Research on Phase Difference Rate of Change for Passive Location with Variant Posture of the Observer

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Abstract: - Technology of passive location has broad prospects in applications. In this paper, phase difference rate of change for single observer-passive location is introduced based on existing methods. We can obtain the direction of the target with phase difference information of two orthogonal interferometers on the observer and the radial distance with corresponding phase difference rate of change. Then the target can be located with high speed and precision. When flying posture of the observer is changed， the target can be located directly in the original coordinate system according to measurement data. Related locating expressions are given. Simulations show that this method is effective.

Key-Words: - phase difference, rate of change, single observer, passive location, variant posture, direction, radial distance

1 Introduction

Passive location technology is prior to active location technology in far distance, secret receiving and low probability of detection. The existing and working abilities of corresponding systems in complex electromagnetic environment can be improved with these advantages. So the passive location technology has broad prospects in ocean saving, location and tracking of weapon control systems, electronic scouting, measurement and control of spacecrafts, satellite location. etc. Single observer passive location technology works with only one observing platform. Its superiorities are few equipments and low cost. But this technology is difficult to realize because the information obtained is insufficient compared to multi-observer location technology. Traditional methods of single observer-passive location technology include bearing only (BO), time only (TO), Doppler only (DO), bearing/time (BT) and bearing/Doppler (BD).The shortcomings of these methods are long location time, low precision, high sensitivity on direction measurement error and high demands impersonally on measurement equipments.

A new method for single observer passive location - phase difference rate of change method is introduced in this paper on the applying background of location to remote immovable ground targets before launching weapons. In this method, targets can be located with high speed and precision through phase difference information received by two orthogonal interferometers on the observer and corresponding rate of change information. When flying posture of the observer is changed，the target can be located directly in the original coordinate system according to measurement data in the condition of variant posture. Related locating expressions are given. Simulations show that this method is effective.

2 Location Principle for Phase Difference Rate of Change with Variant Posture

In this method, the direction and radial distance of targets can be obtained through phase difference information received by two orthogonal interferometers on the observer and corresponding rate of change. Then targets can be located. Detailed demonstrations are as follows:

The curvature of the earth can be neglected when targets are immovable. Suppose the plane observer is flying horizontally along a beeline with a constant velocity and without any posture change. A three-dimensional orthogonal coordinate system is built on a fixed position. OX follows the direction of fuselage axis towards the head. OY points to the direction of the left airfoil. OZ is vertical up to XOY according to the right-hand relation. Location（x_O， y_O， z_O）and velocity

($\dot{x}_O$, $\dot{y}_O$, $\dot{z}_O$) of the observer in this coordinate system can be obtained through navigation equipments. The flying posture of the observer is changed when t=i. Deflexions of the fuselage relative to the original coordinate system are: roll angle-θ_i , pitching angle-η_i , yaw angle-γ_i . Direction of corresponding angle velocity vector is identical with OX, OY and OZ respectively. So a new coordinate system O-X' Y' Z' comes into being with the same setup as O-XYZ. The rotating matrix from O-XYZ to O- X' Y' Z' is:

$$H_i = \begin{bmatrix} \cos\eta\cos\gamma & \cos\theta\sin\gamma+\sin\theta\sin\eta\cos\gamma \\ -\cos\eta\sin\gamma & \cos\theta\cos\gamma-\sin\theta\sin\eta\sin\gamma \\ \sin\eta & -\sin\theta\cos\eta \end{bmatrix}_i \quad (1)$$

It's easy to validate that $H_i^T = H_i^{-1}$. Suppose:

$$P_i = \dot{H}_i \quad (2)$$

Two orthogonal interferometers (three antennas altogether) are equipped on the observer. One antenna is placed on the ventral point of intersection of the fuselage and the head, that is, O'. One antenna is placed towards the tail along the fuselage axis. The third antenna is placed on the left airfoil towards the fuselage. When t=i, the azimuth angle of the target relative to the observer in the new coordinate system O-X' Y' Z' (the fiducial axis is OY' axis) is β_i' , and corresponding pitching angle (the fiducial plane is X' Y' Z' plane) is ε_i'. Suppose the radiating frequency of the target is changeless and direction information of the target obtained by the observer all comes from phase difference information received by two interferometers. In the condition of no phase ambiguity, geometrical connection that two interferometers receive electromagnetic waves from the target is shown in figure 1.

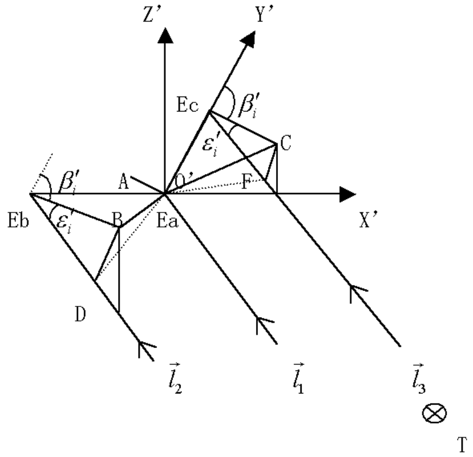


Fig. 1

In this figure, Ea and Eb are two antennas of the interferometer placed on the fuselage axis, and its baseline is d_x. Ea and Ec are two antennas of the interferometer placed on the airfoil axis, and its baseline is d_y . $\vec{l}_1$, $\vec{l}_2$ and $\vec{l}_3$ represent the direction of electromagnetic waves received respectively by Ea, Eb, Ec. Considering the distance between the target and the observer is much farther than d_x and d_y , suppose $\vec{l}_1 // \vec{l}_2 // \vec{l}_3$. EaA, EbB, EcC represent the projections of $\vec{l}_1$, $\vec{l}_2$ and $\vec{l}_3$ on the X' Y' Z' plane, with EaB□EbB, BD□EbD, EaC□EcC and CF□EcF. Then, EaD□EbD, EaF□EcF. So, EbD is the wave route difference between Ea and Eb. EcF is the wave route difference between Ea and Ec according to figure 1,

In RtΔEaBEb,

$$EbB = EaEb * \cos\angle EaEbB = d_x \cos(\beta_i' - 90^0) = d_x \sin\beta_i'$$

In RtΔBDEb,

$$\angle BEbD = \varepsilon_i' \quad , \quad EbD = EbB * \cos\varepsilon_i' = d_x \sin\beta_i' \cos\varepsilon_i' \quad (3)$$

and in RtΔEaCEc,

$$EcC = EaEc * \cos\angle EaEcC = d_y * \cos(180^0 - \beta_i') = -d_y \cos\beta_i'$$

In RtΔCFEc,

$$\angle CEcF = \varepsilon_i' \quad , \quad EcF = EcC * \cos\angle CEcF = -d_y \cos\beta_i' \cos\varepsilon_i' \quad (4)$$

The phase difference of electromagnetic waves received by Ea and Eb is:

$$\phi_{xi}' = \omega_{T*}\Delta t_{x'} = 2\pi f_T * \frac{d_x}{c} \sin\beta_i' \cos\varepsilon_i' = k_x f_T \sin\beta_i' \cos\varepsilon_i' \quad (5)$$

Corresponding rate of change is:

$$\dot{\phi}'_{xi} = k_x f_T (\dot{\beta}'_i \cos\beta'_i \cos\varepsilon'_i - \dot{\varepsilon}'_i \sin\beta'_i \sin\varepsilon'_i) \quad (6)$$

The phase difference of electromagnetic waves received by Ea and Ec is:

$$\phi'_{yi} = \omega_{T*} \Delta t_{y'} = 2\pi f_T * \frac{d_y}{c}(-\cos\beta'_i \cos\varepsilon'_i)$$
$$= -k_y f_T \cos\beta'_i \cos\varepsilon'_i \qquad k_y = \frac{2\pi d_y}{c} \quad (7)$$

Corresponding rate of change is:

$$\dot{\phi}'_{yi} = k_y f_T (\dot{\beta}'_i \sin\beta'_i \cos\varepsilon'_i + \dot{\varepsilon}'_i \cos\beta'_i \sin\varepsilon'_i) \quad (8)$$

Where ω_T is the angle frequency of electromagnetic waves received by the observer, and f_T is the corresponding frequency. $\Delta t_{x'}$ is the arriving time difference between Ea and Eb, $\Delta t_{y'}$ is the arriving time difference between Ea and Ec. C represents the velocity of light. From the four expressions we can obtain:

$$\cos\varepsilon'_i \sin\beta'_i = \phi'_{xi} / k_x f_T ,$$
$$\frac{d(\cos\varepsilon'_i \sin\beta'_i)}{dt} = \dot{\phi}'_{xi} / k_x f_T ,$$
$$\cos\varepsilon'_i \cos\beta'_i = -\phi'_{yi} / k_y f_T ,$$
$$\frac{d(\cos\varepsilon'_i \cos\beta'_i)}{dt} = -\dot{\phi}'_{yi} / k_y f_T ,$$
$$\sin\varepsilon'_i = \frac{\sqrt{k_x^2 k_y^2 f_T^2 - k_y^2 \phi'^2_{xi} - k_x^2 \phi'^2_{yi}}}{k_x k_y f_T} ,$$
$$\frac{d\sin\varepsilon'_i}{dt} = -\frac{k_y^2 \phi'_{xi}\dot{\phi}'_{xi} + k_x^2 \phi'_{yi}\dot{\phi}'_{yi}}{k_x k_y f_T \sqrt{k_x^2 k_y^2 f_T^2 - k_y^2 \phi'^2_{xi} - k_x^2 \phi'^2_{yi}}}$$

Suppose the position of the observer in O-XYZ is $(x_{Oi},\ y_{Oi},\ z_{Oi})$ with velocity $(\dot{x}_{Oi},\ \dot{y}_{Oi},\ \dot{z}_{Oi})$ when t=i. Its position in O-X' Y' Z' is $(x'_{Oi},\ y'_{Oi},\ z'_{Oi})$ with velocity $(\dot{x}'_{Oi},\ \dot{y}'_{Oi},\ \dot{z}'_{Oi})$. The position of the target in O-XYZ is $(x_{Ti},\ y_{Ti},\ z_{Ti})$ and radial distance r_i from the observer. Its position in O-X' Y' Z' is $(x'_{Ti},\ y'_{Ti},\ z'_{Ti})$ and radial distance r'_i from the observer. So we can obtain:

$$\begin{bmatrix} x'_{Oi} \\ y'_{Oi} \\ z'_{Oi} \end{bmatrix} = H_i * \begin{bmatrix} x_{Oi} \\ y_{Oi} \\ z_{Oi} \end{bmatrix} \quad (9)$$

$$\begin{bmatrix} x'_{Ti} \\ y'_{Ti} \\ z'_{Ti} \end{bmatrix} = \begin{bmatrix} x'_{Oi} + r'_i \cos\varepsilon'_i \sin\beta'_i \\ y'_{Oi} + r'_i \cos\varepsilon'_i \cos\beta'_i \\ z'_{Oi} - r'_i \sin\varepsilon'_i \end{bmatrix} = H_i * \begin{bmatrix} x_{Ti} \\ y_{Ti} \\ z_{Ti} \end{bmatrix}$$
$$= H_i * \begin{bmatrix} x_{Oi} + r_i \cos\varepsilon_i \sin\beta_i \\ y_{Oi} + r_i \cos\varepsilon_i \cos\beta_i \\ z_{Oi} - r_i \sin\varepsilon_i \end{bmatrix} \quad (10)$$

Then:

$$\begin{bmatrix} r'_i \cos\varepsilon'_i \sin\beta'_i \\ r'_i \cos\varepsilon'_i \cos\beta'_i \\ -r'_i \sin\varepsilon'_i \end{bmatrix} = H_i * \begin{bmatrix} r_i \cos\varepsilon_i \sin\beta_i \\ r_i \cos\varepsilon_i \cos\beta_i \\ -r_i \sin\varepsilon_i \end{bmatrix}$$

Considering the radial distance between the target and the observer is not changed with rotating of the coordinate system, that is $r_i = r'_i$, above expression can be simplified:

$$\begin{bmatrix} \cos\varepsilon_i \sin\beta_i \\ \cos\varepsilon_i \cos\beta_i \\ -\sin\varepsilon_i \end{bmatrix} = H_i^{-1} * \begin{bmatrix} \cos\varepsilon'_i \sin\beta'_i \\ \cos\varepsilon'_i \cos\beta'_i \\ -\sin\varepsilon'_i \end{bmatrix} =$$
$$H_i^T * \begin{bmatrix} \cos\varepsilon'_i \sin\beta'_i \\ \cos\varepsilon'_i \cos\beta'_i \\ -\sin\varepsilon'_i \end{bmatrix} \quad (11)$$

The direction information of the target can be obtained:

$$\sin\varepsilon_i = -h_{13i}\frac{\phi'_{xi}}{k_x f_T} + h_{23i}\frac{\phi'_{yi}}{k_y f_T} +$$
$$h_{33i}\frac{\sqrt{k_x^2 k_y^2 f_T^2 - k_y^2 \phi'^2_{xi} - k_x^2 \phi'^2_{yi}}}{k_x k_y f_T} \quad (12)$$

$$\cos\varepsilon_i = [(h_{11i}\frac{\phi'_{xi}}{k_x f_T} - h_{21i}\frac{\phi'_{yi}}{k_y f_T} -$$
$$h_{31i}\frac{\sqrt{k_x^2 k_y^2 f_T^2 - k_y^2 \phi'^2_{xi} - k_x^2 \phi'^2_{yi}}}{k_x k_y f_T})^2$$
$$+ (h_{12i}\frac{\phi'_{xi}}{k_x f_T} - h_{22i}\frac{\phi'_{yi}}{k_y f_T} -$$
$$h_{32i}\frac{\sqrt{k_x^2 k_y^2 f_T^2 - k_y^2 \phi'^2_{xi} - k_x^2 \phi'^2_{yi}}}{k_x k_y f_T})^2]^{1/2} \quad (13)$$

$$\sin\beta_i = \cos^{-1}\varepsilon_i * (h_{11i}\frac{\phi'_{xi}}{k_x f_T} - h_{21i}\frac{\phi'_{yi}}{k_y f_T} - h_{31i}\frac{\sqrt{k_x^2 k_y^2 f_T^2 - k_y^2\phi'^2_{xi} - k_x^2\phi'^2_{yi}}}{k_x k_y f_T}) \tag{14}$$

$$\cos\beta_i = \cos^{-1}\varepsilon_i * (h_{12i}\frac{\phi'_{xi}}{k_x f_T} - h_{22i}\frac{\phi'_{yi}}{k_y f_T} - h_{32i}\frac{\sqrt{k_x^2 k_y^2 f_T^2 - k_y^2\phi'^2_{xi} - k_x^2\phi'^2_{yi}}}{k_x k_y f_T}) \tag{15}$$

the derivative of (11) is:

$$\begin{bmatrix} \dot\beta_i\cos\varepsilon_i\cos\beta_i - \dot\varepsilon_i\sin\varepsilon_i\sin\beta_i \\ -\dot\beta_i\cos\varepsilon_i\sin\beta_i - \dot\varepsilon_i\sin\varepsilon_i\cos\beta_i \\ -\dot\varepsilon_i\cos\varepsilon_i \end{bmatrix} = P_i^T * \begin{bmatrix} \cos\varepsilon'_i\sin\beta'_i \\ \cos\varepsilon'_i\cos\beta'_i \\ -\sin\varepsilon'_i \end{bmatrix} + H_i^T \begin{bmatrix} \dot\phi'_{xi}/k_x f_T \\ -\dot\phi'_{yi}/k_y f_T \\ -\dot\varepsilon'_i\cos\varepsilon'_i \end{bmatrix} \tag{16}$$

So: $$\dot\beta_i = \frac{Q_{1i}\cos\beta_i - Q_{2i}\sin\beta_i}{\cos\varepsilon_i},$$

$$\dot\varepsilon_i = \frac{-Q_{1i}\sin\beta_i - Q_{2i}\cos\beta_i}{\sin\varepsilon_i} \tag{17}$$

where,

$$Q_{1i} = p_{11i}\frac{\phi'_{xi}}{k_x f_T} - p_{21i}\frac{\phi'_{yi}}{k_y f_T} - p_{31i}\frac{\sqrt{k_x^2 k_y^2 f_T^2 - k_y^2\phi'^2_{xi} - k_x^2\phi'^2_{yi}}}{k_x k_y f_T} + h_{11i}\frac{\dot\phi'_{xi}}{k_x f_T} - h_{21i}\frac{\dot\phi'_{yi}}{k_y f_T} + h_{31i}\frac{k_y^2\phi'_{xi}\dot\phi'_{xi} + k_x^2\phi'_{yi}\dot\phi'_{yi}}{k_x k_y f_T\sqrt{k_x^2 k_y^2 f_T^2 - k_y^2\phi'^2_{xi} - k_x^2\phi'^2_{yi}}} \tag{18}$$

$$Q_{2i} = p_{12i}\frac{\phi'_{xi}}{k_x f_T} - p_{22i}\frac{\phi'_{yi}}{k_y f_T} - p_{32i}\frac{\sqrt{k_x^2 k_y^2 f_T^2 - k_y^2\phi'^2_{xi} - k_x^2\phi'^2_{yi}}}{k_x k_y f_T} + h_{12i}\frac{\dot\phi'_{xi}}{k_x f_T} - h_{22i}\frac{\dot\phi'_{yi}}{k_y f_T} + h_{32i}\frac{k_y^2\phi'_{xi}\dot\phi'_{xi} + k_x^2\phi'_{yi}\dot\phi'_{yi}}{k_x k_y f_T\sqrt{k_x^2 k_y^2 f_T^2 - k_y^2\phi'^2_{xi} - k_x^2\phi'^2_{yi}}} \tag{19}$$

Thus, we have acquired direction and derivative information of the target in O-XYZ according to measurement data with variant posture. In addition, there are some relations in O-XYZ:

$$tg\beta_i = \frac{x_{Ti} - x_{Oi}}{y_{Ti} - y_{Oi}},$$

$$tg\varepsilon_i = \frac{z_{Oi} - z_{Ti}}{\sqrt{(x_{Ti} - x_{Oi})^2 + (y_{Ti} - y_{Oi})^2}}$$

$$\dot\beta_i = \frac{(\dot x_{Ti} - \dot x_{Oi})(y_{Ti} - y_{Oi}) - (x_{Ti} - x_{Oi})(\dot y_{Ti} - \dot y_{Oi})}{(x_{Ti} - x_{Oi})^2 + (y_{Ti} - y_{Oi})^2} = \frac{\dot x_i\cos\beta_i - \dot y_i\sin\beta_i}{r_i\cos\varepsilon_i} \tag{20}$$

$$\dot\varepsilon_i = \frac{-\dot z_i r_{pi} + \dot r_{pi} z_i}{x_i^2 + y_i^2 + z_i^2} = \frac{-\dot z_i\cos\varepsilon_i - (\dot x_i\sin\beta_i + \dot y_i\cos\beta_i)\sin\varepsilon_i}{r_i} \tag{21}$$

so:

$$r_i = \frac{\dot x_i\cos\beta_i - \dot y_i\sin\beta_i}{\dot\beta_i\cos\varepsilon_i} = \frac{-\dot z_i\cos\varepsilon_i - (\dot x_i\sin\beta_i + \dot y_i\cos\beta_i)\sin\varepsilon_i}{\dot\varepsilon_i} \tag{22}$$

Then position of the target in original coordinate system O-XYZ can be calculated:

$$\begin{aligned} x_{Ti} &= x_{Oi} + r_i\cos\varepsilon_i\sin\beta_i \\ &= x_{Oi} + \dot\beta_i^{-1}(\dot x_i\cos\beta_i - \dot y_i\sin\beta_i)\sin\beta_i \\ &= x_{Oi} + \dot\varepsilon_i^{-1}[-\dot z_i\cos\varepsilon_i - \\ &\quad (\dot x_i\sin\beta_i + \dot y_i\cos\beta_i)\sin\varepsilon_i]\cos\varepsilon_i\sin\beta_i \end{aligned} \tag{23}$$

$$\begin{aligned} y_{Ti} &= y_{Oi} + r_i \cos\varepsilon_i \cos\beta_i \\ &= y_{Oi} + \dot{\beta}_i^{-1}(\dot{x}_i \cos\beta_i - \dot{y}_i \sin\beta_i)\cos\beta_i \\ &= y_{Oi} + \dot{\varepsilon}_i^{-1}[-\dot{z}_i \cos\varepsilon_i - \\ &\quad (\dot{x}_i \sin\beta_i + \dot{y}_i \cos\beta_i)\sin\varepsilon_i]\cos\varepsilon_i \cos\beta_i \end{aligned} \tag{24}$$

$$\begin{aligned} z_{Ti} &= z_{Oi} - r_i \sin\varepsilon_i \\ &= z_{Oi} - \dot{\beta}_i^{-1} \cos\varepsilon_i^{-1}(\dot{x}_i \cos\beta_i - \dot{y}_i \sin\beta_i)\sin\varepsilon_i \\ &= z_{Oi} - \dot{\varepsilon}_i^{-1}[-\dot{z}_i \cos\varepsilon_i - (\dot{x}_i \sin\beta_i + \dot{y}_i \cos\beta_i)\sin\varepsilon_i]\sin\varepsilon_i \end{aligned} \tag{25}$$

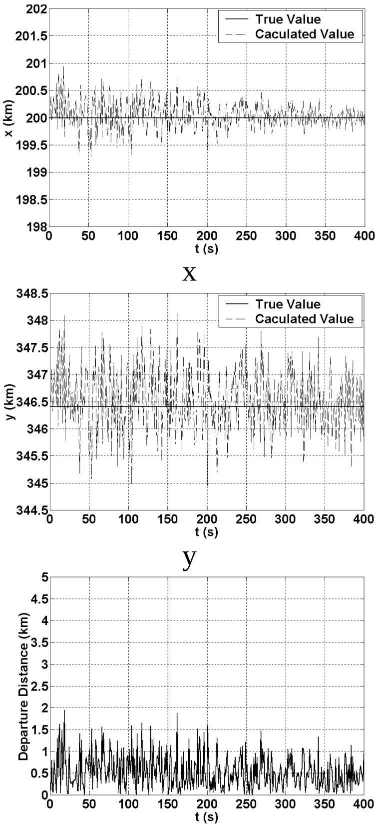
where $\sin\beta_i$, $\cos\beta_i$, $\sin\varepsilon_i$, $\cos\varepsilon_i$, $\dot{\beta}_i$, $\dot{\varepsilon}_i$ are calculated respectively through （14）,（15）,（12）,（13）and（17）.

It is deserving to note that the whole process of location is based on two ideal suppositions. These suppositions include: (1) frequency of electromagnetic waves radiated by the targets is known and changeless, (2) rate information of change for phase differences received by the two interferometers is known. The researching progress for this location method without above-mentioned suppositions will be introduced in subsequent papers.

3 Simulation

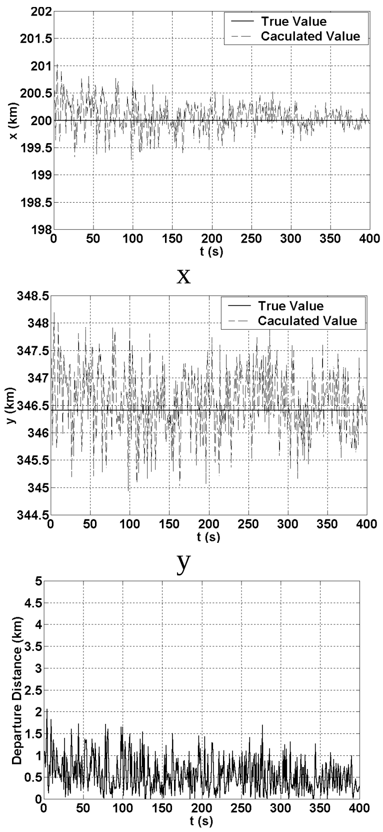
With the restriction of paper length and simulation condition, simulations are given when posture change of the observer is completed instantly. Corresponding measurement data are: $f_T = 3\times10^9 H_Z$, $d_x = 10m$, $d_y = 5m$. Measurement precisions are: $\sigma_\phi = 35mrad$, $\sigma_{\dot{\phi}} = 7.5mrad/s$, $\sigma_V = 0.1m/s$, $\sigma_p = 16m$, $\sigma_{f_T} = 10^4 H_Z$.

Suppose position of the target in O-XYZ is $(200, 200\sqrt{3}, 0)_{km}$ in simulations. And jumping-off point of the observer in O-XYZ is: $(1,1,8)_{km}$ with velocity $H^{-1}*(300,0,0)_{m/s}$ .Simulation outcomes are shown in figure 2.



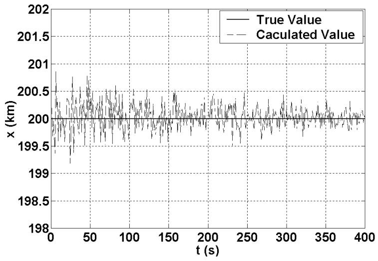
$\sqrt{(x-\hat{x})^2+(y-\hat{y})^2}$

Fig. 2 (a)

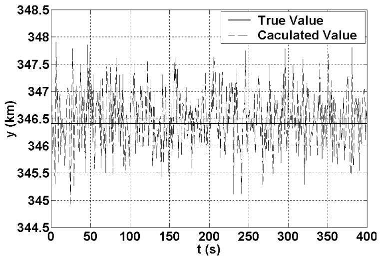


$\sqrt{(x-\hat{x})^2+(y-\hat{y})^2}$

Fig. 2 (b)



x



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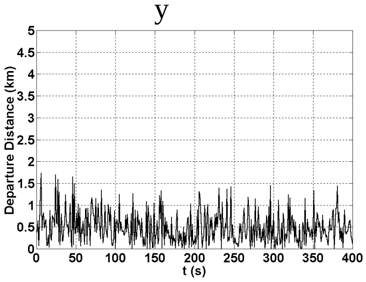

$\sqrt{(x-\hat{x})^2+(y-\hat{y})^2}$

Fig. 2 (c)

Fig. 2 (a) $\theta = 30^o$, $\eta = -5^o$, $\gamma = 0^o$ (corresponding deflexion of the observer: the left airfoil is up and the head is up) (b) $\theta = 0^o$, $\eta = 0^o$, $\gamma = -5^o$ (corresponding deflexion of the observer: yaw towards the right horizontally) (c) $\theta = 30^o$, $\eta = -5^o$, $\gamma = -5^o$ (corresponding deflexion of the observer: the left airfoil is up, the head is up and the observer yaws towards the right).

According to above simulations, the horizontal departure distance is less than 2.2km in given measurement precision. In most cases, the departure distance is less than 1km. And applying needs can be met.

4 Conclusion

In this paper, a new method for single observer passive location - phase difference rate of change method is introduced based on the applying requirement of location to remote immovable ground targets before launching weapons. Principle and some location expressions are given when flying posture of the observer is changed. Simulations show that this method is effective. After further improvement, it has broad prospects in applications with high concealment, speed and precision.

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