A New Dynamic Phenomenon in Nonlinear Circuits: State-Space Analysis of Chaotic Beats

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***Abstract**: –*In this paper a new dynamic phenomenon in nonlinear circuits driven by two sinusoidal signals is investigated. By exploiting state-space analysis, the paper shows that the application of signals with slightly different frequencies enables the phenomenon of chaotic beats to be obtained. Moreover, the power spectral density and the Lyapunov exponents are computed, with the aim of confirming the chaotic nature of the beats. Finally, in order to analyze in detail the beats phenomenon, the case of two sinusoidal signals with equal frequencies is discussed.

Key-Words: - Dynamic phenomenon, nonautonomous circuit, chaotic beats, state-space analysis.

1 Introduction

The study of dynamic phenomena in nonlinear systems is an important research topic. In particular, in recent years several complex behaviors have been observed, including quasi-periodic oscillations, period-adding sequences, subharmonics and torus breakdown to chaos [1]-[6]. Furthermore, intermittent transitions, generation of multi-scroll attractors and synchronization properties have been found in nonlinear circuits [7]-[10].
Referring to nonautonomous systems, in [11] their behavior has been investigated in the presence of two sinusoidal inputs characterized by slightly different frequencies. Note that such behavior has been widely studied in linear systems and has been called "beats" [12]. Namely, when two waves characterized by slightly different frequencies interfere, the frequency of the resulting waveform is the average of the frequencies of the two waves, whereas its amplitude is modulated by an envelope, the frequency of which is the difference between the frequencies of the two waves [12]. By generalizing this concept, in [11] the generation of chaotic beats in *nonlinear systems* with very small nonlinearities has been studied.

Based on these considerations, the aim of this paper is to investigate the phenomenon of beats in *nonlinear circuits*, rather than in nonlinear systems. To this purpose, by exploiting the early results obtained for the circuit proposed in [15], a detailed state-space analysis of the chaotic beats is carried out. The paper is organized as follows. In Section 2, based on the nonlinear circuit illustrated in [15], it is shown that the application of two sinusoidal signals (characterized by large equal amplitudes and slightly different frequencies) enable chaotic beats to be generated. In particular, a detailed state-space analysis of chaotic beats and corresponding envelopes is carried out. Additionally, the power spectral density and the Lyapunov exponents are reported, with the aim of confirming the chaotic nature of the phenomenon. Finally, Section 3 analyzes the behaviour of the proposed circuit driven by two sinusoidal signals with equal frequencies.

2 Generating Chaotic Beats

2.1 Proposed nonlinear circuit

Based on the approach developed in [15], the circuit considered herein (Fig.1) contains two external periodic excitations, a capacitor, an inductor, a linear resistor and a nonlinear element, namely, the Chua's diode.

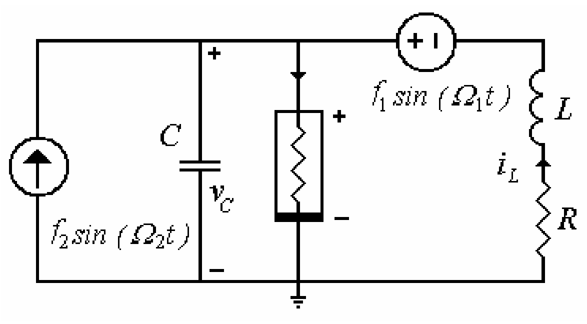


Fig.1 The considered circuit for generating chaotic beats.

The state equations for the voltage v_C across the capacitor C and the current i_L through the inductor L can be written as:

$$\begin{bmatrix} \frac{dv_C}{dt} \\ \frac{di_L}{dt} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{C} \\ -\frac{1}{L} & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} v_C \\ i_L \end{bmatrix} + \begin{bmatrix} -\frac{1}{C} \\ 0 \end{bmatrix} g(v_C) + \begin{bmatrix} \frac{1}{C} & 0 \\ 0 & \frac{1}{L} \end{bmatrix} \begin{bmatrix} f_2 \sin(\Omega_2 t) \\ f_1 \sin(\Omega_1 t) \end{bmatrix} \quad (1)$$

where f_1 and f_2 are the amplitudes of the periodic excitations whereas Ω_1 and Ω_2 are their angular frequencies. The nonlinearity

$$g(v_C) = G_b v_C + (G_a - G_b)(|v_C + B_p| - |v_C - B_p|)/2 \quad (2)$$

is the mathematical representation of the piece-wise linear characteristic of the Chua's diode [13]. Rescaling Eq.s (1)-(2) as $v_C = x_1 B_p$, $i_L = x_2 B_p/R$, $\omega_1 = \Omega_1 CR$, $\omega_2 = \Omega_2 CR$, $t = \tau CR$ and then redefining τ as t, the following set of dimensionless equations are obtained:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\beta & -\beta \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \end{bmatrix} g(x_1) + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} F_2 \sin(\omega_2 t) \\ F_1 \sin(\omega_1 t) \end{bmatrix} \quad (3)$$

where $\beta = R^2 C/L$, $F_1 = f_1 \beta / B_p$ and $F_2 = f_2 \beta / B_p$. Furthermore,

$$g(x_1) = bx_1 + (a - b)(|x_1 + 1| - |x_1 - 1|)/2, \quad (4)$$

where $a = G_a R$ and $b = G_b R$. Note that the dynamics of system (3)-(4) depend on the parameters β, a, b, ω_1, ω_2, F_1 and F_2.

2.2 Generation of chaotic beats

For the present analysis the parameters $a = -1.27$ and $b = -0.68$ are fixed, whereas the remaining parameters β, F_1, F_2, ω_1 and ω_2 have to be properly chosen. Since the aim of the paper is to develop a detailed analysis of the chaotic beats in the considered circuit, several numerical simulations are carried out for different values of the bifurcation parameters β, F_1, F_2 and for slightly different values of the frequencies ω_1 and ω_2. In particular, it is interesting to analyze the circuit behavior for $\beta = 0.680044$, $F_1 = F_2 = 200$, $\omega_1 = 0.49$ and $\omega_2 = 0.50$. To this purpose, Fig. 2 illustrates the time behavior of the state variable x_1, giving a first idea about the occurrence of chaotic beats.

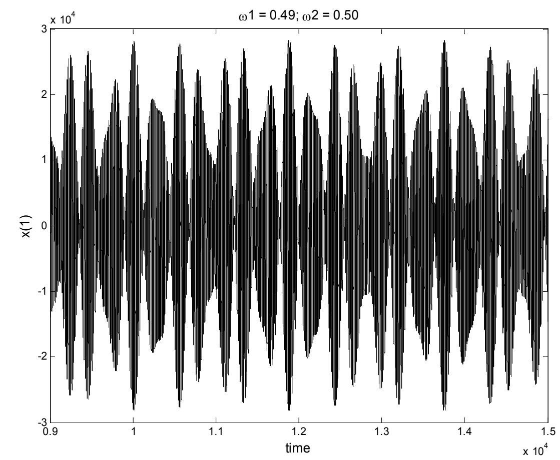


Fig.2 Time behavior of x_1 for $t \in [9000, 15000]$.

Moreover, by computing the power spectral density of the envelope of the signal x_1 (Fig.3), it is possible to confirm the chaotic nature of the beats phenomenon.

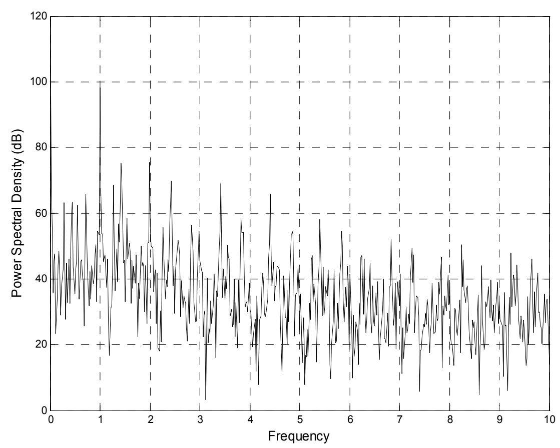


Fig.3 Power spectral density of the envelope of the signal x_1.

Additionally, the Lyapunov exponents of system (3) are calculated. In particular, the 2th-order time-periodic nonautonomous system (3) is converted into a 3th-order autonomous system by appending an extra state variable [14]. Thus, since sinusoidal forcing terms in Eq.(3) are treated as a parameter, a null exponent is obtained. Namely, the Lyapunov exponents are:

$$\lambda_1 = 1.8893\text{e}{-}005, \quad \lambda_2 = 0.00000,$$
$$\lambda_3 = -2.3767\text{e}{-}005. \quad (5)$$

Notice that the presence of one positive Lyapunov exponent further confirms the chaotic dynamics of the considered circuit.

2.3 State-space analysis of chaotic beats

In order to analyze in detail the chaotic amplitude modulation of the beats, several phase portraits are carried out in the (x_1, x_2)-state space at different time units. Such instants have been chosen by considering the most significant amplitude changes in the time behavior of the state variable x_1 (see Fig. 4 and consider instants t = 105, 307, 440, 545, 655, 777). The corresponding phase portraits are reported in Fig. 5(a)-(f).

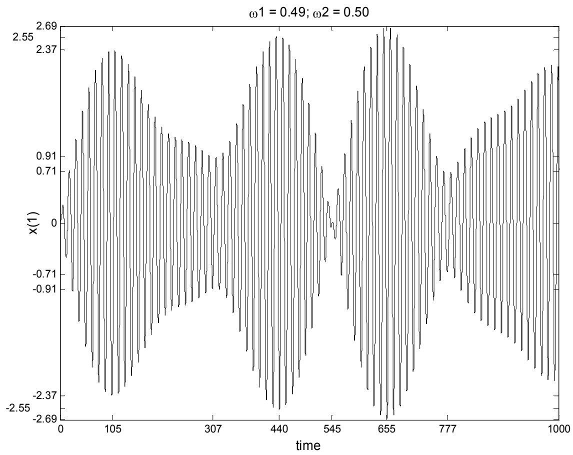
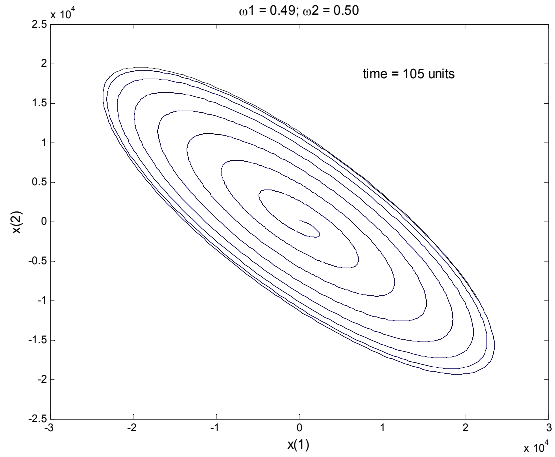
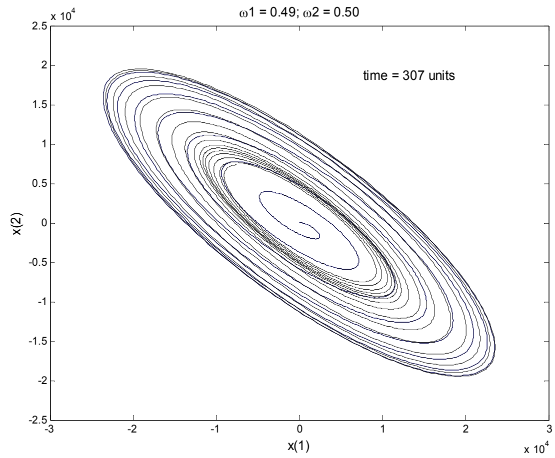


Fig.4 Time behavior of the state variable x_1 for $t \in [0, 1000]$: the amplitude reaches relative maximum or minimum values for t = 105, 307, 440, 545, 655, 777.

In particular, Fig.4 and Fig.5(a) show that the circuit dynamics start from the origin and expand until x_1 approximately reaches the values ±23700. Then, Fig.4 and Fig.5(b) illustrate that the trajectory of variable x_1 shrinks back until the values ±9100 are approximately reached. In Fig.4 and Fig.5(c) the dynamics expand again until they reach the values ±25500, whereas in Fig.4 and Fig.5(d) the trajectory goes toward the origin. Successively, in Fig.4 and Fig.5(e) the values ±26900 are approximately reached. Finally, Fig.4 and Fig.5(f) show that the dynamics contract again, until the values ±7100 are reached.



(a)



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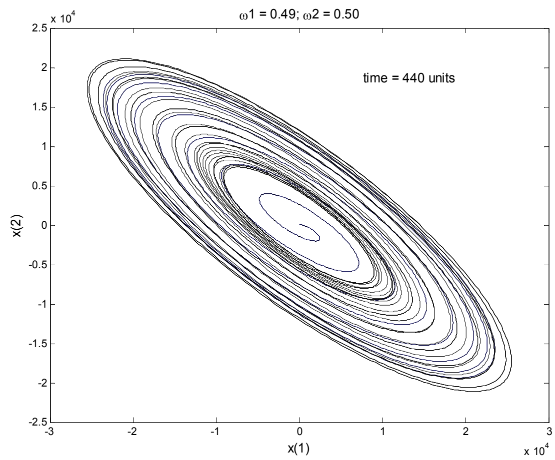
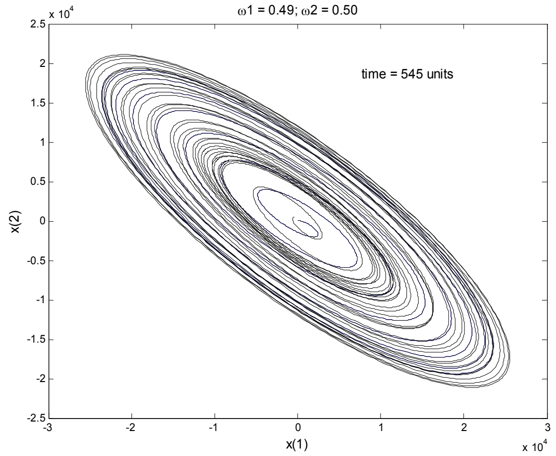
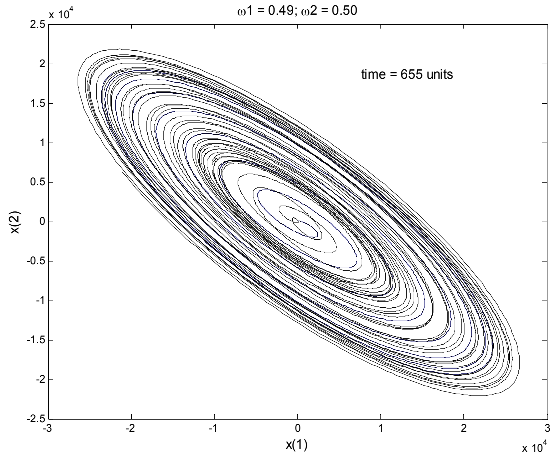


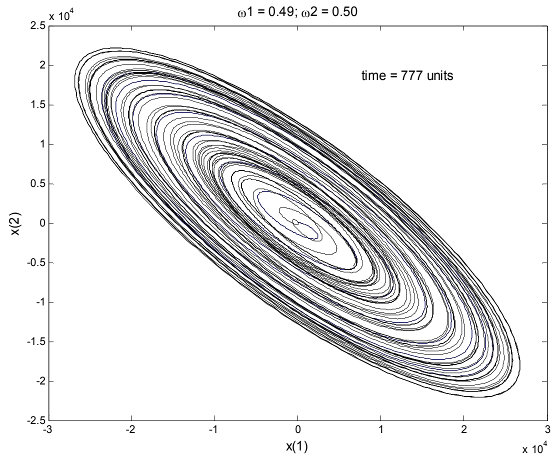
(c)



(d)



(e)



(f)

Fig.5 Phase portraits in the (x_1, x_2)-state space. (a): time t = 105; (b): time t = 307; (c): time t = 440; (d): time t = 545; (e): time t = 655; (f): time t = 777.

These expanding and contracting behaviors go on *chaotically* for increasing times. An example of the attractor, obtained for large time, is reported in Fig.6, where the maximum/minimum values of x_1 are ±28400.

Based on the state-space analysis developed herein, it can be concluded that for the parameter values given by:

$$\beta = 0.680044,\ a = -1.27,\ b = -0.68, \tag{6}$$

$$F_1 = F_2 = 200, \tag{7}$$

$$\omega_1 = 0.49,\ \omega_2 = 0.50, \tag{8}$$

the suggested circuit is able to generate the new phenomenon of chaotic beats.

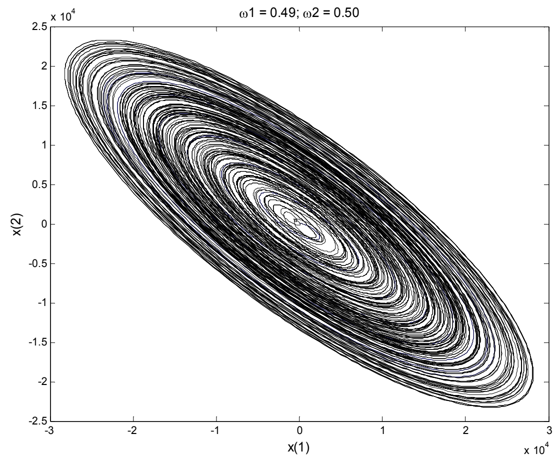


Fig.6 Phase portraits of the attractor in the (x_1, x_2)-state space for large t.

3 Generating Periodic Beats

This Section try to answer to the following question: are the chaotic beats preserved when equal frequencies $\omega_1 = \omega_2$ are chosen in the considered nonlinear circuit? To this purpose, the dynamics of system (3) are analyzed for the parameter values (6)-(7), whereas equal frequencies $\omega_1 = \omega_2 = 0.49$ are chosen. The resulting time waveforms of the state variable x_1 are reported in Figure 7(a)-(b) for different resolutions of the time scale. Differently from the previous Section, Figure 7 clearly highlights that in this case the expanding and contracting behavior goes on *periodically* for increasing times. More precisely, Figure 7(a) highlights the presence of beats due to a periodic envelope, whereas Figure 7(b) reveals in the signal x_1 also the presence of a fundamental frequency. Based on the analysis developed through this Section, it can be concluded that for the parameter values

(6)-(7) and equal frequencies

$$\omega_1 = \omega_2 = 0.49, \qquad (9)$$

the chaotic beats are not preserved. However, notice that in this case the considered nonlinear circuit exhibit *periodic beats*, similarly to the phenomenon obtained in linear systems.

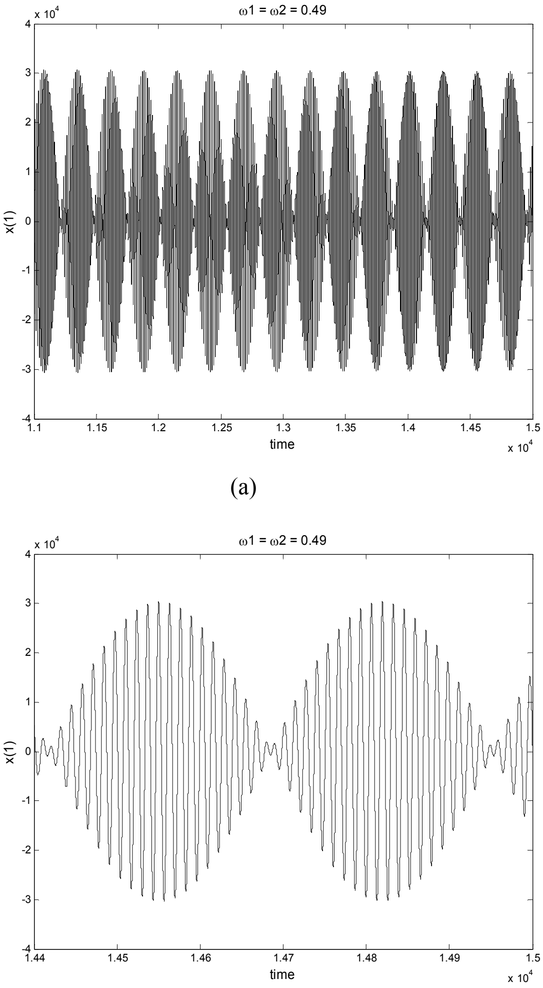


Fig.7 Behaviors of the state variable x_1 for different resolutions of the time scale; (a): $t \in [11000, 15000]$; (b): $t \in [14400, 15000]$.

4 Conclusions

In this paper a new dynamic phenomenon in nonlinear circuits driven by two sinusoidal signals has been investigated. In particular, state-space analysis has shown that the application of signals with slightly different frequencies enables *chaotic beats* in nonautonomous circuits to be obtained. Finally, it has been shown that the application of signals with equal frequencies enables *periodic beats* to be obtained.

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