A Proposal for Knowledge Extension in a Fuzzy Decision Support System

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Abstract: In this work is presented an algorithm that generalizes a fuzzy set describing existing knowledge in a model or rule, given another fuzzy set representing new evidence. The algorithm proceeds by extending the focal elements of the initial set and from the resulting possibility measure reconstructs a new generalized fuzzy set. Since the algorithm is based on simple set operations it can be applied to information defined over non numerical spaces.

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1. Introduction

In the present work we consider the process of generalization by which existing knowledge structures are expanded in the light of new information. More formally, consider a space X and a knowledge representation scheme that defines a class C based on a property s, i.e.

$$C = \{x : x \in X \quad and \quad s(x)\ is\ true\}$$

A generalization process $C = \{x : x \in X \quad and \quad s(x)\ is\ true\}$ will then extend C to C_g so that the domain of property s will be extended to include new elements, that is

$$\exists\ y \in X : y \notin C \quad and \quad y \in C_g$$

For the fuzzy set case,, let X be the space of definition of a fuzzy set F in a model or a rule, with support set $S_F = \{x \in X : m_F(x) \neq 0\}$ Suppose further that G represents a data fuzzy set having support set $S_G = \{x \in X : m_G(x) \neq 0\}$.

Let by T denote the set of elements in X that belong to the support set of G, but not too the support set of F, and therefore have zero membership in it, i.e.

$$T = \{x \in X : m_G(x) \neq 0 \quad and \quad m_F(x) = 0\}$$

The generalization algorithm will then extend F to a new fuzzy set S, containing all the elements in S_F plus elements from T, so that

$$S_S = S_F \cup T' \text{ and } \varnothing \neq T' \subseteq T$$

The next section describes the proposed generalization algorithm.

2. An Algorithm for Generalizing a Fuzzy Set

Consider a fuzzy set F, representing existing information, the corresponding possibility measure $\{F_i, m_i\}$ (where $F_1 \subseteq F_2 \subseteq ... \subseteq F_n$ the focal elements assigned the basic probability masses $m_1, m_2, ..., m_n$ respectively), and let G be a fuzzy set representing new evidence and $G_1 \subseteq G_2 \subseteq ... \subseteq G_m$ the focal elements of the corresponding possibility measure [2].

Suppose that there exists index k in $\{1,2,...,n\}$, such that for some F_k in $F_1, F_2, ..., F_n$

$$G_1 \cap F_k = \varnothing \quad \text{and} \quad G_1 \cap F_{k+1} \neq \varnothing .$$

i.e. the set G_1 containing the elements with full membership in the concept described by G and the set F_k are disjoint. Then using the possibility measure with focal elements

$$F_1, F_2, ..., F_k \cup G_1, F_{k+1} \cup G_2, ..., F_n \cup G_m$$

and the basic probability masses $m_1, m_2, ..., m_n$, we can generate a new fuzzy set S.

In the case where $m - n + k - 1 \neq 0$, the last focal element F_n is aligned with a focal element G_j different from the last G_m, then the final focal element to generate the generalized set S becomes

$$F_n \cup G_j \cup G_{j+1} \cup ... \cup G_m$$

The above algorithm results in a set S that is an extension of F, towards the support set of G, while preserving the existing information in F. If by S_F, S_G, and S_S we denote the support sets of F, G and S respectively, it is obvious that

$$S_F \subseteq S_S$$

Over these sets a probability distribution can be defined [1], as follows: For each $x \in S_F$ let F_p be the smallest focal elements such that

$$\{x\} \cap F_p \neq \varnothing$$

Since $F_p \subseteq ... \subseteq F_n$, the value of the probability distribution at the point x can be defined as

$$p(x) = \sum_{p}^{n} \frac{m_p}{|F_p|}$$

$|F_p|$ being the cardinality of F_p and m_p the mass associated with it.

After the generalization of F, resulting at the set S, the value of the distribution at x will now become

$$p'(x) = \sum_{r}^{n} \frac{m_r}{|S_r|}$$

where again S_r is the smallest focal element such that $\{x\} \cap S_r \neq \varnothing$. This defines a uniform probability distribution over each focal element.

The amount $Dp = p(x_0) - p'(x_0)$ where x_0 an element of the core of F, gives an indication of the degree of the generalization performed.

The above algorithm can be parameterized as follows: Suppose that the fuzzy set F is updated given G, producing the set S. If now S is further updated given again the set G as new evidence, then this outcome can be considered as F updated given G with a parameter equal to 2. This corresponds to a more intense generalization of the initial knowledge. This procedure if repeated converges when the smallest focal element G_1 is unified with F_1 to produce the first focal element that will generate S. The number of repetitions needed to reach the convergence state, constitutes an upper limit for the values that the parameter can take.

In that way is provided a means for gradual updating using a parameter with an intuitive meaning: It can take integer values which correspond to the number of repetitions of the new evidence given by the set G. The higher the number of repetitions the greater will be the generalization of the set F. The procedure reaches a convergence state, at which no further generalization is possible.

The number of iterations till the convergence state can also be considered as an index of the similarity between the fuzzy sets F and G [3].

The following example illustrates the algorithm.

Given a fuzzy set

$$F = 10/0.1, 20/0.4, 30/0.8, 40/1, 50/0.7, 60/0.3, 70/0.2$$

considered a piece of existing knowledge, and the set

$$G = 10/0.1, 20/0.2, 30/0.6, 40/0.8, 50/1, 60/0.6, 70/0.3$$

representing new evidence, the algorithm will proceed by taking the union of the respective focal elements

$$\{40\} : 0.2$$
$$[\{40,30\} \cup \{50\}] : 0.1$$
$$[\{40,30,50\} \cup \{50,40\}] : 0.1$$
$$[\{40,30,50,20\} \cup \{50,40,30,60\}] : 0.1$$
$$[\{40,30,50,20,60\} \cup \{50,40,30,60,70\}] : 0.1$$
$$[\{40,30,50,20,60,70\} \cup \{50,40,30,60,70,20\}] : 0.1$$
$$[\{40,30,50,20,60,70,10\} \cup \{50,40,30,60,70,20,10\}] : 0.1$$

This results in the possibility measure

$$\{40,50\} : 0.2$$
$$\{40,30,50\} : 0.1$$
$$\{40,30,50,60\} : 0.3$$
$$\{40,30,50,60,20,70\} : 0.1$$
$$\{40,30,50,60,20,70\} : 0.1$$
$$\{40,30,50,60,20,70,10\} : 0.1$$
$$\{40,30,50,60,20,70,10\} : 0.1$$

or, after adding probability masses assigned to identical sets.

$$\{40,50\} : 0.2$$
$$\{40,30,50\} : 0.1$$
$$\{40,30,50,60\} : 0.3$$
$$\{40,30,50,60,20,70\} : 0.2$$
$$\{40,30,50,60,20,70,10\} : 0.2$$

This can generate the generalized fuzzy set

$$M1 = 10/0.1, 20/0.4, 30/0.8, 40/1, 50/0.8, 60/0.4, 70/0.3$$

If now we apply the algorithm with a parameter 2 to F, the outcome will be the set:

$$M2 = 10/0.1, 20/0.4, 30/0.8, 40/1, 50/1, 60/0.4, 70/0.3$$

Diagrammatic representations of the fuzzy sets are given in Figures 1, 2 and 3.

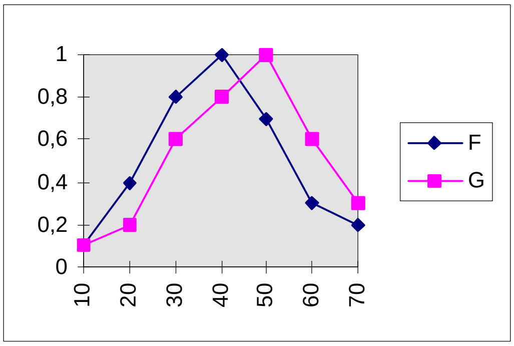

Figure 1

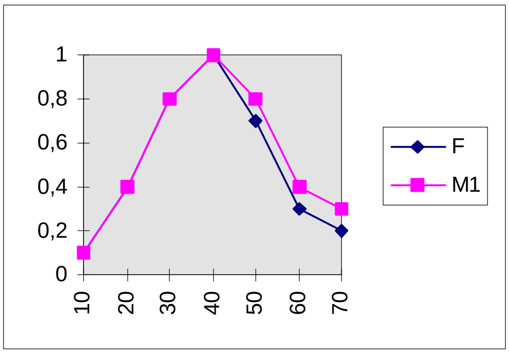

Figure 2

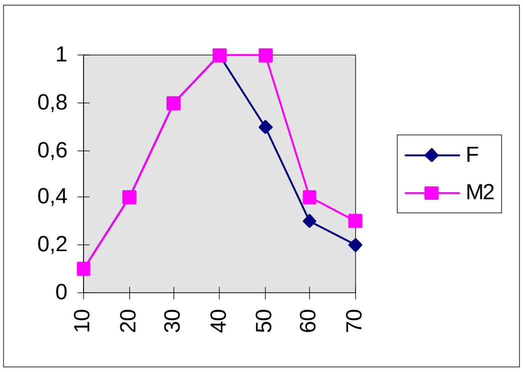

Figure 3

3. Discussion of the Characteristics of the Generalization Algorithm

If a fuzzy set is regarded as a representation model for uncertain knowledge, then some of its characteristics play an important role in any attempt for its semantic interpretation. As such we can coonsider:

- The support set comprising the elements with non-zero membership to the concept described by the fuzzy set.
- The core, i.e. the elements with full membership in the fuzzy set.
- The spread of the set, giving an indication of its fuzziness.

It seems therefore necessary that these characteristics be taken into account when the set is extended. Moreover, the corresponding features of the new evidence could influence the behavior of the extension in which case the process becomes context sensitive. Algorithms operating on a fuzzy set in a point-wise way, although undoubtedly affected by its features, they do not provide for a holistic and intuitive treatment of the fuzzy set. There are even cases where a counter intuitive behavior is possible, such as allowing a single membership value (for instance a zero value) to influence the overall result as in some proposals for aggregation operators[4].

By extending the constituent focal elements of the possibility measure underlying the fuzzy set as proposed in the present work, simple operations from classical set theory can be utilized to operate on a fuzzy set. Additionally the number of repetitions of the same evidence till the process converges, depends on the features of the two fuzzy sets, and therefore the context becomes a determining factor.

The proposed algorithm is not symmetric. If by $G(B/A)$ we denote the process of generalizing the existing knowledge B given new evidence A, then

$$G(B/A) \neq G(A/B)$$

except in the case where $A = B$.

This is often the case in human knowledge updating, especially in situations of important safety considerations, where the existing knowledge may be the product of careful collection of valuable expertise or the coding of domain experts judgment. The proposed algorithm assigns to the data already in the knowledge base a higher degree of importance over the newly presented evidence, implementing in a sense a monotonic updating scheme. The use of the repetition of the same piece of knowledge as a means for determining the degree of generalization, and as such as a parameter, has an intuitive interpretation: The extension of a fuzzy set with a parameter higher than one, indicates the presence of persistent information, which justifies the greater change in the initial knowledge towards this piece of evidence.

The upper limit of this parameter – the number of repetitions till convergence state – is also an index of the conceptual distance between the entities represented by the fuzzy sets. According to this interpretation the convergence state produces a new fuzzy set that includes the previous two.

Finally, the application of the algorithm can be extended to fuzzy sets defined over non-numerical domains, since it is based on simple set theoretic operations.

References

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