Applications of Gabor Filters in Image Processing Systems

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Abstract: - Gabor's theory implies that the information in image can represent in terms of the amplitudes of functions that are localized in both space and frequency.
We study applications of Gabor functions that are considered in computer vision systems, such as contour detection, stereo matching process and object recognition. It is known that structural information even in grey scale images resides in contours. The problem of extracting contours from images is approached with the use of Gabor filters. These filters have optimal properties relating to simultaneous localisation in space, spatial-frequency and orientation.
Matching algorithms using image properties maximize a similarity measure between two images. This similarity may apply to features images derived from images. The main problem of stereoscopic vision is to find corresponding parts of the left image and the right image. Gabor filters have been used to resolve the correspondence problem.
The problem of recognition of free-form objects in real-world scenes is still a difficult. The magnitude of Gabor features can be used as objects features. The output of this filter is a vector that describes the structure around the image location. This vector are classified into N types form a set of regions. Each region is characterised by its index which describes image in terms of the Gabor filter response.

Key-Words: - Gabor functions, Gabor filters, stereo matching process, feature extraction.

1 Introduction

In 1928 Hartley [5] concluded that "*the total amount of information which may be transmitted ... is proportional to the product of frequency range which is transmitted and the time which is available for the transmission*".
In 1946 Gabor presented an approach to characterize a time function in time and frequency. Gabor showed how to represent time-varying signals in terms of functions that are localized in both time and frequency. Information contained in a visual scene is pre-processed in the retina and represented in the brain in highly compressed form for further processing. Gabor's theory implies that the information in image can represent in terms of the amplitudes of functions that are localized in both space and frequency. In a joint space/spatial-frequency representation for images, frequency is viewed as a local frequency that can vary with position throughout the image.
Gabor functions are Gaussians modulated by complex sinusoids. In its general form, the two-dimensional Gabor function and its Fourier transform can be written as [1], [2]

$$g(x,y;u_0,v_0) = \exp(-[\frac{x^2}{2\sigma_x^2}+\frac{y^2}{2\sigma_y^2}]+2\pi i[u_0x+v_0y]) \quad (1)$$

$$G(u,v) = \exp(-2\pi^2(\sigma_x^2(u-u_0)^2+\sigma_y^2(v-v_0)^2)) \quad (2)$$

where σ_x and σ_y define the widths of the gaussian in the spatial domain and (u_0, v_0) is the frequency of the complex sinusoid.
We study applications of Gabor functions that are considered in computer vision systems, such as contour detection, stereo matching process and object recognition.
It is known that structural information even in grey scale images resides in contours. The problem of extracting contours from images is approached with the use of Gabor filters. These filters have optimal properties relating to simultaneous localisation in space, spatial-frequency and orientation.
Matching algorithms using image properties maximize a similarity measure between two images. This similarity may apply to features images derived from images. The main problem of stereoscopic vision is to find corresponding parts of the left image and the right image. Gabor filters have been used to resolve the correspondence problem.
The problem of recognition of free-form objects in real-world scenes is still a difficult.
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2 Stereo matching process

We do not assume that the cameras are parallel. If (u_l, v_l) and (u_r, v_r) are the coordinates in the left and the right images respectively, of the perspective projection of an object point $P(X, Y, Z)$ then [7]:

$$u_l = f \cdot \frac{x_l}{z_l + f} \qquad v_l = f \cdot \frac{y_l}{z_l + f} \tag{3}$$

$$u_r = f \cdot \frac{x_r}{z_r + f} \qquad v_r = f \cdot \frac{y_r}{z_r + f} \tag{4}$$

The image coordinate systems are translated by vector $[a, b, c]^T$ and rotated (by the rotation matrix R determined by the three Euler angles ϕ, θ, ψ) such that

$$\begin{bmatrix} x_r \\ y_r \\ z_r \end{bmatrix} = [R] \cdot \begin{bmatrix} x_l - a \\ y_l - b \\ z_c - c \end{bmatrix} \tag{5}$$

Assuming that $a = 0$ $\psi = 0$ i.e. the cameras optic axes are coplanar, and $\phi = 0$ (cameras not rotate around the z - axis)

$$\begin{bmatrix} x_r \\ y_r \\ z_r \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix} \cdot \begin{bmatrix} x_l - a \\ y_l - b \\ z_c - c \end{bmatrix} \tag{6}$$

We want to express z_l in terms of v_l and v_r (horizontal disparities). For $Z \equiv z_l$

$$Z = \frac{f}{\xi} \cdot [(b - v_l) \cdot (v_r \sin\theta + f \cos\theta) + \\ + f v_r + c(f \cos\theta - v_r \cos\theta)] \tag{7}$$

where: f is the focal length of the lens,
θ is the convergence angle,
$\xi = \sin\theta(v_l v_r + f^2) + f \cos\theta(v_l - v_r)$.

For $c = \theta = 0$

$$Z = f \cdot \frac{b - v_l}{v_l - v_r} \tag{8}$$

For computing stereo disparities we assume matching occurs only there where edge point features exist.
The matching begins by location feature points in each image f at multiple resolutions. Let $W(x, y)$ denote the output of the feature detection filter at (x, y). $W(x, y)$ is defined by the convolution of $f(x, y)$ with Gabor elementary function $h(x, y)$ i.e. $W(x, y) = h(x, y) f(x, y)$. The left feature point at $L = (u_l, v_l)$ matches a right feature point at $R = (u_r, v_r)$. The disparity of a match (L, R) is the horizontal pixel distance $|v_l - v_r|$.
Feature points from image are extracted by uses Gabor wavelet decomposition and local scale interaction. Feature points are points of curvature maxima in the image. To determine a high spatial curvature point the response from a larger sized cell

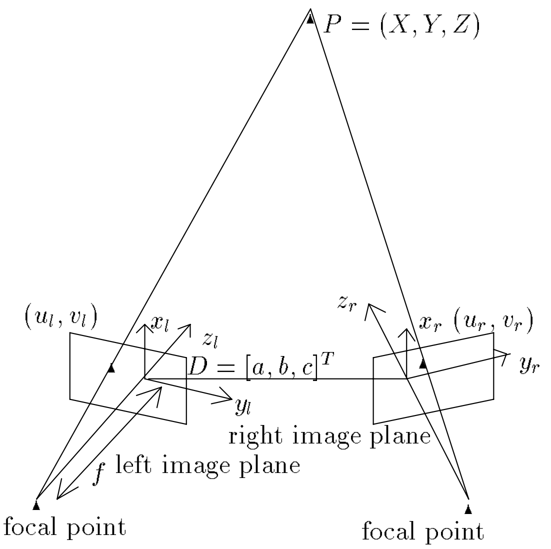

Figure 1: Stereo geometry

is subtracted from the smaller cell. Feature point detection utilizes a simple mechanism uses the responses of filter from different frequency channels [3], [10]

$$I_{i,j}(x,y,\theta) = s(||W_i(x,y,\theta) - \gamma W_j(x,y,\theta)||) \tag{9}$$

where $\gamma = \alpha^{-2(i-j)}$ is the normalizing factor, and s is sigmoid nonlinearity function

$$s(x) = \frac{1}{1+\exp(-\beta x)}$$

and $W_i(x,y,\theta)$ is done by [4]:

$$\begin{aligned} W(x,y,\theta,\alpha) &= f(x,y) \otimes \Phi(x,y,\theta) \\ &= f(x,y) \otimes \Phi(\alpha^j x, \alpha^j y, \theta) \end{aligned} \tag{10}$$

where $\otimes$ denotes convolution, $f(x,y)$ is an image, $\Phi(x,y,\theta)$ is a wavelet representation of the Gabor function, and W is the filter output. The feature vector at each spatial location (x,y) is specified as

$$\mathbf{W}(x,y,\theta) = \{W(x,y,\theta,\alpha)\}_{\theta,\alpha} \tag{11}$$

The parameter values used in our experiment are $\alpha = 2, \beta = 4, i = -2, j = -5$.We used two vector feature points $\mathbf{I}^q_{i,j}$ where $q = l, r$. The epipolar constraint limits the search space from 2D image plane to a single 1D horizontal line. The disparity vector is characterized by a cost function

$$U_d = \sum_x |I^l_{i,j}(x,y,\theta) - I^r_{i,j}(x+d_x,y,\theta)|^2 \tag{12}$$

3 Gabor filters

Gabor filters are filters with Gabor functions as impulse response. They have been employed in a number of applications, notably in the area of the edge detection, texture segmentation and stereo matching process.

3.1 Gabor filter for edge detection

Gabor functions are Gaussians modulated by complex sinusoids. In its general form, the two-dimensional Gabor function and its Fourier transform can be written as shown in eqn. 1 and eqn. 2.
When the complex impulse response in eqn. 1 is expanded:

$$g(x,y;u_0,v_0) = g_r(x,y;u_0,v_0) + i g_i(x,y;u_0,v_0) \tag{13}$$

where

$$\begin{aligned} g_r(x,y;u_0,v_0) &= \exp[-(\frac{x^2}{2\sigma_x^2} + \frac{y^2}{2\sigma_y^2})] \times \\ &\times \cos[2\pi(xu_0 + yv_0)] \end{aligned} \tag{14}$$

and

$$\begin{aligned} g_i(x,y;u_0,v_0) &= \exp[-(\frac{x^2}{2\sigma_x^2} + \frac{y^2}{2\sigma_y^2})] \times \\ &\times \sin[2\pi(xu_0 + yv_0)] \end{aligned} \tag{15}$$

are the real part and the imaginary part referred to as cosine and sine, respectively. Cosine part is even symmetric with respect to an oriented axis, define by , and is strongly sensitive to features with symmetry property along the same oriented axis, useful for detecting oriented blocs or roof edges. Sine part is odd symmetric version along the same oriented axis, useful for detecting oriented step edges.
One of the attractive aspects of using Gabor filters are orientation selectivity. Structural information even in gray-scale images resides in lines and edges. Projections of the original image into a specified discrete basis set of 2D Gabor functions gives information for local line/edge position and direction.

The parameters of the Gabor filters were studied:
- the orientation of the grid, denoted by θ, with

$\theta = \{0^0, 30^0, 45^0, 60^0, 90^0\}$,
- the scale of the Gaussian denoted by σ, with $\sqrt{2}\sigma = \{3\sqrt{2}, 4\sqrt{2}, 5\sqrt{2}, 6\sqrt{2}, 10\sqrt{2}\}$ pixels, and $\lambda = \frac{\sqrt{2}\sigma}{2}$,
- the ratio between the scale and the grid wave length, denoted by $\frac{\sigma}{\lambda}$ with $\lambda = \{2, 3, 4\}$ for a constant scale ($\sqrt{2}\sigma = 4\sqrt{2}$) and a constant orientation ($\theta = 45^0$).
The edge representation was computed by convolving original image with the Gabor functions.

3.2 Wavelet representation of Gabor functions

The Gabor functions form a complete though nonorthogonal basis set. Like the Fourier series, a function $g(x, y)$ can easily be expanded using the Gabor function. Consider the following wavelet representation of the Gabor function:

$$\Phi_\lambda(x, y, \theta) = \exp[-(\lambda^2 x'^2 + y'^2) + i\pi x'] \tag{16}$$

where

$$x\prime = x\cos\theta + y\sin\theta$$

$$y\prime = -x\sin\theta + y\cos\theta$$

where λ is the spatial aspect ratio and $\theta$is the preferred orientation.
In the experiments, λ is the set to 1, and θ is discretized into four orientations. The orientation parameter θ determines the direction of the edges.
Family of wavelets can be generated by translations $\Phi(x - x_0, y - y_0, \theta)$ and dilations $\Phi(\alpha^j x, \alpha^j y, \theta)$, where (x_0, y_0) and α^j are the translation and scale parameters, respectively. The resulting family of wavelets is given by

$$\{\Phi[\alpha^j(x - x_0), \alpha^j(y - y_0), \theta_k]\} \tag{17}$$

for $\alpha \in R$, $j = \{0, -1, -2, \cdots\}$,
and $\theta_k = \frac{k\phi}{N}$, $N = 4$, $k = \{0, 1, 2, 3\}$.

We assume the following filter structure for analyzing images [4]

$$W(x, y, \theta, \alpha) = F(x, y) \otimes \Phi(x, y, \theta) =$$
$$= F(x, y) \otimes \Phi(\alpha^j x, \alpha^j y, \theta) \tag{18}$$

where $\otimes$ denotes convolution, $F(x, y)$ is an image, $\Phi(x, y, \theta)$ is a wavelet representation of the Gabor function, and W is the filter output.
We call the filtering operator shown in eqn.(48) a Gabor filter.
The feature vector at each spatial location (x, y) is specified as

$$\mathbf{W}(x, y, \theta) = \{W(x, y, \theta, \alpha)\}_{\theta,\alpha} \tag{19}$$

4 Object detection

In object recognition for each pixel location (X, Y) we compute the magnitude Gabor features

$$G_{mag}(X, Y) =$$
$$\sum_{x=-w/2}^{x=w/2-1} \sum_{y=-w/2}^{y=w/2-1} f(X + x, Y + y) g(x, y; u_0, v_0) \tag{20}$$

The input image with 256 x 256 pixels and 256 greylevel was divided into a set 8 x 8 non-overlapping blocks and used Gabor filter we obtained 32 x 32 x 5 (number of orientations) features for each image. Next these features are the input vectors for a k-nearest neighbour classifier. The Gabor vector that describes the structure around the image location are quantised into N regions. Each region is characterised by code-book index which describes the corresponding image .

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Figure 2: Input image

Figure 3: Edge detection using Gabor filtering.

Figure 4: Edge detection with another set of parameters.