PI CONTROLLER AUTOTUNING USING FAULT DETECTION AND ISOLATION

Ferenc Szigeti
Departamento de Sistemas de Control
Escuela de Ingeniería de Sistemas
Facultad de Ingeniería, ULA, Mérida, Venezuela

Edmary Altamiranda
Sección Automatización
Superintendencia de Tecnología, Gerencia Técnica
PDVSA PEQUIVEN, El Tablazo, Zulia, Venezuela
CSCC'99 Proc.pp.7131-7138

Abstract: **In this paper, a scheme of on-line automatic tuning is proposed for PI controllers. This method is based on FDI filters, designed by Luenberger observers for the linearized plant model, in closed loop. The isolation of the PI controller gains can be achieved by reescaling of the state variables. The gain parameters of the new diagnostic model are involved in the failure functions. Then, the tuning law is obtained by imposition of a simple asymptotically stable differential equation, guaranteeing that the failure function tends to zero. Our method was applied successfully for chemical process control, stabilizing the pH in a plant unit.**

I. INTRODUCTION

Fault detection and isolation has been used in the industry since the 70´s. The first methods were designed for change point detection contributing to improvement of the safety of plants operations. However, these detection systems, principally, became part of alarm systems.

Improving the fault detection and isolation it has thought in detection of incipient changes. This kind of techniques need more precise knowledge about the process and plants operation, hence fault detection and isolation for incipient changes are based on models, which can be obtained using identification methods, statistical methods or through laws which depend on the nature of phenomenon . The method of fault detection and isolation used in this paper is developed in [1], improving the FDI filter design, based on C-A invariant subspace algorithms, see [3] and [7]. The linearized plant model is extended, adding the PI dynamic to the state equations. Disturbances and errors of the linearizations requires a dynamic tuning of the PI controllers. To achieve this goal, the error of the required PI controller gains with respect to the actual gain values will be interpreted as failure. Unfortunately, the extended diagnostic model doesn´t allow us to isolate the gain errors. Hence, the state variables will be rescaled with the purpose that the new diagnostic model allows the isolation of the gain errors. The error functions of the new diagnostic model depend on the actual PI controller gains error, which can be forced to zero, imposing that the gain errors to be the solution of a simple asymtotically stable differential equation. The dynamic autotunig is computed from this imposed differential equation.

II. DIAGNOSTIC MODELS

The model of any process in study can be obtained using identification methods, statistical methods or through laws of the nature, which govern the phenomenon, that characterizes the process.

Consider the behavior of a RL circuit represented by the following equation:

$$\dot{x} = -\frac{R}{L}x + \frac{1}{L}u \qquad (1)$$

where ***x*** *is* the current, L the inductance, R the resistance and ***u is*** the input voltage. Suppose that the nominal condition of the resistance is lost in time and its nominal value is defined by

$$R_0 = R(t) + \Delta R(t) \qquad (2)$$

ΔR(t) *is* related to the deviation respect to the nominal value. Using the equations (1) and (2) a diagnostic model with output **y*(t)*** and fault ***v(t),*** can be obtained

$$\dot{x} = -\frac{R_0}{L}x + \frac{1}{L}u + v \qquad (3)$$

$y = x$, where $v = \frac{1}{L}\Delta R(t)$.

III. THE GENERALIZED DIAGNOSTIC MODEL

The example showed in section II, leads to non-classic diagnostic models with the following structure.

$$\dot{x}(t) = Ax(t) + Bu(t) + L_D v(t) \qquad (4)$$
$$y(t) = Cx(t) + L_O v(t)$$

where

$$L_D v(t) = \sum_{j=1}^{J} M_j\left(\frac{d}{dt}\right) L_j v_j(t)$$
$$L_O v(t) = \sum_{j=1}^{J} \tilde{M}_j\left(\frac{d}{dt}\right) \tilde{L}_j v_j(t) \qquad (5)$$

where the dimension of the matrixes are $A \in R^{n\times n}, B \in R^{n\times p}$,
$C \in R^{q\times n}, \tilde{L}_j \in R^{q\times p_j}, j = 1,\ldots,J$.
The dimension of the matrixes
$M_j\left(\frac{d}{dt}\right) \in R^{n\times n}\left[\frac{d}{dt}\right], \tilde{M}_j\left(\frac{d}{dt}\right) \in R^{q\times q}\left[\frac{d}{dt}\right]$

with linear differential operators in its entries

IV FAULT DETECTION AND ISOLATION FILTER DESIGN

In this paper, a method to fault detection and isolation in linear system is used. The mentioned method is based on observers and developed in [1]. Let $O \subset R^n$ be the observable subspace: $O = \sum_{i=0}^{n-1} \mathrm{Im}\left(A^{*^i} C^*\right)$.Let us denote by P yhe orthogonal projection of R^n onto *O*. It is shown that if a system is detectable then the necessary conditions

1. The weak separation

$$\mathrm{Im}(PL_i) \cap \left(\sum_{j\neq i} \mathrm{Im}(PL_j)\right) = \{0\} \qquad (6)$$

2. The fault observability, see [4]

$$Ker(PL_j) = \{0\} \qquad (7)$$

are also sufficient to design a FDI filter if the inequality q $\geq \sum p_i$ holds. Detectability means that the system on the unobservable subspace is asymptotically stable. It is known that Luenberger Observers let intact the unobservable subspace, see [7].

A Design Methods

The first step is to define the observer according to the generalized diagnostic model. The input ***u*** and the output ***y*** of the diagnostic system are inputs of the observer and the observer state $\hat{x}(t)$ is an estimated of the state $x(t)$:

$$\hat{x}(t) = A\hat{x}(t) + Bu(t) + D(y(t) - C\hat{x}(t)) \qquad (8)$$
$$\hat{y}(t) = C\hat{x}(t)$$

The gain matrix $D \in R^{n\times q}$ is designed in order to the estimation error be asymptotically stable when there are no faults.

$$\dot{e}(t) = (A - DC)e(t), \eta(t) = Ce(t) \qquad (9)$$

There exists a gain matrix $D \in R^{n \times q}$, such that the error equation is asymptotically stable if and only if the unobservable dynamic is asymptotically stable, see [7].

In presence of faults, the error equation is expressed in the following way

$$\dot{e}(t) = (A - DC)e(t) + (L_D - DL_O)v(t) \quad (10)$$
$$\eta(t) = Ce(t) + L_O v(t)$$

The next step is to find the transfer function for the error system considering the faults as inputs

$$\eta(s) = (C(sI - A + DC)^{-1}(L_D - DL_O) + L_O)v(s) \quad (11)$$

In [1] it is proved that the gain matrix D can be chosen such that:

1. The error system is asymptotically stable
2. The transfer function in (11) is of full rank , that means, its rank is $\sum p_i$. Hence, the transfer function is left invertible with left inverse G(s)

$$v(s) = G(s)\eta(s) \quad (12)$$

The transfer matrix G(s) could contain terms which are not realizable or unstable, therefore the next step is the cancelation of the non desired poles in each term. Let α_i, unstable poles from (12) with multiplicity n_i, then (12) is multiplied by $\dfrac{\prod_{\alpha_i}(S - \alpha_i)^{n_i}}{(s+1)^{\sum n_i}}$ I. If additionally (12) has poles in infinite (is improper) (12) is multiplied by $\dfrac{1}{(s+1)^{n_0}}$, where n_0 is the multiplicity of this poles in infinite. These terms constitute a postfilter matrix

$$H(s) = \frac{\prod_{\alpha}(s - \alpha_i)^{n_i}}{(s+1)^{n_0 + \sum n_i}} I \quad (13)$$

The H(s) defines the transfer matrix from the inputs *v*(s) into new outputs *w*(s):

$$w(s) = H(s)v(s) \quad (14)$$

The realization of (14) in the state space form is

$$\begin{aligned} \dot{z}_1 &= v(t) - z_1 \\ \dot{z}_2 &= z_1 - z_2 \\ &. \\ \dot{z}_k &= z_{k-1} - z_k \\ w(t) &= Mz(t) \end{aligned} \quad (15)$$

where $k = n_0 + \sum n_i$. This system is, essentially, multiple integration. When the error system is stabilized for t $< t_0$ then $v_j(t) \neq 0$ in $t \in [t_0, t_0 + \tau]$, if and only if $w_j(t)$ is not zero in the same interval. This system is called a detector.

V PI AUTOTUNING USING FAULT DETECTION AND ISOLATION

A. Diagnostic Model.

Let us consider the following linear time-invariant system

$$\dot{x} = Ax + Bu \quad (16)$$
$$y = Cx$$

$A \in R^{n \times n}, x \in R^n, B \in R^{n \times 1}, C \in R^{1 \times n}$,

with the control law $u = -k_P y - k_I \int y(t)dt$, assuming set point = 0. If a new state $z = \int y(t)dt$ is defined, the closed loop system can be rewritten as

$$\begin{pmatrix} \dot{x} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} A - Bk_p C & -Bk_I \\ C & 0 \end{pmatrix} \begin{pmatrix} x \\ z \end{pmatrix}. \quad (17)$$

If the nominal values of the controller parameters are defined by

$$k_{P0} = k_p + \Delta k_p \quad (18)$$
$$k_{I0} = k_I + \Delta k_I$$

the diagnostic model of the closed loop system is written into

$$\begin{pmatrix} \dot{x} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} A - Bk_{p0} C & -Bk_{I0} \\ C & 0 \end{pmatrix} \begin{pmatrix} x \\ z \end{pmatrix} + \begin{pmatrix} B \\ 0 \end{pmatrix} v_1 + \begin{pmatrix} B \\ 0 \end{pmatrix} v_2 \quad (19)$$

where $v_1 = (\Delta k_p C)x$ and $v_2 = (\Delta_{kI})z$

The fault direction are not linear independent, hence, a new diagnostic model is required to isolate these faults.

If the state vector is redefined as $x_1 = k_I z, x_2 = k_p Cx = k_p \dot{z}$ and $x_3 = x$, using equation (25) a new diagnostic model is obtained

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \tilde{A} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + L_D \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \quad (20)$$

where

$$\tilde{A} = \begin{pmatrix} 0 & \frac{k_{I0}}{k_{P0}} & 0 \\ -k_{P0}CB & -k_{P0}CB & k_{P0}CA \\ -B & -B & A \end{pmatrix}$$

$$L_D = \begin{pmatrix} -\frac{d}{dt} & \frac{k_{I0}}{k_{P0}} \\ 0 & -\frac{d}{dt} \\ 0 & 0 \end{pmatrix}$$

$\tilde{A} \in R^{(n+2)\times(n+2)}, x_1 \in R, x_2 \in R, x_3 \in R^n$

defining the outputs as $y_1 = z, y_2 = \dot{z}$

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \tilde{C} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + L_O \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \quad (21)$$

where

$$\tilde{C} = \begin{pmatrix} \frac{1}{k_{I0}} & 0 & 0 \\ 0 & \frac{1}{k_{P0}} & 0 \end{pmatrix} \quad L_O = \begin{pmatrix} \frac{1}{k_{I0}} & 0 \\ 0 & \frac{1}{k_{P0}} \end{pmatrix}$$

for this diagnostic model, the faults are defined $v_1 = \Delta k_I z, v_2 = \Delta k_P \dot{z}$

B. Adaptive Tuning of PI Controller Parameters

After getting the diagnostic model, the following step is to design the Luenberger observer

$$\hat{x}(t) = \tilde{A}\hat{x}(t) + Bu(t) + D(y(t) - \tilde{C}\hat{x}(t)) \quad (21)$$
$$\hat{y}(t) = \tilde{C}\hat{x}(t)$$

with gain matrix D such that $\tilde{A} - D\tilde{C}$ be asymptotically stable and the matrix $G(s) = (C(sI - \tilde{A} + D\tilde{C})^{-1}(L_D - DL_O) + L_O)$ have full rank. It is easy to verify that $G(s) \in R^{2\times 2}$, because there are two faults related to the controller parameters. The following step is to invert G(s) to find $v = G^{-1}(s)\eta$. If the system is not realizable it will be necessary a postfilter matrix, getting the detector system *w*. In order to find in an adaptive way the controller parameters, it is necessary to have the expression for *v*, from the detector equation or from the error term. In both cases the result could be non causal, but in a practical sense, it would mean that it is required error derivatives or output detector derivatives which are on line computable. Once *v* is obtained, it is possible to get Δk_p and Δk_I from $v_1 = \Delta k_I z, v_2 = \Delta k_P \dot{z}$ where z y $\dot{z}$ are outputs of the diagnostic model which are measurable or computable. The last step is to find the controller parameters such that the faults tend to zero asymptotically

$$\begin{pmatrix} \Delta\dot{k}_P \\ \Delta\dot{k}_I \end{pmatrix} = \begin{pmatrix} -\lambda_1 & 0 \\ 0 & -\lambda_2 \end{pmatrix} \begin{pmatrix} \Delta k_P \\ \Delta k_I \end{pmatrix} \quad (22)$$

using (18) we get

$$\Delta k_p(t) = k_{P0} - k_p(t) \quad (23)$$
$$\Delta k_I(t) = k_{I0} - k_I(t)$$

After derivating (23) and substituting in (22) the dynamic expression is obtained for the controller parameters

VI ACADEMIC EXAMPLE

Consider a particular case (Proportional Controller) in order to control a first order system.

$$y(s) = \frac{K}{\tau s - 1} u(s) \quad (24)$$
$$k_p = -k_p y$$

assuming that the set point is zero. The respective differential equation is the following

$$\dot{y} = (1/\tau)y + (K/\tau)u \qquad (25)$$

If the proportional parameter is defined as

$$k_p(t) = k_{P0} + \Delta k_p(t), \qquad (26)$$

then, the diagnostic model can be expressed in the follwing way

$$\dot{y} = ((1 - Kk_P)/\tau)y + (K/\tau)v_1 \qquad (27)$$
$$v_1 = \Delta k_P y$$

The next step is to compute an observer such that *A-DC<0*, with $D = (1/\tau), A - DC = (-Kk_P/\tau)$ assuming that $K > 0$ and $k_P > 0$. Therefore the observation error is defined by

$$\dot{e} = \frac{-Kk_P}{\tau}e + \frac{K}{\tau}v_1 \qquad (28)$$
$$\eta = Ce.$$

Its transfer function is

$$\eta(s) = \frac{(cK/\tau)}{s + (Kk_P/\tau)}v_1(s). \qquad (29)$$

Inverting the expression in (29), we get

$$v_1(s) = \frac{s + (Kk_P/\tau)}{(cK/\tau)}\eta(s) \qquad (30)$$

The transfer function from η into v_1 is improper, hence, a postfilter is needed:

$$h(s) = \frac{1}{(s+1)}. \qquad (31)$$

The fault detector system is the following

$$w(s) = h(s)v_1(s), \qquad (32)$$
$$w(s) = \frac{1}{(s+1)}\frac{s + (Kk_P/\tau)}{(cK/\tau)}\eta(s).$$

Its respective differential equation

$$\dot{w} = -w + v_1. \qquad (33)$$

In discrete time, is possible to compute v in spite of that the transfer function from η into v_1 is improper because we only need error or output detector derivatives. For

$$\Delta k_P(t) = v_1(t)/y(t) \qquad (34)$$

it is desired that

$$\Delta\dot{k}_P = -\lambda\Delta k_P \qquad (35)$$

derivting (26) and using (35) is obtained

$$\dot{k}_P(t) = \Delta\dot{k}_P(t) = -\lambda\Delta k_P, \lambda > 0 \qquad (36)$$

This is a dynamical expression for the controller gain. To the computationally implementation of this autotuning, it is necessary to discretize the differential equations

$$\hat{x}(k) = \frac{\hat{x}(k-T) + (T/\tau)y(k)}{(1 - (T(-Kk_{P0}+1)/\tau) + (TC/\tau))}$$
$$e(k) = y(k) - \hat{x}(k),\ \eta = Ce$$
$$w(k) = \frac{w(k-T) + (T/KC)(\eta(k) - \eta(k-T)) + (Tk_{P0}/C)\eta(k)}{(1+T)}$$
$$v_1(k) = \frac{\eta(k) - \eta(k-T)}{T}(\tau/KC) + \eta(k)k_{P0}$$
$$\Delta k_P(k) = \frac{v_1(k)}{y(k)} \qquad (37)$$
$$k_P(k) = k_P(k-T) + T*\lambda*\Delta k_P(k)$$

where *T* must be chosen such that discretization be efficient .

Let the nominal model be

$$G(s) = \frac{1}{0.1s - 1}u(s), \qquad (38)$$
$$k_p = -1.5y, \quad y(0) = 0.01,$$

setting *T=0.01.*

First Case of Simulation

Nominal Conditions:
K = 1,
$\tau = 0.1$ for t >700 and t <450.

Fault Conditions:
K = 1,
$\tau = 0.5$ for $450 \le t \le 700$.

In this case there is no unstability in the fault condition, however the detector system detects the fault. When adaptive gain is applied, the fault is corrected guaranteeing the closed loop specifications. See Figures 1 and 2.

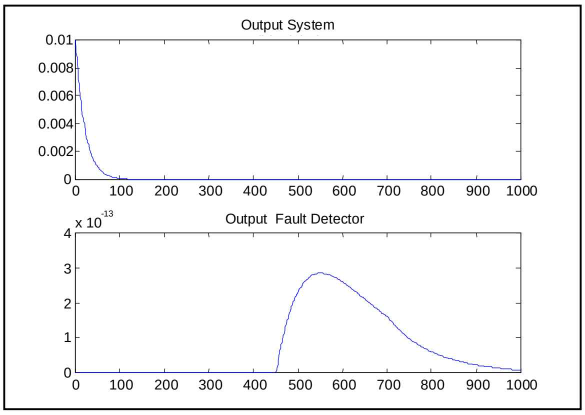

Fig.1 System response without adptive gain. First Case

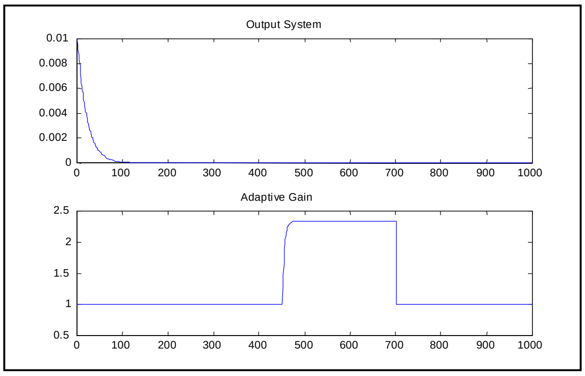

Fig2. System response with adaptive gain. First Case

Second Case of Simulation

Nominal Conditions:
K = 1,
$\tau = 0.1$ for t >700 and t <450.
Fault Conditions:
K = 0.1,
$\tau = 0.1$ for $450 \le t \le 700$.

In this case the fault conditions cause loss of stability which is detected by the fault detector. When adaptive gain is applied are guaranteed the stability and closed loop specifications. See Figures 3 and 4.

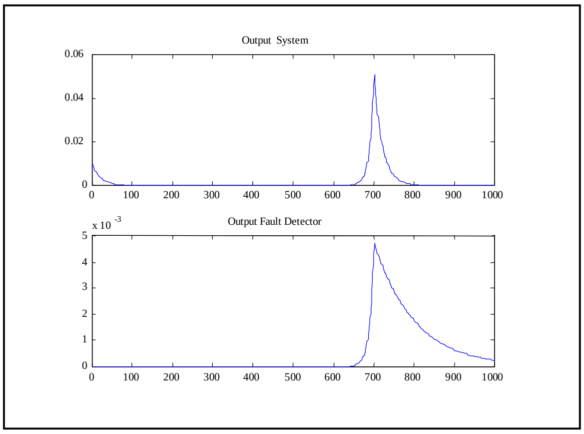

Fig.3 System response without adptive gain. Second Case

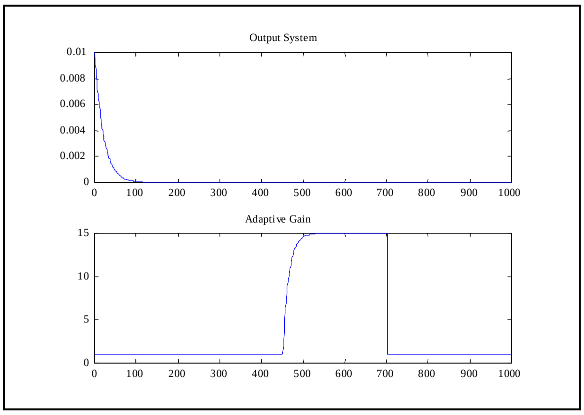

Fig.4. System response with adaptive gain. Second Case

VI. INDUSTRIAL APPLICATION

The autotuning of PI controllers proposed in this paper, was implemented to solve a pH control problem in a chemical plant (Chlorine Plant) with satisfactory results. The problem is to control the pH in the Depleted Brine ($NaCL + H_2O$) with 200 gpl of $NaCl$ approximately. This Brine comes to the process unit with pH close to 2 and the objective is to obtain a pH close to 8.5 by adding of Caustic Soda (NaOH) with an approximated concentration of 12%.

The difficulties that there exist to control the pH are related with multiples disturbances originated in different units of the plant which affect the system in a direct and indirect way. Changes in the soda concentration, affect the system producing deviations in its dynamic respect to the nominal behavior. Because of this fact it is hard to control the system with the required specifications.

The original control strategy was designed to compensate the nonlinearities related to the process gain using a PI controller with adaptive gain. The tuning of the proportional gain was achieved according to the information obtained from titration data assuming that the soda concentration doesn't change. This fact causes that the strategy isn´t robust with respect to such changes.

Our method was applied to correct deformations in the characteristic curve, caused by changes in the soda concentration and other disturbances.

The first step to design the new strategy was the system identification in open loop using step change methods in order to get the system model with nominal conditions.

Defining the state ***y* = *(pH – set point*)** and ***u* = *NaOH Flow*** the obtanied linear aproximation is the following

$$\dot{y}(t) = -\frac{1}{\tau} y(t) + K(t)u(t) , \qquad (39)$$
$$u(t) = -K_P y(t)$$

K(t) depends on the pH and on the operational conditions. In nominal operational conditions the characteristic process gain curve is the following

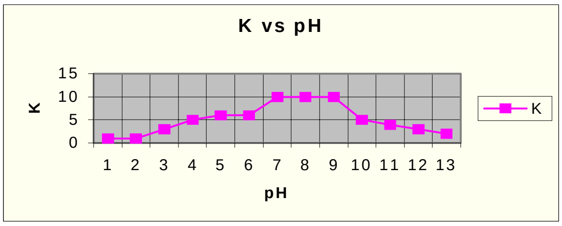

Fig5. Characteristic Curve K vs pH

The characteristic curve could be deformed in presence of disturbances That deformation can be interpreted as a deviation of the controller proportional gain respect to a nominal value. The purpose is to autotune the proportional gain in order to compensate such deformations. To achieve this goal the proportional gain is defined by

$$K_P(t) = \left(\frac{1}{K(t)}\right)\tilde{K}(t) , \qquad (40)$$

where the term $\frac{1}{K(t)}$ is used to compensate the nonlinearities of the nominal system and $\tilde{K}(t)$ is used to correct the deformations produced in the curve through gain autotuning using the proposed method. The expression for the closed loop system is given by

$$\dot{y}(t) = ((1-\tilde{K}(t))/\tau)y(t) + (1/\tau)v_1(t) \quad (41)$$
$$v_1(t) = \Delta\tilde{K}y(t)$$

Following the steps exposed in section VI for the academic example, similar expressions can be obtained for the detector filter and the control law that guarantees the closed loop specifications correcting the deformations of the nominal curve.

The following graphics show the result of the method, applied to pH control.

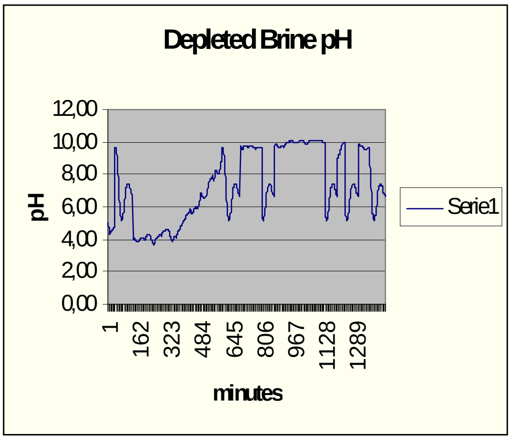

Fig.6 System Response without Autotunig

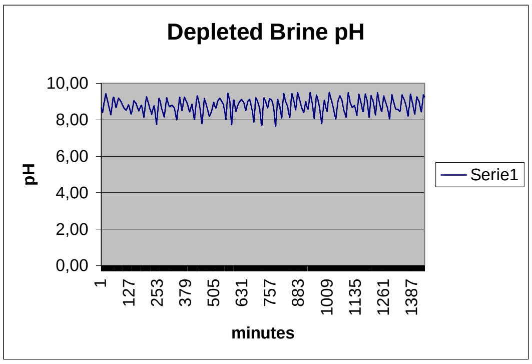

Fig.7 System Response with Autotuning

VII CONCLUSION

Using the fault detection and isolation method developed in [1] based on FDI filters, designed by Luenberger observers, was possible to synthesize a scheme of on-line autotunig for PI controllers. Disturbances and error of the linearizations can be compensated by autotunig the controller parameters. The tuning law is obtained by imposition of asymptotically stable differential equation that guarantees the required closed loop specifications. This method was applied succesfully to solve a pH control problem in a Clorine Plant guaranteeing the closed loop specification in spite of the nonlinearities related to the nominal model, the disturbances coming from others unit of the plant an deviation in the soda concentration.

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