An adaptive robust approach to control rigid robots

O. Barambones and V. Etxebarria
Departamento de Electricidad y Electrónica
Facultad de Ciencias
Universidad del País Vasco
Apdo. 644. 48080 Bilbao (Spain)
E-mail oscar@we.lc.ehu.es

Abstract: -In this paper it is presented a robust adaptive control scheme for mechanical manipulators. The design basically consists, on the one hand, of an adaptive controller which implements a feedback linearization control law that compensate the modeled dynamics, and, on the other hand, of an adaptive sliding-mode control law that overcome the unmodelled dynamics and noise. Also it is proved that the resulting closed-loop system is stable and that the trajectory-tracking control objective is asymptotically achieved. Finally, some simulations results are also provided to evaluate the design.

Key-Words: Robotics, Non-linear systems, Robust control, Adaptive Control.

1 Introduction

Since robotic manipulators are inherently nonlinear systems with time-varying inertia and gravitational loads, adaptive controllers have been proposed as a feasible technique to achieve consistent performance in the presence of configuration and payload variations (Ortega and Spong 1989). However, as it is well known, the adaptive control systems may have robustness problems under non-ideal conditions such as unmodelled dynamics and noise.

On the other hand, variable structure control has been successfully used to design controllers to handle systems with unmodelled uncertainties and bounded external disturbances. This robust properties have done the sliding control particularly successful in robotic applications (Slotine and Sastry 1983, Spong 1992). However, its main drawback is the existence of high-frequency chattering owing to system inertia and delay of switchings, which may result in the excitation of unmodelled high frequency dynamics (Slotine and Li 1991) which is undesirable for most practical applications.

The idea of combining adaptive and robust control methods as a way to improve the performance and robustness to modeling imprecissions of control systems has been explored by some researchers (Slotine and Coetsee 1986). It should also be noted that this adaptive approach allows to introduce a priori modeling information in the control design because of the direct relation between the physical and the controller parameters. This constitutes an advantage over some recently developed neural designs (Liao 1998) for similar problems, since in this case there is not a direct relation between the plant and the neural controller parameters.

This paper presents an adaptive control scheme which consist of a feedback linearization law for the modeled dynamics of the robot, together with a robust sliding control with an adaptive sliding gain to avoid the necessity of the prior knowledge of the unmodelled dynamics and noise bounds. From this point of view, the adaptive control is used to estimate the parameters for the modeled dynamics of the robot, and the sliding control is used to overcome the uncertainties. As a whole, the scheme is proved to be stable, and it is shown that the outputs of the closed-loop system asymptotically track the desired reference trajectories.

2 Problem statement

The vector equations of motion of a n-link robot manipulator can be written as (Spong and Vidyasagar 1989):

$$\mathcal{T} = M(\Theta)\ddot{\Theta}+C(\Theta,\dot{\Theta})\dot{\Theta}+G(\Theta)+F(\Theta,\dot{\Theta})+D(t) \quad (1)$$

where $\mathcal{T}$ is a $n \times 1$ vector of joint torques; $\Theta, \dot{\Theta}$ and $\ddot{\Theta}$ are the $n \times 1$ vectors of joints positions, speed and accelerations, respectively; $M(\Theta)$ is the $n \times n$ mass matrix of the manipulator; $C(\Theta,\dot{\Theta})$ is an $n \times n$ vector of centrifugal and Coriolis terms which is chosen so that the matrix $\dot{M}-2C$ is skew-symmetric; $G(\Theta)$ is an $n \times 1$ vector of gravitational terms, $F(\Theta,\dot{\Theta})$ is an $n \times 1$ vector of friction terms, and $D(t)$ is an $n \times 1$ vector whose elements represent the dynamic uncertainties caused by unmodelled dynamics and noise.

The equation of motion (1) form a set of coupled nonlinear ordinary differential equations which are quite complex, even for simple manipulators. One of the most widely used techniques to design a trajectory-following control system for such a device is the so-called computed-torque control using feedback linearization (Craig 1986). This control technique performs well if the model of the system is accurately known.

On the other hand, if there are parametric uncertainties in the model of the system, or if the parameters of the model are time-varying, one may use an adaptive algorithm (Craig 1988, Slotine and Li 1992) to estimate these parameters, but this control technique is not robust under unmodelled dynamics or noise which are present in many practical situations. In these circumstances, it looks apparent that some sort of change in the control law should be done, so that the system becomes robust under these uncertainties.

3 Sliding adaptive control

To compensate for the above described non-idealities in the closed loop system, in this section it is proposed a sliding adaptive control scheme. In the sliding control theory, the switching gain must be constructed so as to attain the sliding condition (Slotine and Li 1992). In order to meet this condition a suitable choice of the sliding gain vector should be made to compensate for the non-idealities. For selecting the sliding gain vector, an upper bound of the unmodelled dynamics and noise magnitudes should be known, but in practical applications there are situations in which these bounds are unknown. A solution could be to choose a sufficiently high value for the sliding gain, but on the other hand, a too high value for this could cause undesirable vibrations and instabilities in the robot's motion. One possible way to overcome this difficulty is to estimate the gain and to update it by some adaptation law, so that the sliding condition is achieved.

To carry out this idea the following assumption is stated:

($\mathcal{A}1$) There exits an unknown finite non-negative gain vector $\rho = [\rho_1, \ldots, \rho_n]^T$ such that

$$\rho \geq D_{max} + \eta \qquad \eta > 0 \tag{2}$$

where $\quad D_{max} \geq |D(t)| \qquad \forall\, t$

Note that this condition implies that the unmodelled dynamics and noise magnitudes are bounded.

Given a proper definition of the unknown parameter vector, it is possible to obtain the following linear dependence (Slotine and Li 1992)

$$M(\Theta)\ddot{\Theta} + C(\Theta,\dot{\Theta})\dot{\Theta} + G(\Theta) + F(\Theta,\dot{\Theta}) = Y(\Theta,\dot{\Theta},\ddot{\Theta})A$$

where A is an r-dimensional vector containing the system dynamical parameters and $Y(\Theta,\dot{\Theta},\ddot{\Theta})$ is an $n \times q$ matrix often refered to as regresor matrix, whose elements are nonlinear known functions.

Let us now define the control input to be of the form:

$$\begin{aligned} \mathcal{T} &= \hat{M}(\Theta)\ddot{\Theta}_r + \hat{C}(\Theta,\dot{\Theta})\dot{\Theta}_r + \hat{G}(\Theta) + \\ &\quad + \hat{F}(\Theta,\dot{\Theta}) - \hat{\mathcal{P}}\,\mathrm{sgn}(S) \\ &= Y(\Theta,\dot{\Theta},\dot{\Theta}_r,\ddot{\Theta}_r)\hat{A} - \hat{\mathcal{P}}\,\mathrm{sgn}(S) \end{aligned} \tag{3}$$

where $\hat{(\cdot)}$, denotes the estimates of $(\cdot)$; $\dot{\Theta}_r = \dot{\Theta}_d - \lambda E, E = \Theta - \Theta_d, \lambda > 0$, and Θ_d is the desired trajectory vector; $\hat{\mathcal{P}}$ is a $n \times n$ diagonal matrix, $\hat{\mathcal{P}} = \mathrm{diag}(\hat{\rho}))$; S is a surface vector defined by $S = \dot{E} + \lambda E = \dot{\Theta} - \dot{\Theta}_r$, and the function $\mathrm{sgn}(S)$ is the usual sign function.

The dynamical parameters of the system, and the elements of the switching gain vector are updated according to the following laws respectively:

$$\dot{\hat{A}} = -,\ Y^T(\Theta,\dot{\Theta},\dot{\Theta}_r,\ddot{\Theta}_r)\,S \qquad ,\ > 0 \tag{4}$$

$$\dot{\hat{\rho}} = |S| \qquad \hat{\rho}(0) = [0,\ldots,0]^T \tag{5}$$

The control diagram of this sliding adaptive control is illustrated in Fig.(1)

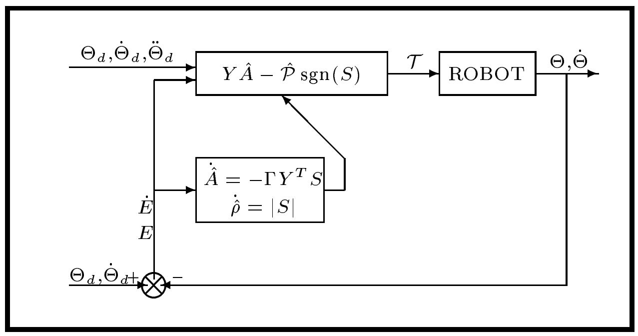


Figure 1: Proposed robust adaptive control scheme

Under these conditions the following stability result can be formulated:

Theorem 1 *Consider the robotic manipputator given by (1). Then, if assumption ($\mathcal{A}1$) is verified, the control law (3) with the adaptation laws (4) and (5) lead the closed loop outputs Θ and their derivatives $\dot{\Theta}$ to track asymptotically the desired trajectories Θ_d and their derivatives $\dot{\Theta}_d$. Therefore, the tracking error $E = \Theta - \Theta_d$ and its derivative tends to zero as t tends to infinity.*

Proof: Define the Lyapunov function candidate:

$$V = \frac{1}{2}\left[S^T M S + \tilde{A}^T,\ ^{-1}\tilde{A} + \tilde{\rho}^T\tilde{\rho}\right] \tag{6}$$

Its time-derivative is calculated as:

$$
\begin{aligned}
\dot{V} &= S^T M \dot{S} + \frac{1}{2} S^T \dot{M} S + \tilde{A}^T ,\ ^{-1} \dot{\tilde{A}} + \tilde{\rho}^T \dot{\tilde{\rho}} \\
&= S^T M \ddot{\Theta} - S^T M \ddot{\Theta}_r + \frac{1}{2} S^T \dot{M} S + \\
&\quad + \tilde{A}^T ,\ ^{-1} \dot{\hat{A}} + \tilde{\rho}^T \dot{\hat{\rho}} \\
&= S^T \left[\mathcal{T} - C(S + \dot{\Theta}_r) - G - F - D \right] - \\
&\quad S^T M \ddot{\Theta}_r + \frac{1}{2} S^T \dot{M} S + \tilde{A}^T ,\ ^{-1} \dot{\hat{A}} + \tilde{\rho}^T \dot{\hat{\rho}} \\
&= S^T \left[\mathcal{T} - M \ddot{\Theta}_r - C \dot{\Theta}_r - G - F - D \right] + \\
&\quad + \frac{1}{2} S^T \left[\dot{M} - 2C \right] S + \tilde{A}^T ,\ ^{-1} \dot{\hat{A}} + \tilde{\rho}^T \dot{\hat{\rho}} \\
&= S^T \left[\mathcal{T} - YA - D \right] + \tilde{A}^T ,\ ^{-1} \dot{\hat{A}} + \tilde{\rho}^T \dot{\hat{\rho}} \\
&= S^T \left[Y\hat{A} - \hat{\mathcal{P}} \operatorname{sgn}(S) - YA - D \right] + \\
&\quad + \tilde{A}^T ,\ ^{-1} \dot{\hat{A}} + \tilde{\rho}^T \dot{\hat{\rho}} \\
&= S^T Y \tilde{A} - \hat{\rho}^T |S| - D^T S - \tilde{A}^T Y^T S + \\
&\quad (\hat{\rho}^T - \rho^T)|S| \\
&= -D^T S - D^T_{max} |S| - \eta^T |S| \\
&\leq -\eta^T |S| \qquad (7)
\end{aligned}
$$

Since V is clearly a positive definite and decrescent function, equation (7) implies that the states S, $\tilde{A}$ and $\tilde{\rho}$ are bounded, and as the states A and ρ are also bounded consequently Θ, $\dot{\Theta}$, $\hat{A}$ and $\hat{\rho}$ are bounded.

The closed loop dynamics can be written in the form:

$$
\begin{aligned}
M\ddot{\Theta} + C\dot{\Theta} + G + F + D &= Y\hat{A} - \hat{\mathcal{P}} \operatorname{sgn}(S) \\
M\dot{S} + CS + D - YA &= Y\hat{A} - \hat{\mathcal{P}} \operatorname{sgn}(S) - YA \\
M\dot{S} + CS + D &= Y\tilde{A} - \hat{\mathcal{P}} \operatorname{sgn}(S) \qquad (8)
\end{aligned}
$$

Since Y, $\tilde{A}$, D, C, $\hat{\rho}$ and S are bounded and as M^{-1} exists because of the uniform positive definiteness of M, from equation (8) it is concluded that $\dot{S}$ is bounded. In eqn.(7) it can be seen that the boundedness of $\dot{S}$ implies that $\ddot{V}$ is also bounded and then $\dot{V}$ is a uniformly continuous function. So Barbalat's lemma let us conclude that $\dot{V} \to 0$ as $t \to \infty$ which from eqn.(7) implies that $S \to 0$ as $t \to \infty$, and then under definition of S it is concluded that $E, \dot{E} \to 0$ as $t \to \infty$.

4 Modifications of the control law

A frequently encountered problem in sliding control is that the control signal given by eqn.(3) is not smooth since the sliding control law is discontinuous across the sliding surfaces, which causes the chattering phenomenon. Chattering is undesirable in practice, since it involves high control activity and further may excite high-frequency dynamics. This situation can be avoided by smoothing out the control chattering within a thin boundary layer of thickness $\beta > 0$. On the other hand, it is well known that when in an adaptive control system the signals are not persistently exciting the parameter drift phenomenon may appear (Slotine and Li 1992).

In this way, some modifications should be done in the control law (3) and in the adaptations laws (4),(5) to overcome the above mentioned problems:

To smooth the control law (3), the sign function included in it is replaced by a saturation function, so that it becomes:

$$
\mathcal{T} = Y\hat{A} - \hat{\mathcal{P}} \operatorname{sat}\left(\frac{S}{\beta}\right) \qquad (9)
$$

where $\beta = [\beta_1, \ldots, \beta_n]^T \quad \beta_i > 0 \quad i = 1, \ldots, n$ are the thicknesses of the boundary layers for each sliding surface associated with each joint, the saturation vector is defined by $\operatorname{sat}\left(\frac{S}{\beta}\right) = \left[\operatorname{sat}\left(\frac{s_1}{\beta_1}\right), \ldots, \operatorname{sat}\left(\frac{s_n}{\beta_n}\right)\right]^T$ and the saturation function is defined in the usual way.

It should be noted that the possible choice of different values ρ_i and β_i for each surface s_i lead to greater flexibility in dealing with the different uncertainty levels appearing for each joint of the manipulator. Moreover, this allows to design the controller using different precision criteria for each joint.

To avoid the parameter drift phenomenon, the parameter adaptation laws are modified to:

$$
\begin{aligned}
\dot{\hat{A}} &= -,\ Y^T(\Theta, \dot{\Theta}, \dot{\Theta}_r, \ddot{\Theta}_r)\, S_o \qquad (10) \\
\dot{\hat{\rho}} &= |S_o| \qquad \hat{\rho}(0) = [0, \ldots, 0]^T \qquad (11)
\end{aligned}
$$

where S_o is defined by $S_o = S - B \operatorname{sat}\left(\frac{S}{\beta}\right)$ with $B = \operatorname{diag}(\beta)$.

It is interesting to point out that s_{o_i} is a measure of the distance of the ith component of the surface vector S to the interval $[-\beta_i, \beta_i]$, and that $\dot{s}_{o_i} = \dot{s}_i$ when s_i is outside the interval, while $\dot{s}_{o_i} = 0$ otherwise.

Before presenting the main stability result,the following assumption $\mathcal{A}\,2$ which take the place of the previous assumption $\mathcal{A}\,1$ is stated:

($\mathcal{A}\,2$) There exits an unknown finite non-negative gain vector $\rho = [\rho_1, \ldots, \rho_n]^T$ such that

$$
\rho \geq D_{max} + C\beta_{max} + \eta \qquad \eta > 0 \qquad (12)
$$

where $D_{max} \geq |D(t)| \quad C\beta_{max} \geq |C(t) \cdot \beta| \quad \forall\, t$

Under these conditions, the following theorem holds:

Theorem 2 *Consider the robotic maniputator given by (1). Then, if assumption ($\mathcal{A}\,2$) is verified, the control law (9) with the adaptation laws (10) and (11) lead the closed loop outputs Θ and their derivatives $\dot{\Theta}$*

to track asymptotically the desired trajectories Θ_d and their derivatives $\dot{\Theta}_d$. Moreover, the the tracking error vector $E = \Theta - \Theta_d$ can be made as small as desired by choosing adequately small boundary layers β_i.

Proof Define the following Lyapunov function candidate:

$$V = \frac{1}{2}\left[S_o^T M S_o + \tilde{A}^T,\ ^{-1}\tilde{A} + \tilde{\rho}^T\tilde{\rho}\right] \qquad (13)$$

whose time-derivative is:

$$\begin{aligned}
\dot{V} &= S_o^T M\dot{S}_o + \frac{1}{2}S_o^T\dot{M}S_o + \tilde{A}^T,\ ^{-1}\dot{\tilde{A}} + \tilde{\rho}^T\dot{\tilde{\rho}} \\
&= S_o^T M\dot{S} + \frac{1}{2}S_o^T\dot{M}S_o + \tilde{A}^T,\ ^{-1}\dot{\tilde{A}} + \tilde{\rho}^T\dot{\tilde{\rho}} \\
&= S_o^T M(\ddot{\Theta} - \ddot{\Theta}_r) + \frac{1}{2}S_o^T\dot{M}S_o + \tilde{A}^T,\ ^{-1}\dot{\hat{A}} + \tilde{\rho}^T\dot{\hat{\rho}} \\
&= S_o^T\left[\mathcal{T} - YA - D - CB\,\mathrm{sat}\left(\frac{S}{\beta}\right)\right] + \\
&= +\tilde{A}^T,\ ^{-1}\dot{\hat{A}} + \tilde{\rho}^T\dot{\hat{\rho}} \\
&= S_o^T Y\tilde{A} - \hat{\rho}^T|S_o| - D^T S_o - S_o^T CB\,\mathrm{sat}\left(\frac{S}{\beta}\right) - \\
&= -\tilde{A}^T Y^T S_o + (\hat{\rho}^T - \rho^T)|S_o| \\
&\le -D^T S_o - S_o^T C\beta - \left[D_{max} + C\beta_{max} + \eta\right]^T|S_o| \\
&\le -\eta^T|S_o| \qquad (14)
\end{aligned}$$

Since V is clearly a positive definite and decrescent function, equations (14),(13) imply that the states S_o, $\tilde{A}$ and $\tilde{\rho}$ are bounded, and as the states A and ρ are also bounded, consequently Θ, $\dot{\Theta}$, $\hat{A}$ and ρ are all bounded.

Using an analogous reasoning as it was utilized in the previous section, one can conclude that $\dot{S}$ is bounded and then as $\dot{S}_o = \dot{S}$ if $S_o \ge \beta$, or $\dot{S}_o = 0$ otherwise, it is deduced that $\dot{S}_o$ is also bounded. Thus from eqn.(14) $\ddot{V}$ is bounded and then $\dot{V}$ is a uniformly continuous function, so Barbalat's lemma let us conclude that $\dot{V} \to 0$ as $t \to \infty$ which from eqn.(14) implies that $S_o \to 0$ as $t \to \infty$ or equivalently S converges to the interval $[-\beta, \beta]$ asymptotically, so under the definition of S, the tracking error of each joint ($e_i = \theta - \theta_d$) converge to a small size depending on the thickness β_i for each joint.

5 Simulation examples

In this section we will consider the control of the simple planar manipulator with two revolute joints shown in Figure 2. Let us fix the notation as follows: For each link i (i=1,2) θ_i denotes the joint angle; m_i denotes the mass; l_i denotes the length; l_{c_i} denotes the distance from the previous joint $(i-1)$ to the center of mass of link i; and I_i denotes the moment of inertia of link i about an axis perpendicular to the plane, passing through the center mass of link i.

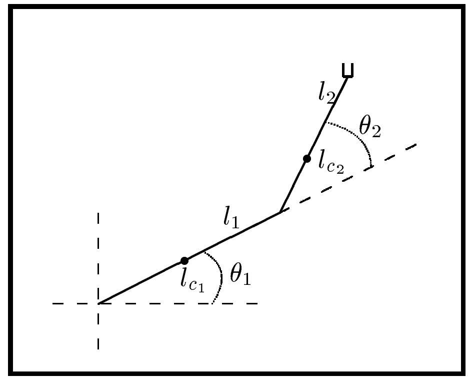


Figure 2: The simple planar manipulator with two revolute joints.

Using the well-known Lagrangian equations in classical dynamics, one can show that the dynamic equations of the robot are:

$$\begin{aligned}
\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix} &= \begin{bmatrix} d_{11} & d_{12}\\ d_{21} & d_{22}\end{bmatrix}\begin{bmatrix}\ddot{\theta}_1\\ \ddot{\theta}_2\end{bmatrix} + \begin{bmatrix} g_1\\ g_2\end{bmatrix} + \begin{bmatrix} f_1\\ f_2\end{bmatrix} \\
&+ \begin{bmatrix} c\dot{\theta}_2 & c\dot{\theta}_1 + c\dot{\theta}_2\\ -c\dot{\theta}_1 & 0\end{bmatrix}\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\end{bmatrix} \qquad (15)
\end{aligned}$$

where the coefficients d_{ij}, c, g_i, f_i are:

$$\begin{aligned}
d_{11} &= m_1 l_{c_1}^2 + I_1 + m_2[l_1^2 + l_{c_2}^2 + 2l_1 l_{c_2}\cos\theta_2] + I_2 \\
d_{22} &= m_2 l_{c_2}^2 + I_2 \\
d_{12} &= d_{21} = m_2 l_1 l_{c_2}\cos\theta_2 + m_2 l_{c_2}^2 + I_2 \\
c &= -m_2 l_1 l_{c_2}\sin\theta_2 \\
g_1 &= m_1 l_{c_1} g\cos\theta_1 + m_2 g[l_{c_2}\cos(\theta_1 + \theta_2) + l_1\cos\theta_1] \\
g_2 &= m_2 l_{c_2} g\cos(\theta_1 + \theta_2) \\
f_1 &= v_1\dot{\theta}_1 + k_1\,\mathrm{sgn}(\theta_1) \\
f_2 &= v_2\dot{\theta}_2 + k_2\,\mathrm{sgn}(\theta_2)
\end{aligned}$$

where g is the gravity acceleration, and v_i and k_i are the viscous and Coulomb friction coefficients, respectively.

Using a proper parametrization, the dynamic equations of the robot can be put in linear dependence ($\mathcal{T} = YA$), where the elements of Y are:

$$\begin{aligned}
&y_{11} = \ddot{\theta}_{r1} && y_{12} = \ddot{\theta}_{r2} \\
&y_{13} = (2\ddot{\theta}_{r1} + \ddot{\theta}_{r2})\cos\theta_2 - [\dot{\theta}_2\dot{\theta}_{r1} + (\dot{\theta}_1 + \dot{\theta}_2)\dot{\theta}_{r2}]\sin\theta_2 \\
&y_{14} = g\cos\theta_1 && y_{15} = g\cos(\theta_1 + \theta_2) \\
&y_{21} = 0 && y_{22} = \ddot{\theta}_{r1} + \ddot{\theta}_{r2} \\
&y_{23} = \cos\theta_2\ddot{\theta}_{r1} + \sin\theta_2\dot{\theta}_1\dot{\theta}_{r1} && y_{24} = 0 \\
&y_{25} = g\cos(\theta_1 + \theta_2)
\end{aligned}$$

and the unknown dynamical parameters are :

$$\begin{aligned}
&a_1 = I_1 + m_1 l_{c_1}^2 + m_2 l_1^2 + I_2 + m_2 l_{c_2}^2 \\
&a_2 = m_2 l_{c_2}^2 + I_2 \qquad a_3 = m_2 l_1 l_{c_2} \qquad (16) \\
&a_4 = m_1 l_{c_1} + m_2 l_1 \qquad a_5 = m_2 l_{c_2}
\end{aligned}$$

It should be noted here that the friction terms have not been modeled. Therefore in this example, the friction terms are the unmodelled dynamics of the system. This is a realistic assumption since the dynamic friction terms are unknown in real applications, and they are difficult to model.

In all the subsequent examples the following values for the robot's parameters will be assumed (SI units):

$$\begin{array}{llll} m_1 = 2 & m_2 = 1.2 & I_1 = 0.25 & I_2 = 0.1 \\ l_1 = 1 & l_{c_1} = 0.4 & l_2 = 0.7 & l_{c_2} = 0.3 \\ v_1 = 0.5 & v_2 = 0.4 & k_1 = 0.3 & k_2 = 0.2 \end{array} \quad (17)$$

Using the values presented in eqn.(17), the real dynamical parameters a_i are:

$$A = [1.98 \;\; 0.21 \;\; 0.36 \;\; 2.00 \;\; 0.36]^T \quad (18)$$

In the following examples a square wave trajectory, whose steps have been replaced by cubic trajectories to smooth the control signal will be used. This trajectory has been chosen because it is frequently used in real manipulation tasks.

In these examples it is used the modified sliding adaptive control scheme for mechanical manipulators presented in section 4. The controller parameters have been chosen as:

$$\lambda = \text{diag}(10\;10) \quad \beta = [0.1\;0.15]^T \quad , \; = \text{diag}(1\;1\;1\;1\;1)$$

In the first example, shown in Fig.(3), it is assumed that there is not any knowledge of the dynamical parameters, that is, all the parameters are initialized as 0. Figures 3 (c) and (d) show that, after a small time, both links track the desired references with a small error, which maximum value is chosen by means of the β parameter.

In the second example illustrated in Fig.(4), it is assumed that some a priori information of the dynamical parameters is available, and therefore these are incorrectly initialized with an error around 50 %:

$$\hat{A}(0) = [1.5 \;\; 0.1 \;\; 0.2 \;\; 1.5 \;\; 0.2]^T \quad (19)$$

Comparing Figures (3) and (4), it can be seen that in the latter, the control efforts are smaller, due to the prior knowledge of the system which is taken into account by the dynamical parameter initialization.

6 Conclusions

In this paper a robust adaptive control scheme for robotic manipulators has been presented. It is used an adaptive feedback linearization control strategy which can adaptively compensate the non-linear assumed dynamics of the model, together with an also adaptive robust sliding control which overcomes the problems that are likely to appear because of the unmodelled dynamics and noise. Moreover the adaptation of the sliding gain avoids having to know a bound for the unmodelled dynamics and noise. The design has been proved to guarantee the closed loop stability and the asymptotic elimination of the tracking errors. Finally, by means of simulation examples, it has been shown that the proposed control scheme performs reasonably well in practice, and that the tracking control objective is achieved. It is shown also that if prior knowledge of system is introduced in the design, the control is improved as expected. From this point of view the presented sliding adaptive approach has advantages over similar neural control schemes that have been appearing lately in the literature, because in this adaptive design there is a direct relation between the physical and the controller parameters, while in the neural approach this relation does not exist in general, and cannot be exploited in the control design.

Acknowledgments

The authors are grateful to CICYT for partial support of this work through project TAP98-0540. O. Barambones also wishes to thank the Basque Government for a scholarship.

References

[1] Liao, T.L, 1998, Adaptive robust neural tracking control of a class of unknown nonlinear systems. *International Journal of System Science*, **29**, 731-743.

[2] Ortega, R. and Spong, M.W, 1989, Adaptive motion control of rigid robots: a tutorial. *Automatica*, **25**, 251-265.

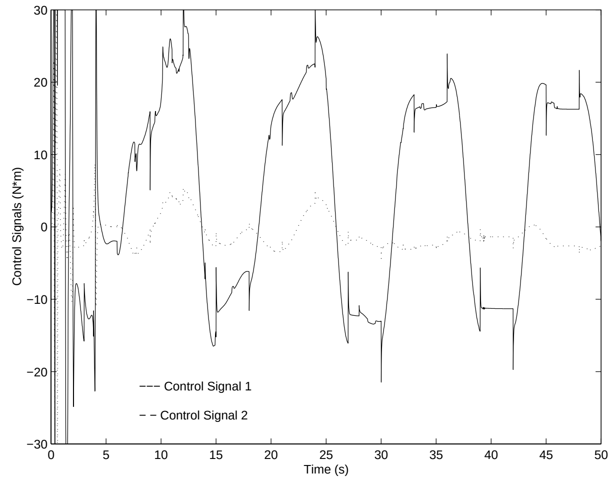
[3] Slotine, J.J.E. and Coetsee, J.A., 1986, Adaptive sliding controller synthesis for nonlinear systems. *International Journal of Control*, **43**, 1631-1651.

[4] Slotine, J.J.E. and Li, W., 1991, *Applied nonlinear control.* (Englewood Cliffs, New Jersey, USA: Prentice-Hall).

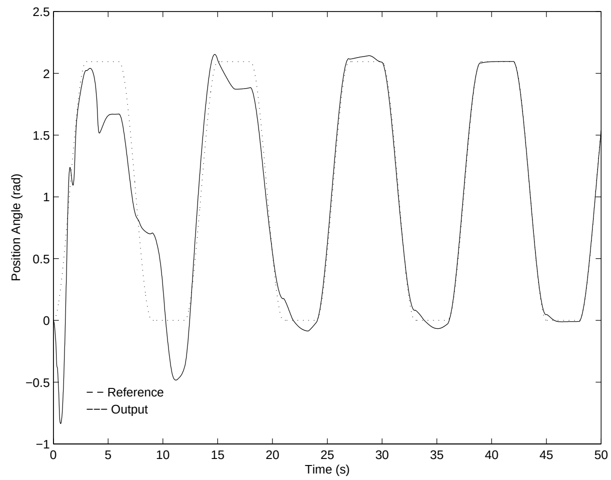
[5] Slotine, J.J.E. and Sastry, S.S., 1983, Tracking control of nonlinear systems using sliding surfaces with applications to robot manipulators. *International Journal of Control*, **38**, 465-492.

[6] Spong, M.W. and Vidyasagar, M., 1989, *Robot dynamics and control.* (New York, USA: John Wiley and Sons).

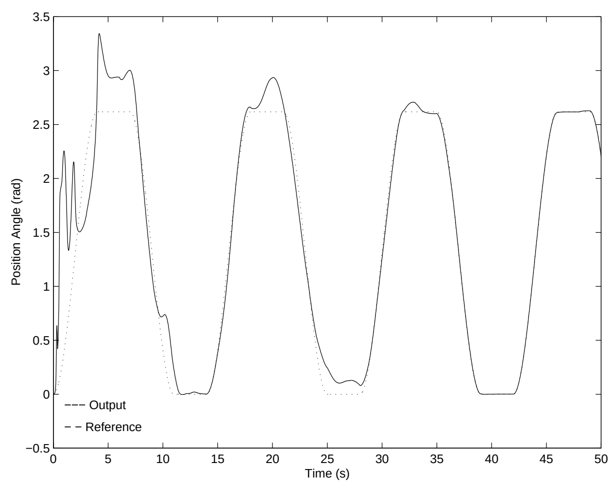
[7] Spong, M., 1992, On the robust control of robot manipulator. *IEEE Transactions on Automatic Control*, **7**, 1782-1786.



(a)

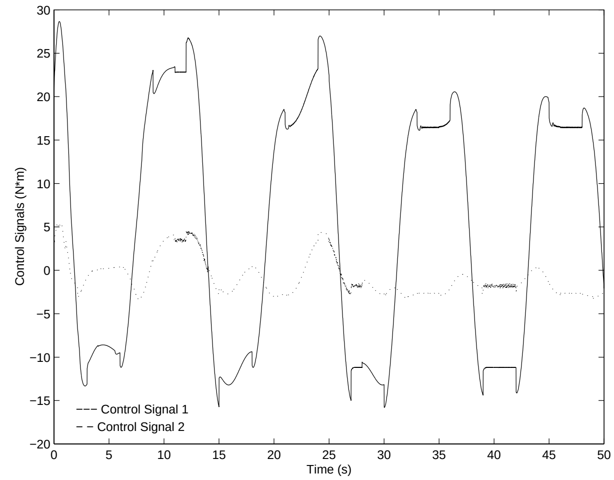


(b)

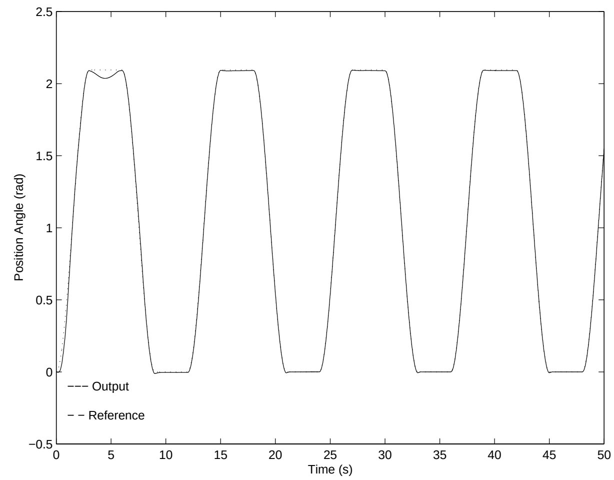


(c)

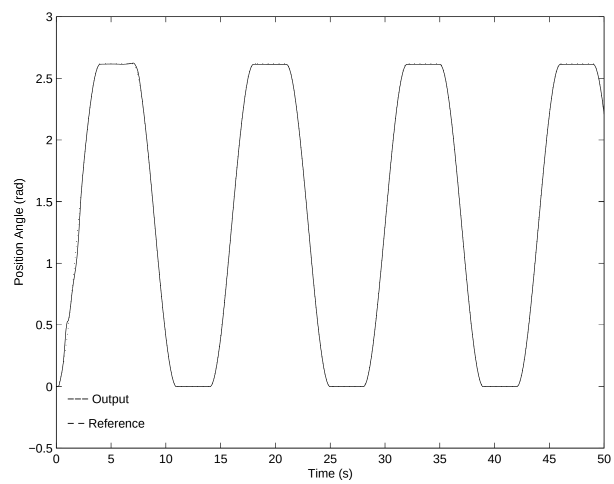
Figure 3: Simulation results for example 1: (a):Control signals for both joints ; (b):Output and reference signal for the first joint; (c):Output and reference signal for the second joint.



(a)



(b)



(c)

Figure 4: Simulation results for example 2: (a):Control signals for both joints ; (b):Output and reference signal for the first joint; (c):Output and reference signal for the second joint.