Singularity Analysis of Nearly General Stewart Platforms

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Abstract: In this paper, the singularity study of general Stewart Platforms is presented. Based on the kinematic equations, a new method to formulate the Jacobian matrix in simplified form is developed. The geometrical characteristics of singular configuration is given. It is pointed out that the singular configurations correspond to the repeated solutions of direct kinematics equations. A comprehensive theoretical proof with illustrative numerical examples is given.

Key-Words: Continuation Method, Jacobian Matrix,Parallel Manipulator,Repeated Solution,Robotics,Singularity

1 Introduction
During the past few years, parallel manipulator systems have become one of the research attentions in robotics. This popularity is due to the fact that parallel manipulators possess some specific advantages over serial manipulators, namely higher rigidity and load-carrying capacity, better dynamic performance and a simple inverse position kinematics. The best known fully parallel manipulator is the Stewart Platform which was introduced as an aircraft simulator by Stewart [1] in 1965. The general form of the Stewart Platform, as shown in Fig.1, consists of two platforms which are connected to each other by six legs. All the six joints in both the base and the top platforms are respectively randomly placed in two planes. Each leg has a spherical joint at one end and a universal joint (or spherical joint) at the other. These joints are passive. There is an actively controlled prismatic joint between the two ends of each leg. One of the platforms, called the top platform, has six degrees of freedom relative to the other platform which is usually referred to as the base platform.

One of the important problems in robot kinematics is to determine the singularity conditions for the manipulator. For the parallel manipulators, many authors have studied the singularity problem. Hunt [2] used screw theory to discuss the singularity problem and pointed out one singular configuration for the Stewart Platform. Fichter [3] studied a prototype Stewart platform and found two distinct singular configurations. One configuration is the same as Hunt’s. The other configuration is reached by rotating the device ±90 degrees about its z-axis while keeping the platform parallel to the base. Cleary and Arai [4] presented a prototype parallel link manipulator and shown a singular configuration for their prototype. Ma and Angeles [5] classified the singularities into three categories, namely, the Architecture, Configuration and Formulation singularities. In their paper, Ma and Angeles focused on the first category, namely, architecture singularity, and reported several groups of singular architectures.

It is well-known that the Jacobian matrix takes a very important role in the singularity studies of robotics. Since the determinant of Jacobian matrix is zero at a singular position, one method to compute the singular configurations is to form the Jacobian matrix symbolically and take the determinant. The determinant can then be equated to zero and the solution of this equation will give the singular positions. In [4, 5], the expression of Jacobian matrix was given. But each elements of the Jacobian matrix is quite complicated as shown in [4]. In this paper, we present a new method to form the Jacobian matrix in simplified form.

In [6], we studied the singularity analysis of a class of the Stewart Platforms and found a phe-
nomenon, namely, the singular configurations always correspond to the repeated solution of the direct kinematics equations. In this paper, it is pointed out that this is a general conclusion for Stewart Platforms, and a clear proof is given.

2 Kinematics of General Stewart Platform

In [7], Zhang and Song formed the kinematics equations of general Stewart platform and derived the closed-form solution. It is shown that the resultant equation is a 20th order polynomial with one variable. The present paper is based on this mathematical model.

The geometry of a general Stewart platform is illustrated in Fig. 1. For the convenience of analysis, two coordinate systems are used. A fixed coordinate frame O-XYZ (or S) is attached to the base platform and its origin coincides with the spherical joint B1. The X and Y axes are set to be in the base platform and the Z axis is upward and perpendicular to the base. The coordinate system O1-X1Y1Z1 (or S1) is fixed at the moving platform with its origin coincident with P1, X1 and Y1 axes lying in the moving platform, Z1 being upward and perpendicular to the platform. The homogeneous transformation matrix T representing the position and orientation of the top platform relative to the base is described as:

\[
T = \begin{bmatrix}
    R & p \\
    0 & 1
\end{bmatrix}
\]

where \( p = [x, y, z]^T \) is a 3 \times 1 matrix denoting the position vector of \( O_1 \) with respect to \( S \), \( 0 \) is a 1 \times 3 zero matrix, and \( R \) is a 3 \times 3 directional cosine matrix representing the orientation of \( S_1 \) with respect to \( S \) and can be expressed as:

\[
R = \begin{bmatrix}
    l_x & m_x & n_x \\
    l_y & m_y & n_y \\
    l_z & m_z & n_z
\end{bmatrix} = \begin{bmatrix} 1 & m & n \end{bmatrix}
\]

(2)

Among the nine elements of the directional cosine matrix, there are only three independent ones and the remaining six can be determined by the following equations:

\[
l_x^2 + l_y^2 + l_z^2 = 1 \quad (3)
\]

\[
m_x^2 + m_y^2 + m_z^2 = 1 \quad (4)
\]

\[
l_x m_x + l_y m_y + l_z m_z = 0 \quad (5)
\]

\[
n_x = l_y m_z - l_z m_y \quad (6)
\]

\[
n_y = l_z m_x - l_x m_z \quad (7)
\]

\[
n_z = l_x m_y - l_y m_x \quad (8)
\]

Instead of using the rotation submatrix \( R \) to describe the orientation, we can use three Euler angles, yaw \( \gamma \), pitch \( \beta \), and roll \( \alpha \). The orientation represented by the Roll-Pitch-Yaw angles is given by:

\[
R = \begin{bmatrix}
    l_x & m_x & n_x \\
    l_y & m_y & n_y \\
    l_z & m_z & n_z
\end{bmatrix} =
\begin{bmatrix}
    C\gamma C\beta & -S\gamma C\alpha + S\gamma S\beta S\alpha & S\gamma S\alpha + C\gamma S\beta C\alpha \\
    S\gamma C\beta & C\gamma S\alpha + S\gamma S\beta S\alpha & -C\gamma S\alpha + S\gamma S\beta C\alpha \\
    -S\beta & C\beta S\alpha & C\beta C\alpha
\end{bmatrix}
\]

(9)

where \( \sin \alpha \equiv S\alpha, \cos \alpha \equiv C\alpha, \sin \beta \equiv S\beta, \cos \beta \equiv C\beta, \sin \gamma \equiv S\gamma, \cos \gamma \equiv C\gamma \).

2.1 Velocities of the Platform

Let us define the position, orientation, linear velocity, and angular velocity vectors of the platform with respect to the reference frame, respectively, as:

\[
p = (x, y, z)^T \quad \mathbf{\Phi} = (\alpha, \beta, \gamma)^T \\
\mathbf{v} = (v_x, v_y, v_z)^T \quad \mathbf{\Omega} = (\omega_x, \omega_y, \omega_z)^T
\]

(10)

where the superscript “\( T \)” denotes the transpose operation on vectors and matrices. The linear velocity of the platform with respect to the reference
frame is equal to the time derivative of the position:

\[ \mathbf{v} = \frac{d\mathbf{p}}{dt} = \dot{\mathbf{p}} \quad (11) \]

Since the inverse of a direction cosine matrix is equivalent to its transpose, the instantaneous angular velocities of the platform coordinate frame about the principal axes of the reference frame can be obtained from (2) as [8]:

\[
\mathbf{R} \frac{d\mathbf{R}^T}{dt} = - \frac{d\mathbf{R}}{dt} \mathbf{R}^T = \begin{bmatrix}
0 & -\omega_z & \omega_y \\
\omega_z & 0 & -\omega_x \\
-\omega_y & \omega_x & 0
\end{bmatrix} \mathbf{R} \quad (12)
\]

From the above equation, the relation between \((\omega_x, \omega_y, \omega_z)^T\) and \(\frac{d\mathbf{R}}{dt}\) can be found:

\[
\frac{d\mathbf{R}}{dt} = \begin{bmatrix}
0 & -\omega_z & \omega_y \\
\omega_z & 0 & -\omega_x \\
-\omega_y & \omega_x & 0
\end{bmatrix} \mathbf{R} \quad (13)
\]

### 2.2 Kinematics Equations

For a given Stewart platform, the position of the center of each spherical joint \(B_i\) with respect to \(S\) is known and can be expressed as:

\[ B_i(a_i, b_i, 0, 1), \quad i = 1, \ldots, 6. \quad (14) \]

Similarly, the position of \(P_i\) with respect to \(S\) is also known and can be expressed as:

\[ P_i(p_i, q_i, 0, 1), \quad i = 1, \ldots, 6. \quad (15) \]

By using the transformation matrix \(T\), the position of \(P_i\) relative to \(S\) can be expressed as:

\[
(p_i l_x + q_i m_x + x, p_i l_y + q_i m_y + y, \\
p_i l_z + q_i m_z + z, 1), \quad i = 1, 2, \ldots, 6.
\quad (16)
\]

Since the positions of \(P_i\) and \(B_i\) have been expressed at the same coordinate system \(O-XYZ\), the distance equation between two points can be applied to derive one constraint equation for each leg. Hence, we have:

\[
(p_i l_x + q_i m_x + x - a_i)^2 + (p_i l_y + q_i m_y + y - b_i)^2 + (p_i l_z + q_i m_z + z)^2 = l_i^2, \quad i = 1, 2, \ldots, 6. \quad (17)
\]

where \(l_i\) is the length of the \(i\)th leg which connects the joints \(P_i\) and \(B_i\). It should be noticed that \(n_x, n_y, \) and \(n_z\) do not appear in the above constraint equations due to the fact that \(P_i\)'s locate at a planar platform. Hence, there are only nine unknowns involved in the kinematics equations.

For \(i = 1\), since \(a_1 = b_1 = p_1 = q_1 = 0\), Eq. (17) becomes:

\[\begin{align*}
x^2 + y^2 + z^2 &= l_1^2 \quad (18)
\end{align*}\]

For \(i = 2, 3, \ldots, 6\), Eq. (17) can be rearranged and simplified by substituting Eqs. (3)-(5) and (18) into it:

\[
\begin{align*}
p_i w_1 + q_i w_2 - a_i x - b_i y - q_i a_i m_x &= \\
P_i a_i l_x + P_i b_i l_y + q_i b_i m_y + \\
(l_i^2 - l_1^2 - a_i^2 - b_i^2 - p_i^2 - q_i^2)/2, \quad i = 2, \ldots, 6 \quad (19)
\end{align*}
\]

where \(w_1\) and \(w_2\) are intermediate variables and defined as:

\[
\begin{align*}
w_1 &= l_x x + l_y y + l_z z \quad (20) \\
w_2 &= m_x x + m_y y + m_z z \quad (21)
\end{align*}
\]

### 3 Formulation of Jacobian Matrix

Upon differentiating the constraint Eqs. (18) and (19) with respect to time, we obtain:

\[
x \dot{x} + y \dot{y} + z \dot{z} = l_i \dot{l}_i \quad (22)
\]

\[
\begin{align*}
p_i \dot{w}_1 + q_i \dot{w}_2 - a_i \dot{x} - b_i \dot{y} - q_i a_i \dot{m}_x &= \\
P_i a_i \dot{l}_x + P_i b_i \dot{l}_y + q_i b_i \dot{m}_y + l_i \dot{l}_i - l_1 \dot{l}_1, \quad i = 2, \ldots, 6 \quad (23)
\end{align*}
\]

Considering (23) as a linear equation in \(\dot{w}_1, \dot{w}_2, \dot{x}, \dot{y}, \dot{m}_x, \dot{l}_x, \dot{l}_y, \dot{m}_y\), we can express \(\dot{w}_1, \dot{w}_2, \dot{x}, \dot{y}, \dot{m}_x\) in terms of \(\dot{l}_x, \dot{l}_y, \dot{m}_y\) through the following matrix manipulation:
\[
\begin{bmatrix}
\dot{w}_1 \\
\dot{w}_2 \\
\dot{x} \\
\dot{y} \\
\dot{m}_x \\
\dot{m}_y
\end{bmatrix} = \mathbf{K}^{-1} \begin{bmatrix}
p_2 & q_2 & -a_2 & -b_2 & -q_{2a_2} \\
p_3 & q_3 & -a_3 & -b_3 & -q_{3a_3} \\
p_4 & q_4 & -a_4 & -b_4 & -q_{4a_4} \\
p_5 & q_5 & -a_5 & -b_5 & -q_{5a_5} \\
p_6 & q_6 & -a_6 & -b_6 & -q_{6a_6}
\end{bmatrix}
\begin{bmatrix}
\dot{w}_1 \\
\dot{w}_2 \\
\dot{x} \\
\dot{y} \\
\dot{m}_x \\
\dot{m}_y
\end{bmatrix}
\]

\[
\mathbf{K}^{-1} = \begin{bmatrix}
-l_1 & l_2 & 0 & 0 & 0 & 0 \\
-l_1 & 0 & l_3 & 0 & 0 & 0 \\
-l_1 & 0 & 0 & l_4 & 0 & 0 \\
-l_1 & 0 & 0 & 0 & l_5 & 0 \\
-l_1 & 0 & 0 & 0 & 0 & l_6
\end{bmatrix}
\]

where

\[
\mathbf{J} = \begin{bmatrix}
1 & 0_{1 \times 3}
\end{bmatrix}
\]

Substituting Eqs. (28)-(30) into (27) and together with (22), the relationship between input velocities of the joints and output velocities of the platform can be found as

\[
\mathbf{J} \begin{bmatrix}
\dot{\mathbf{v}} \\
\dot{\Omega}
\end{bmatrix} = \begin{bmatrix}
\dot{l}_1 \\
\dot{l}_2 \\
\dot{l}_3 \\
\dot{l}_4 \\
\dot{l}_5 \\
\dot{l}_6
\end{bmatrix}
\]

where

\[
\mathbf{J} = \begin{bmatrix}
x & y & z & 0 & 0 & 0 \\
l_x & l_y & l_z & J_{24} & J_{25} & J_{26} \\
m_x & m_y & m_z & J_{34} & J_{35} & J_{36} \\
0 & 0 & 0 & J_{44} & J_{45} & J_{46} \\
0 & 0 & 0 & J_{54} & J_{55} & J_{56} \\
0 & 0 & 0 & J_{64} & J_{65} & J_{66}
\end{bmatrix}
\]

Then Eq. (24) becomes

\[
\begin{bmatrix}
\dot{w}_1 \\
\dot{w}_2 \\
\dot{x} \\
\dot{y} \\
\dot{m}_x \\
\dot{m}_y
\end{bmatrix} = \mathbf{K}^{-1}
\begin{bmatrix}
F_1 & G_1 & H_1 \\
F_2 & G_2 & H_2 \\
F_3 & G_3 & H_3 \\
F_4 & G_4 & H_4 \\
F_5 & G_5 & H_5
\end{bmatrix}
\begin{bmatrix}
\dot{w}_1 \\
\dot{w}_2 \\
\dot{x} \\
\dot{y} \\
\dot{m}_x \\
\dot{m}_y
\end{bmatrix}
\]

Differentiating Eqs. (20) and (21) yields

\[
\dot{w}_1 = l_x \dot{x} + l_y \dot{y} + l_z \dot{z} + x \dot{l}_x + y \dot{l}_y + z \dot{l}_z \quad (28)
\]

\[
\dot{w}_2 = m_x \dot{x} + m_y \dot{y} + m_z \dot{z} + x \dot{m}_x + y \dot{m}_y + z \dot{m}_z \quad (29)
\]

By equating the elements of the first two columns in Eq. (13), we obtain

\[
\det(\mathbf{J}) = 0. \quad (34)
\]
4 Geometrical Characteristics of the Singular Configuration

In [7], Zhang and Song derived the closed-form solution of the forward position analysis of the nearly general Stewart platform. They derived 21 dependent constraint equations with 21 unknown variables that can be represented in matrix form:

\[ AX = 0 \]  

where \( A \) is a 21 \( \times \) 21 matrix and its elements are polynomials of \( m_y \); \( X \) is a 21 \( \times \)1 matrix of unknown variables and is defined as:

\[
X = \begin{bmatrix}
I_y, I_y^2, I_y^3, l_x, l_y, l_z, I_y^2 l_x, I_y l_x^2, I_x l_y, I_x l_y l_x, I_x l_y^2, l_x^2, l_x l_y, l_y l_x, l_y, l_x, 1
\end{bmatrix}
\]

If we treat Eq. (35) as a system of linear, homogeneous equations, the condition for consistency is that the determinant of \( A \) is zero. That is:

\[ \det(A) = 0 \]  

From Eq. (37), a 20th polynomial equation of \( m_y \) can be derived in the form of:

\[ f = a_0 m_y^{20} + a_1 m_y^{19} + \ldots + a_{19} m_y + a_{20} = 0 \]  

where \( a_i (i = 0, 1, \ldots, 20) \) are parameters related to structure parameters and six input variables \( l_1, \ldots, l_6 \).

After \( m_y \) has been determined from Eq. (38), all the elements of \( A \) can be calculated and \( l_y \) and \( l_x \) can be uniquely determined by using the first 20 equations in Eq. (35), which can be expressed in matrix form as:

\[ BY = C \]

where \( B \) is formed by eliminating the last row and the last column of \( A \); \( Y \) contains the first 20 elements of \( X \); and \( C \) is formed by changing the signs of the first 20 elements of the last column of \( A \).

**Theorem 1** The singular configuration corresponds to a repeated solution of the direct kinematics equations.

**Proof.** Differentiating Eq. (38), yields

\[
(20a_0 m_y^{19} + \ldots + 2a_{18} m_y + a_{19})\dot{m}_y = -a_0 m_y^{20} - \ldots - a_{19} m_y - \dot{a}_{20}
\]

where \( \dot{m}_y, \ldots, \dot{a}_{20} \) are functions of \( l_1, \ldots, l_6; \dot{l}_1, \ldots, \dot{l}_6 \).

From Eq. (40), it can be seen that \( \dot{m}_y \) is undefined if and only if

\[ 20a_0 m_y^{19} + \ldots + 2a_{18} m_y + a_{19} = 0 \]  

Now assume \( \dot{m}_y \) has been determined.

Since \( l_y \) and \( l_x \) can be uniquely expressed in terms of \( m_y \), \( \dot{l}_y, \dot{l}_x \) can be determined by differentiating \( l_y \) and \( l_x \) respectively.

Once \( \dot{m}_y, \dot{l}_y, \) and \( \dot{l}_x \) have been determined, \( \dot{w}_1, \dot{w}_2, \dot{x}, \dot{y}, \) and \( \dot{m}_z \) can be determined from Eq. (27). Based on Eqs. (22), (28) and (29), \( \dot{z}, \dot{l}_z, \dot{m}_z \) can be determined if \( z \neq 0 \).

Hence \[
\begin{bmatrix}
\dot{v} \\
\Omega
\end{bmatrix}
\]
is undefined if and only if \( \frac{\partial f}{\partial m_y} = 0 \) or \( z = 0 \).

Therefore when \( \frac{\partial f}{\partial m_y} = 0 \) or \( z = 0 \), the velocities of the platform cannot be defined, consequently the mechanism becomes singular.

From [9], we know when \( f = 0, \frac{\partial f}{\partial m_y} = 0 \), Eq. (38) has multiple solutions. This means that the singular configuration corresponds to a repeated solution of the direct kinematics equations. Therefore at the singular position, one pair of branch of the direct kinematic problem meet.

5 Example

Let us consider a Stewart Platform in which the six joints in both the base and the top platforms located in two planes and arranged in semi-regular hexagons (see Fig. 2 and Fig. 3).

The structure parameters are chosen as follows: \( R = 1, R_0 = 2, \beta = \frac{\pi}{12}, \beta_0 = \pi/4 \).

where \( R \) and \( R_0 \) represent the radii of top and base platforms respectively, \( \beta \) and \( \beta_0 \) are the half
angles of $\angle P_1 O_1 P_0 (= \angle P_3 O_1 P_2 = \angle P_1 O_1 P_3)$ and $\angle B_1 O_B_0 (= \angle B_3 O B_2 = \angle B_4 O B_3)$ respectively.

Attach a fixed Cartesian coordinate frame $O-XYZ$ to the base platform with the $Z$ axis pointing vertically upwards and a moving Cartesian coordinate $O_1-X_1 Y_1 Z_1$ to the top platform with $Z_1$ axis normal to the platform.

The coordinates of $P_i$ and $B_i$ can be calculated as follows:

$$p_1 = 0, \quad q_1 = 0$$

$$p_2 = R \cos(\pi/6 - \beta) - R \sin(\beta)$$

$$q_2 = R \sin(\pi/6 - \beta) + R \cos(\beta)$$

$$p_3 = R \sin(\pi/3 - \beta) - R \sin(\beta)$$

$$q_3 = R \cos(\pi/3 - \beta) + R \cos(\beta)$$

$$p_4 = -R \sin(\pi/3 - \beta) - R \sin(\beta)$$

$$q_4 = q_3$$

$$p_5 = -R \cos(\pi/6 - \beta) - R \sin(\beta)$$

$$q_5 = q_2$$

$$p_6 = -2R \sin(\beta) \quad q_6 = 0$$

$$a_1 = 0, \quad b_1 = 0$$

$$a_2 = R \cos(\pi/6 - \beta_0) - R \sin(\beta_0)$$

$$b_2 = R \sin(\pi/6 - \beta_0) + R \cos(\beta_0)$$

$$a_3 = R \sin(\pi/3 - \beta_0) - R \sin(\beta_0)$$

$$b_3 = R \cos(\pi/3 - \beta_0) + R \cos(\beta_0)$$

$$a_4 = -R \sin(\pi/3 - \beta_0) - R \sin(\beta_0)$$

$$b_4 = b_3$$

$$a_5 = -R \cos(\pi/6 - \beta_0) - R \sin(\beta_0)$$

$$b_5 = b_2$$

$$a_6 = -2R \sin(\beta_0) \quad b_6 = 0$$

Substituting $p_i, q_i, a_i,$ and $b_i$ into (25) we can obtain $K,$ then substituting $K$ into (26) we can obtain $F_1, G_i, H_i,$ finally substituting $F_1, G_i, H_i$ into (32) we obtain the Jacobian matrix as follows:

$$\mathbf{J} = \begin{bmatrix}
    x & y & z & 0 & 0 & 0 \\
    l_x & l_y & l_z & J_{24} & J_{25} & J_{26} \\
    m_x & m_y & m_z & & & \\
    1 & 0 & 0 & J_{44} & J_{45} & J_{46} \\
    0 & 1 & 0 & J_{51} & J_{55} & J_{56} \\
    0 & 0 & 0 & J_{64} & J_{65} & J_{66}
\end{bmatrix}
$$

where

$$J_{24} = 5.2779l_z - yl_x + xl_y$$

$$J_{25} = 1.4142l_z + xl_x - xl_y$$

$$J_{26} = -1.4142l_y - 5.2779l_x - xl_y + yl_x$$

$$J_{34} = -1.4142l_z - 0.5176m_z$$

$$- ym_z + zm_y$$

$$J_{35} = -1.9319l_z + xm_z - zm_y$$

$$J_{36} = 1.9319l_y + 1.4142l_x$$

$$+ 0.5176m_x - xm_y + ym_x$$

$$J_{44} = 0.9659l_z$$

$$J_{45} = -0.2588l_z$$

$$J_{46} = 0.2588l_y - 0.9659l_x$$

$$J_{54} = 0.2588l_z - 1.9319m_z$$

$$J_{55} = -0.9659l_z$$

$$J_{56} = 0.9659l_y - 0.2588l_x + 1.9319m_x$$

$$J_{64} = l_z$$

$$J_{65} = m_z$$

$$J_{66} = -l_x - m_y$$

By inspection of matrix (42), we can get two singular configurations:

$$\mathbf{T}_1 = \begin{bmatrix}
    1 & 0 & 0 & 0 & 0 & 0 \\
    0 & 0.3477 & 0.9376 & 0 & 0 & 0 \\
    0 & -0.9376 & 0.3477 & 3 & 0 & 0
\end{bmatrix}
$$

$$\mathbf{T}_2 = \begin{bmatrix}
    0.5755 & 0 & -0.8178 & 0 & 0 & 0 \\
    0.8178 & 0 & 0.5755 & 3.5 & 0 & 0 \\
    0 & 0 & 0 & 1 & 0 & 0
\end{bmatrix}
$$

The Jacobian matrices corresponding to the two singular positions are (here we omit the 3rd and 4th digits after decimal point):

$$\mathbf{J}_1 = \begin{bmatrix}
    0 & 0 & 3.00 & 0 & 0 & 0 \\
    1.00 & 0 & 0 & 0 & -3.00 & -5.27 \\
    0 & 0.34 & 0 & -0.93 & 1.52 & 0 \\
    1.00 & 0 & 0 & 0 & 0 & -0.96 \\
    0 & 1.00 & 0 & 1.81 & 0 & -0.25 \\
    0 & 0 & 0 & 0 & -0.93 & -1.34
\end{bmatrix}
$$

$$\mathbf{J}_2 = \begin{bmatrix}
    0 & 0 & 0 & 0 & 0 & 0 \\
    0 & 0 & 3.00 & 0 & 0 & 0 \\
    1.00 & 0 & 0 & 0 & -3.00 & -5.27 \\
    0 & 0.34 & 0 & -0.93 & 1.52 & 0 \\
    1.00 & 0 & 0 & 0 & 0 & -0.96 \\
    0 & 1.00 & 0 & 1.81 & 0 & -0.25 \\
    0 & 0 & 0 & 0 & -0.93 & -1.34
\end{bmatrix}
$$
\[
J_2 = \begin{bmatrix}
0 & 0 & 3.50 & 0 & 0 & 0 \\
0.57 & 0 & 0.81 & 4.31 & -0.85 & -3.03 \\
0 & 1.00 & 0 & 2.34 & -1.57 & 0.81 \\
1.00 & 0 & 0 & 0.78 & -0.21 & -0.55 \\
0 & 1.00 & 0 & 0.21 & -0.78 & -0.14 \\
0 & 0 & 0 & 0.81 & 0 & -1.57 \\
\end{bmatrix}
\]

We have developed a continuation method for the forward displacement analysis of this Stewart platform. Given the six leg lengths, we can get all the configurations corresponding to the set of lengths.

For a given singular position, using Eq. (17), we can calculate the six lengths of the Stewart platform. Then using obtained set of lengths, we solve the direct position kinematics. It is found that there is always one pair of solution that are almost the same. Therefore the theorem 1 has been verified further by examples. This means one pair configuration meet at the singular position.

6 Conclusions

This paper deals with the singularity analysis of general Stewart platforms. First, based on the kinematics equations developed by Zhang and Song, we present a method to formulate the Jacobian matrix. Then the geometrical characteristics of the singular configuration is given. It is proved that at the singular position, one pair of configuration with the same lengths meet. Using the new Jacobian matrix formulation method, the Jacobian matrix of a practical Stewart platform is given. Taking the determinant of the Jacobian matrix, several singular configurations are given. By using the continuation method, it is verified that these singular configurations correspond to the repeated solutions of the direct kinematics equations.

References


