defined by the rows of the uncertain matrix ([9]). Some numerical aspects regarding the computation check of $\mathrm{rank}[\cdot]$ and the construction of $\langle\cdot\setminus\cdot\rangle_{\Re}$ and $[\cdot]_{\Re}^{\perp}$ are given in [10], [11].

3.1 Necessary and sufficient conditions

Eq. (2.8) is a non homogeneous uncertain equation. According to [9] the following lemma is derived.

Lemma 3.1. Equation (2.8) is solvable, for an independent of q vector η, if and only if

$$\mathrm{rank}_{\Re}\begin{bmatrix}\Pi(q)\\ \xi(q)\end{bmatrix}=\mathrm{rank}_{\Re}\left[\Pi(q)\right] \tag{2.12}$$ ■

The general solution of equation (2.8), for an independent of the uncertainties vector η, is

$$\eta=\lambda\left[\Pi(q)\right]_{\Re}^{\perp}+\left\langle\,\xi(q)\middle\backslash\Pi(q)\right\rangle_{\Re} \tag{2.13}$$

where $\lambda=[\lambda_1,\ldots,\lambda_v]\in\Re^v$: arbitrary vector and

$$v=m(m+\mu+\zeta_1)-\mathrm{rank}_{\Re}\left[\Pi(q)\right] \tag{2.14}$$

To derive the general form of Γ, Θ and Φ, define

$$\begin{aligned} &\left[(q_0)_1,\ldots,(q_0)_m\middle|(r_0)_1,\ldots,(r_0)_m\middle|(\tilde r_0)_1,\ldots,(\tilde r_0)_m\,\right]\\ &=\left\langle\,\xi(q)\middle\backslash\Pi(q)\right\rangle_{\Re} \end{aligned}\tag{2.15}$$

$$\left[\hat K_1,\ldots,\hat K_m\middle|K_1,\ldots,K_m\middle|\tilde K_1,\ldots,\tilde K_m\right]=[\Pi(q)]_{\Re}^{\perp} \tag{2.16}$$

$$Q_j=\begin{bmatrix}(\hat k_1)_j\\ \vdots\\ (\hat k_m)_j\end{bmatrix},\ S_j=\begin{bmatrix}(k_1)_j\\ \vdots\\ (k_m)_j\end{bmatrix},\ \tilde S_j=\begin{bmatrix}(\tilde k_1)_j\\ \vdots\\ (\tilde k_m)_j\end{bmatrix}\quad j=1,\ldots,v$$

$$Q_0 = \begin{bmatrix} (q_0)_1 \\ \vdots \\ (q_0)_m \end{bmatrix}, S_0 = \begin{bmatrix} (r_0)_1 \\ \vdots \\ (r_0)_m \end{bmatrix}, \tilde{S}_0 = \begin{bmatrix} (\tilde{r}_0)_1 \\ \vdots \\ (\tilde{r}_0)_m \end{bmatrix} \quad (2.17)$$

where $\hat{K}_i$, K_i and $\tilde{K}_i$ are submatrices of dimensions $v\times m$, $v\times\zeta_1$ and $v\times\mu$, respectively, $(q_0)_i \in \Re^{1\times m}$, $(r_0)_i \in \Re^{1\times\zeta_1}$, $(r_0)_i \in \Re^{1\times\mu}$ and where $(k_i)_j$, $(\hat{k}_i)_j$ and $(\tilde{k}_i)_j$ are the *j*-th rows of K_i, $\hat{K}_i$and $\tilde{K}_i$ respectively. Using the above definitions as well as (2.13), the general analytical expressions of the independent of q matrices Γ, Θ and Φ are derived to be

$$\Gamma = Q_0 + \lambda_1 Q_1 + \lambda_2 Q_2 + \cdots + \lambda_v Q_v \quad (2.18)$$

$$\Theta = S_0 + \Sigma_{i=1}^{v} \lambda_i S_i \ , \ \Phi = \tilde{S}_0 + \Sigma_{i=1}^{v} \lambda_i \tilde{S}_i \quad (2.19)$$

Based upon the conditions (2.12) and (2.6) as well as the formula (2.18), the necessary and sufficient conditions for the problem to have a solution can be established. To this end, define

$$\tau_j = \begin{cases} m & j=0 \\ (\tau_{j-1}+1)\tau_{j-1} & j \geq 1 \end{cases}, \ \rho_j = \begin{cases} m & j=0 \\ (\tau_{j-1}+1)\rho_{j-1} & j \geq 1 \end{cases} \quad (2.20)$$

$$N_j(j) = \begin{cases} Q_i & ,j=0,\ \forall i>0 \\ N_j(j-1) \otimes \begin{bmatrix} 0\cdots 0 \\ I_{\tau_{j-1}+1} \end{bmatrix} & ,j\geq 1,\ \forall i>0 \end{cases} \quad (2.21)$$

$$P_j = \begin{cases} Q_0 & ,\ j=0,\ \forall i>0 \\ \tilde{P} & ,\ j\geq 1,\ \forall i>0 \end{cases} \quad (2.22)$$

where $\tilde{P} = P_{j-1} \otimes \begin{bmatrix} 0\cdots 0 \\ I_{\tau_{j-1}+1} \end{bmatrix} + N_{v-j+1}(j-1) \otimes \begin{bmatrix} I_{\tau_{j-1}+1} \\ 0\cdots 0 \end{bmatrix}$.

Theorem 3.1. The necessary and sufficient conditions for the solvability of the RDRRMM problem , via an independent of the uncertainties static measurement output feedback law, are

$$\text{rank}_{\Re} \begin{bmatrix} \Pi(q) \\ \xi(q) \end{bmatrix} = \text{rank}_{\Re}\left[\Pi(q)\right] \quad (2.23)$$

$$\text{rank}[P_v] = \rho_v \quad (2.24)$$

Proof: The condition (2.23) is identical to that in (2.12). If the condition (2.23) is satisfied the problem is reduced to that of finding a vector γ_i $(i=1,\ldots,m)$ such that (2.6) is satisfied with Γ as in (2.18) or equivalently if (2.24) is satisfied. ■

3.2 The feedback matrices

In this subsection the general forms of the matrices G, F_1 and F_2 , will be derived. The following theorem results directly from (2.5), (2.18) and (2.19).

Theorem 3.2. Assume that system (1.1) satisfies the conditions of Theorem 3.1., then the general analytical expressions of the independent of q feedback matrices G and F , are

$$G = (Q_0 + \lambda_1 Q_1 + \lambda_2 Q_2 + \cdots + \lambda_v Q_v)^{-1} \quad (2.25)$$

$$F_1 = (Q_0 + \Sigma_{i=1}^{v} \lambda_i Q_i)^{-1}(\tilde{S}_0 + \Sigma_{i=1}^{v} \lambda_i \tilde{S}_i) \quad (2.26a)$$

$$F_2 = (Q_0 + \Sigma_{i=1}^{v} \lambda_i Q_i)^{-1}(S_0 + \Sigma_{i=1}^{v} \lambda_i S_i) \quad (2.26b)$$

where

$$\det(Q_0 + \lambda_1 Q_1 + \lambda_2 Q_2 + \cdots + \lambda_v Q_v) \neq 0 \quad (2.27) \ ■$$

The condition (2.27) upon the degrees of freedom of λ_i in the controller matrices G, F_1 and F_2 may be viewed as a forbidden hypersurface in the space $\Re^n$ of the arbitrary parameters λ_i. Moving closer to this surface results in the norms of G, F_1 and F_2 approaching infinity. No matter how close to this surface we are, the problem has a solution, since we are not exactly on this surface. However, from a practical point of view, we should choose λ_i such that the norms of G, F_1 and F_2 have suitable values.

4 Conclusions

RDRRMM problem for systems with measurable and nonmeasurable disturbances for systems with nonlinear uncertain structure, via static state feedback, has extensively been solved. The necessary and sufficient conditions have been established. The general analytical expressions of the independent of uncertainties feedback matrices have been derived.

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