Discrete Time Pseudo-linear Anti-windup Controllers

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Abstract: -This brief paper presents the synthesis of discrete time pseudo-linear feedback controllers.
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1 Introduction

The discrete time counterpart of the pseudo-linear anti-windup controller design [1] is briefly presented. The synthesis problem is synopsized in theorem 1, which is given without proof. The general notation and nomenclature is similar to [1]. A detailed presentation of the design method and proofs of results will be presented elsewhere [2].

2 Controller Synthesis

Let the controllable and observable discrete time plant be

$$s(dx) = As(x) + Bs(u) + G_1 w_1 \quad (1)$$

$$y = Cs(x) + G_2 w_2 \quad (2)$$

where $x \in \Re^n$, $u \in \Re^m$, $y \in \Re^p$, $w_1 \in \Re^{k_1}$, $w_2 \in \Re^{k_2}$ are the state, control input, output, state disturbance and output disturbance discrete-time variables of the system, and A, B, C, G_1, G_2 are the associated state space data respectively.

In general $s(q)$ denote the saturation function

$$s(q) := \begin{cases} q_{max}, & q \geq q_{max} \\ q, & q_{min} < q < q_{max} \\ q_{min}, & q \leq q_{min} \end{cases} . \quad (3)$$

The upper and lower saturation limits are denoted as dx^+, dx^- for dx, x^+, x^- for x and u^+, u^- for u. For an unconstrained component, say v, of dx, or x, or u, it is $dx^+_{v1} = e$ and $dx^-_{v1} = -e$, or $x^+_{v1} = e$ and $x^-_{v1} = -e$, or $u^+_{v1} = e$ and $u^-_{v1} = -e$, respectively. e is an adaptive parameter satisfying

$$ke := \begin{cases} sign(k)\infty, k \neq 0 \\ 0, k = 0 \end{cases} \text{ and } t + ke := \begin{cases} t, t \neq 0 \\ ke, t = 0 \end{cases} .$$

Furthermore, define $a, \bar{a} \in \Re^{n \times n}$ as

$a_{ij} := 0$, if (x_{i1} and dx_{i1} : unconstrained) or ($i \neq j$)

$a_{ij} := 1$, if (x_{i1} or dx_{i1} : constrained) and ($i = j$)

$\bar{a}_{ij} := 1$, if (x_{i1} and dx_{i1} : unconstrained) or ($i = j$)

$\bar{a}_{ij} := 0$, if (x_{i1} or dx_{i1} : constrained) or ($i \neq j$).

Also, define $\tilde{A} := A - BB^l A\bar{a}$.

Now, let the following assumptions hold.

Assumption 1: B is full rank and $G_1 \in Ker(B^l)$, where B^l is the left inverse of B.

Assumption 2: Saturation constraints are defined with functions similar to (3), and only for:

a) The actuators' outputs, states, and rate of states.
b) Any state (not actuator state), which is present in an actuator state space equation (i.e. it is present in a differential equation (1), where a control input component is present as well).

As in [1], the objective is to design a feedback controller, for the plant (1), (2) such that the closed loop system is:

(i) Asymptotically stable.
(ii) Optimal in an H_2 sense.

A solution to the above problem can be obtained with theorem 1, which constitutes the controller synthesis problem.

For the design, n[th] (full) order observer-based controllers are used. Such controllers have the general structure

$$dx_c = A_c x_c + B_c y + E_c\left(s(\tilde{u}) - \tilde{u}\right) \quad (4)$$

$$\tilde{u} = C_c x_c \quad (5)$$

$$\tilde{u}^- := max\left(\left|-B^l Aa\right|x^-, \left|B^l a\right|dx^-, u^-\right) \quad (6)$$

$$\tilde{u}^+ := min\left(\left|-B^l Aa\right|x^+, \left|B^l a\right|dx^+, u^+\right) \quad (7)$$

In the present paper, $s(\tilde{u})$ is the radial ellipsoidal saturation function, shown below.

$$s(\tilde{u}) := \begin{cases} \tilde{u}, & \tilde{u}^T R\tilde{u} \le 1 \\ \left(\tilde{u}^T R\tilde{u}\right)^{\frac{-1}{2}} \tilde{u}, & \tilde{u}^T R\tilde{u} > 1 \end{cases} \quad (8)$$

In (8), $R \in \Re^{m\times m}$ is a positive definite matrix.
Under (4)-(8), the closed loop system can be written as

$$s(d\bar{x}) = \bar{A}\, s(\bar{x}) + \bar{B}\left(s(\tilde{u}) - \tilde{u}\right) + \bar{G}_1 w_1 \quad (9)$$

$$\tilde{u}(t) = \bar{C}\, s(\bar{x}(t)) \quad (10)$$

where,

$$\bar{x} := \begin{bmatrix} x \\ x_c \end{bmatrix},\ \bar{A} := \begin{bmatrix} \tilde{A} & BC_c \\ B_c C & A_c \end{bmatrix}, \bar{B} := \begin{bmatrix} B \\ E_c \end{bmatrix},$$

$$\bar{C} := \begin{bmatrix} 0_{m\times n} & C_c \end{bmatrix},\ \bar{G}_1 := \begin{bmatrix} G_1 \\ 0_{n\times k_1} \end{bmatrix} \quad (11)$$

Theorem 1 ([2]): Let the observable and controllable system (1)-(2), with assumptions 1-2 hold. Also let the nonnegative matrices R_1, V_1, and the positive definite matrices R_2, V_2, and suppose that $\left(\tilde{A}, C\right)$ is observable and there are $X, Y, Z \in \mathbf{S}^{n\times n}$ satisfying

$$X = \tilde{A}^T X\tilde{A} - S_X + R_1 \quad (12)$$

$$Y = \tilde{A}Y\tilde{A}^T - \tilde{A}YC^T\left(V_2 + CYC^T\right)^{-1} CY\tilde{A}^T + V_1 \quad (13)$$

$$Z = \tilde{A}_Y^T Z\tilde{A}_Y + S_X \quad (14)$$

where,

$$S_X := \tilde{A}^T XB\left(R_2 + B^T XB\right)^{-1} B^T X\tilde{A} \quad (15)$$

$$\tilde{A}_Y := \tilde{A} - \tilde{A}YC^T\left(V_2 + CYC^T\right)^{-1} C \quad (16)$$

Furthermore define

$$W := \begin{bmatrix} X+Z & -Z \\ -Z & Z \end{bmatrix} \quad (17)$$

$$E_c := B \quad (18)$$

$$C_c := -\left(R_2 + B^T XB\right)^{-1} B^T X\tilde{A} \quad (19)$$

$$B_c := -\tilde{A}YC^T\left(V_2 + CYC^T\right)^{-1} \quad (20)$$

$$A_c := \tilde{A} + BC_c - B_c C \quad (21)$$

$$\bar{R}_1 = \begin{bmatrix} R_1 & 0_{n\times n} \\ 0_{n\times n} & C_c^T R_2 C_c \end{bmatrix} \quad (22)$$

and suppose that $\left(\bar{A}, \bar{R}_1\right)$ is observable. Then the closed loop system (9)-(10) is asymptotically stable if its initial conditions $\bar{x}_o := \begin{bmatrix} x_o & x_{co} \end{bmatrix}^T$ satisfy $\bar{x}_o^T W\bar{x}_o < l_{max}^{-1}\left(\bar{C}^T R\bar{C}W^{-1}\right)$.

Furthermore, the H_2-type cost functional

$$J(\bar{x}_o) := \sum_{t=0}^{\infty}\Big[x(t)^T R_1 x(t) + \tilde{u}(t)^T R_2 \tilde{u}(t)$$
$$+ 2\bar{x}(t)^T \bar{A}^T W\bar{B}\bar{C}\left(\tilde{u}(t) - s(\tilde{u}(t))\right)$$
$$+ \left(\tilde{u}(t) - s(\tilde{u}(t))\right)^T \bar{B}^T W\bar{B}\left(\tilde{u}(t) - s(\tilde{u}(t))\right)\Big] \quad (22)$$

where $t = 0,1,2,\ldots$, is given by $J(\bar{x}_o) = \bar{x}_o^T W\bar{x}_o$.

Remark 2.1: The matrices R_1, R_2, V_1, V_2 in (12)-(16), play the role of penalty matrices. Hence for the deterministic case of (1)-(2), V_1 and V_2 can be set as $V_1 := G_1 G_1^T$ and $V_2 := G_2 G_2^T$. R_1 and R_2 can be selected arbitrarily, or as in [1].

Remark 2.2: The set

$$Y := \left\{\bar{x}_o \in \Re^{2n} : \bar{x}_o^T W\bar{x}_o < l_{max}^{-1}\left(\bar{C}^T R\bar{C}W^{-1}\right)\right\}$$

defines a subset of the domain of attraction of the closed loop system. Theorem 1 is a sufficient condition for asymptotic stability and therefore, it is possible the closed loop system to be asymptotically stable for initial conditions outside Y.

References:

[1] V. A. Tsachouridis and I. Postlethwaite, *Pseudo-linear Anti-wind Controllers for a Single Machine/Infinite Bus Power System under Exciter and Steam Control Valve Saturation*, appearing in the present conference proceedings.

[2] V. A. Tsachouridis and I. Postlethwaite, A New General Method of Designing Anti-wintup Controllers for Systems with Saturation Constraints on the Actuators' Outputs, States and State Rates, t*o be submitted*.