MATHEMATICAL MORPHOLOGY TOOLS FOR GRAY-SCALE IMAGE COMPRESSION

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Abstract - This work introduces a new algorithm for gray-scale image compression using mathematical morphology tools. In order to get a good compression ratio, a bit-plane decomposition is used. Different planes are coded by different methods. The first 4 more significant planes are coded by a combined boundary-sceleton technique, because they are preserving the most important part of information. For the last four least significant planes, taking into account that these planes are carrying less significant information, we can obtain a very good compression ratio by determining the density of probability for black/white pixel occurence and by transmitting only this parameter. Depending on the number of bit-planes coded, bitrates from 0,56 to 1,1 bits/pixel have been obtained. IMACS/IEEE CSCC'99 Proceedings, Pages:7261-7264

1. INTRODUCTION

In the last years more and more interest appeared in using region-based coding approaches for image compression, where the geometrical characteristics of the signal play an important role. Classical linear signal processing tools are not well suited for a geometrical approach and other tools coming from nonlinear signal processing or from computer vision may be attractive for this purpose. The use of mathematical morphology tools for coding is becoming a very active field of research [1-4], due to the fact that mathematical morphology has been developed as a geometrical approach to signal processing, which make it very useful from the beginning in applications involving image analysis.

After the introduction, the second part of the paper describes the bit-plane decomposition. The morphological skeleton is given in the third part. The process of image reconstruction from its minimal skeleton is pointed out in the fourth part. Finally, some results are obtained, and are given with the concluding remarks.

2. BIT-PLANE DECOMPOSITION

Consider an $N \times M$ image, where each pixel value is represented by k bits. By selecting a single bit from the same position of each pixel, an $N \times M$ image binary image called a bit-plane can be formed. Repeating this process for the other bit positions, an original image can be decomposed into a set of k, $N \times M$ bit-planes.

Let us consider a gray-scale image with $k = 8$ (256 gray levels), where a pixel value $x\,[i,j]$ can be written:

$$x[i,j]=a_0[i,j]2^0+a_1[i,j]\,2^1+...+a_7[i,j]\,2^7$$

where $a_n[i,j]\in\{0,1\}$, $n\in\{0,1,...7\}$ represents the n-th bit-plane.

The motivation for this decomposition is that each bit-plane can then be encoded efficiently using binary compression technique. Since the more significant bit-planes generally contain major structural information and are highly compressible, progressively reconstructing an image using the bit -

planes can be a viable technique for this purpose.

We have been used this property of the bit-plane decomposition in the context of the binary mathematical morphology, in order to substantially reduce the existing redundancy in the more significant bit-planes.

The algorithm for the image coding consists of three main steps. The first step is gray-scale image decomposition on bit-planes.

Most of the information is contained in the most significant 4 bit-planes. Therefore, the second step aims at efficiently coding the most significant bit-planes. This is accomplished by means of a method based on a combined morphological boundary- skeleton coding technique. Finally, the least significant 4 bit-planes are lossy coded at a very high compression ratio. This is done by transmitting only information about the probability of occurence for black/white pixels in the respective bit-planes.

3. MORPHOLOGICAL SKELETON

In the case of binary image signals, which are mainly perceived as geometrical patterns, there is a need for representations that emphasize geometric rather than algebraic structure. One such geometric representation is the skeleton. The term skeleton can be obtained, or medial axis, has been used to describe a line-thinned representation of a binary image which summarizes its shape and conveys information about its size, orientation, and connectivity. The skeleton can be obtained by morphological set transformations. To distinguish the skeleton obtained using morphological operators from others, obtained by other approaches, it is generally referred as the morphological skeleton.

The morphological skeleton *SK(X)* of a continuous image object *X*, viewed as a subset of R^2, is defined as the set of the centres of the maximal disks inscriable inside *X*. A disk is maximal if it is not properly contained in any other disk totally included in *X*. Hence, a maximal disk must touch the boundary of the object *X* at least at two different points.

Let $S_r(X)$, $r>0$, denote the *r*-th skeleton subset, i.e. the set of the centres of the maximal disks whose radius is equal to *r*. These skeleton subsets can be obtained by using morphological erosions and openings. Under these assumptions, it was proved that the skeleton *SK(X)* exists and is equal to

$$SK(X)=\bigcup_{r>0} S_r(x)=\bigcup_{r>0} [(X\Theta rB)-(X\Theta rB)_{dB}]$$

where *rB* denotes the open disk of radius *r* and *drB* is a closed disk of infinitesimally small radius *dr*. The boundaries of the eroded sets $(X\Theta rB)$ can be viewed as propagating wave fronts, where the propagating time coincides with the radius *r*. Subtracting from these eroded versions of *X* their opening by *drB* retains only the angular points, which are points of the skeleton. The original set *X* can be reconstructed as the union for all $r>0$ of the subsets $S_r(X)$ dilated by the open disks *rB* respectively.

The dilated skeleton subsets are overlapped, so a boundary constrained minimization is used. A minimal subset of the skeletons is defined as part of the original skeleton, whose points are sufficient for exact reconstruction, but removal of just one of these points will result in partial reconstruction.

Fig.1 illustrates the minimal skeleton obtained for the bit plane 7 of the “Lena” image.

4. ALGORITHM FOR IMAGE RECONSTRUCTION

Based on the theorem of morphological skeleton analysis, the proposed algorithm provides a new technique for image reconstruction by emloying a binary morphological skeleton and a binary boundary structure in place of a series of separate skeleton subsets.

The morphological skeleton of a nonempty and finite region of support binary image sampled in a rectangular grid is obtained by taking the union of every skeleton subset.

The goal is to examine any isolated point of the original image. It is obvious that if the original object contains isolated points with sizes smaller than the structuring element, then the isolated points will be part of its boundary and skeleton structure as well. They actually belong to the first skeleton subset. After the first iteration, the isolated points of the original image are observed and saved, and the following iterations continuously search for the maximal elements of all skeleton points. Once the maximal element of a skeleton point is obtained, the element will be saved as a part of the original object and this skeleton point is discarded prior to the next iteration. The procedure is repeated untill all maximal elements are found.

The original image is then reconstructed by taking the union of all maximal elements and isolated points. Fig.2 presents the result of reconstructing the “Lena” image from its skeletons. Bitrates from 0,56 to 1,1 bits/pixel have been obtained. No essential differences can be noticed between the decoded and original images, because the mathematical morphology tools are very closed to the human visual perception characteristics.

5. CONCLUDING REMARKS

The use of mathematical morphology tools for coding is becoming a very active field of research, due to the fact that it has been developed as a geometrical approach to signal processing. It is very useful in applications involving image analysis.

This paper seeks to provide a new algorithm for gray-scale image compression using mathematical morphology tools. In order to get a good compression ratio, the bit-plane decomposition is used, while different planes are coded by different methods. Depending on the number of coded bit-planes, bitrates from 0,56 to 1,1 bits/pixel have been obtained.

References

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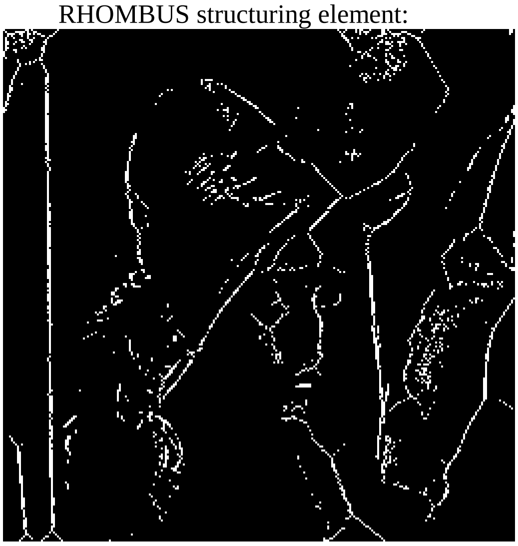
LENA: bit-plane 7

Minimal skeleton:

RHOMBUS structuring element:



SIMPLE structuring element:

Fig. 1 Skeletonization for image Lena

Original

Reconstructed

Fig. 2 Image "Lena" reconstructed from its skeletons