A Shunt Connected DC-Motor Feedback Linearization Technique with On-Line Parameters Estimation

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Abstract

The design of system controllers is usually accomplished based on nominal operating conditions. In particular, system parameters adopted for controller design are those of nominal values and are assumed to be constant for all practical purposes. However, the nominal values of system parameters might vary and have different values due to aging, computer truncation, and environmental changes such as temperature. In this manuscript, feedback linearization technique is used to design state controllers for widely used DC-machines such as, series, shunt and separately excited motors. An on-line parameter estimation technique is incorporated in the controller design to eliminate the effect of parameter changes on the performance of the designed systems. Simulations are used to demonstrate the effectiveness of the on-line parameter estimation technique when parameter variations are to be considered.

1. Introduction

Feedback linearization has gained more attention in the last two decades [1, 2, 3]. In fact, the technique consists of a state and input coordinate transformation followed by a state feedback to eliminate the nonlinearty. DC machines are inherently nonlinear especially when armature reaction and saturation effects are considered. Those machines are often dealt with using linear techniques after linearizing their models in the vicinity of their operating conditions. However, those machines are shown to be feedback linearizable and different authors demonstrated the efficiency of this new technique and its superiority over existing conventional ones [1, 2].

While feedback linearization controllers seem to be very efficient, their implementation needs a perfect knowledge of the model, in order to perfectly eliminate their nonlinearties. In fact, this perfect knowledge requires the exact values of system parameters, which is not often possible. System parameters are, in general, subjected to variations due to component aging, computer truncations, and changes in environmental conditions such as temperature [4]. This problem can be alleviated using a recently proposed fuzzy logic controllers [5]. This manuscript attempts to on-line estimate system parameters that are considered unknown but constant or very slowly time varying. On-line estimation technique is proposed and the feedback controller is constantly fed by the updated values of the parameters allowing a perfect cancellation of the nonlinearities.

2. Shunt Connected DC-Motor Modeling

DC shunt motors have a nonlinear mathematical model even when saturation and armature reaction effects are neglected. A two dimensional dynamical nonlinear model, with the armature voltage and the load torque considered as inputs, was proposed in [1] and is adopted in this manuscript for design and simulation

$$\frac{di_f}{dt} = k_1 i_f + k_4 V \tag{1}$$

$$\frac{d\omega}{dt} = k_2 i_f^2 \omega + k_3 \omega + k_5 i_f V + k_6 T_l \tag{2}$$

where $k_1 = -R_f / L_f$, $k_2 = -k_m^2 /(R_a J)$, $k_3 = -F / J$, $k_4 = 1/ L_f$, $k_5 = k_m / J$, i_f is the field current, ω is the angular velocity, R_f and R_a are the field and armature resistances, respectively. L_f is the field inductance, k_m is the coupling coefficient, F is the friction coefficient, J is the rotor and load inertia constant, V is the input DC voltage, and T_l is mechanical load torque.

3. Feedback Linearization

Feedback linearization technique is used to transform the nonlinear system differential equations into a control canonical form, which is then followed by a linearizing feedback. The feedback linearization conditions are stated and a brief description is given below. More on the feedback linearization technique can be found in [1,2,3]. Lie derivative, which is defined as vector derivative with respect to another vector, of $g \in R^n$ with respect to $f \in R^n$ is given by:

$$[f,g] = ad_f g = \frac{\partial g}{\partial x} f - \frac{\partial f}{\partial x} g \qquad (3)$$

with $\frac{\partial g}{\partial x}$ and $\frac{\partial f}{\partial x}$ are the Jacobean of g and f, respectively. Let's define

$$ad_f^k g = [f, ad_f^{k-1} g] \qquad (4)$$

and

$$ad_f^o g = g \qquad (5)$$

also if h is a scalar field and f is a vector field, then the Lie derivative of h with respect to f is

$$< dh, f >= \frac{\partial h}{\partial x_1} f_1 + \ldots + \frac{\partial h}{\partial x_n} f_n \qquad (6)$$

The two necessary and sufficient conditions for state feedback linearization are:

1. All vectors of the vector field $[g, ad_f g, \ldots, ad_f^{n-1} g]$ are linearly independent which is equivalent to the controllability condition in linear systems.
2. The Lie bracket of any two vectors of the set $U = [g, ad_f g, \ldots, ad_f^{n-1} g]$ is in the set U itself, this second condition is known as involutivity of the set U.

In the case of a multi-input nonlinear system, g is substituted by:

$$G = [g_1, g_2, \ldots, g_m] \qquad (7)$$

and

$$U = [u_1, u_2, \ldots, u_m] \qquad (8)$$

The two mentioned conditions remain the same except that g is substituted by G in the two above conditions

$$rank[g_1, .., g_m, ad_f g_1, .., ad_f g_m, \ldots ad_f^{n-1} g_m] = n$$

$$U = [g_1, \ldots, g_m, ad_f g_1, \ldots, ad_f g_m, ad_f^{n-2} g_1, ..., ad_f^{n-2} g_m]$$

is involutive

If these two conditions are verified, then one can find a transformation such that

$$T(x) = [T_1(x) .. \ T_n(x)]^T = [z_1 \ \ z_2 \ldots.. \ z_n]^T = Z \qquad (9)$$

with

$$< dT_i, f >= T_{i+1}$$
$$i = 1,2,\ldots, n-1 \qquad (10)$$

and

$$Z = T = \begin{pmatrix} T_1 \\ < dT_1, f > \\ < d < dT_1, f >, f > \\ . \\ . \\ . \end{pmatrix} \qquad (11)$$

Which is already in a nonlinear controller canonical form. The control input appears only in the last equation and is used to remove the nonlinearty. Once the nonlinearty is removed, the resulting system can than be dealt with using any linear technique.

4. On-Line Estimation of System Parameters

Marino et al.[4] proposed an experimentally convergent estimator of the rotor resistance of a three phase induction motor. A similar estimator is developed in the following, to estimate the armature and field resistances of the DC shunt motor. Those parameters are chosen, because of their inherent tendency to change due to variation of their temperature. The estimation procedure stands by building an observer model for armature and field resistance estimation that can be written as:

$$\frac{d\hat{i}_f}{dt} = \hat{k}_1 \hat{i}_f + k_4 V + k(i_f - \hat{i}_f) + v_f \quad (12)$$

$$\frac{d\hat{\omega}}{dt} = \hat{k}_2 i_f^2 \hat{\omega} + k_3 \hat{\omega} + k_5 i_f V + k_6 T_l \quad (13)$$

where $\hat{i}_f$, $\hat{\omega}$, $\hat{k}_1$ and $\hat{k}_2$ are the estimate of i_f, ω, k_1 and k_2, respectively. k is an arbitrary positive constant and v_f is an additional signal to be designed latter on.

The error signals are:

$$\tilde{i}_f = i_f - \hat{i}_f \quad (14)$$

$$\tilde{\omega} = \omega - \hat{\omega} \quad (15)$$

$$\tilde{k}_1 = k_1 - \hat{k}_1 \quad (16)$$

$$\tilde{k}_2 = k_2 - \hat{k}_2 \quad (17)$$

Then, the error dynamics are:

$$\frac{d\tilde{i}_f}{dt} = \frac{di_f}{dt} - \frac{d\hat{i}_f}{dt} = \tilde{k}_1 i_f - k\tilde{i}_f - v_f \quad (18)$$

$$\frac{d\tilde{\omega}}{dt} = k_2 i_f^2 \tilde{\omega} + \tilde{k}_2 i_f^2 \hat{\omega} + k_3 \tilde{\omega} \quad (19)$$

To compensate the delay introduced by current measurements, the following filter has been designed as:

$$\frac{d\xi_f}{dt} = -k\tilde{i}_f - v_f \quad (20)$$

with

$$\eta_f = \xi_f - \tilde{i}_f$$

and k is assumed to be a positive number.

Let $v_f = \tilde{k}_1 \eta_f$, then Eq.(18) can be written as

$$\frac{d\tilde{i}_f}{dt} = \tilde{k}_1 i_f - k\tilde{i}_f - \tilde{k}_1 \eta_f \quad (21)$$

Defining the following Lyapunov function as:

$$V = \frac{1}{2}(\tilde{i}_f^2 + \tilde{\omega}^2 + \frac{1}{\gamma_1}\tilde{k}_1^2 + \frac{1}{\gamma_2}\tilde{k}_2^2),$$ where

γ_1 and γ_2 are positive numbers.

The time derivative of the Lyapunov function is:

$$\frac{dV}{dt} = -k\tilde{i}_f^2 + k_2 i_f^2 \tilde{\omega}^2 + k_3 \tilde{\omega}^2 + \tilde{k}_1[\frac{1}{\gamma_1}\frac{d\tilde{k}_1}{dt} - \eta_f \tilde{i}_f + i_f \tilde{i}_f] + \tilde{k}_2[\frac{1}{\gamma_2}\frac{d\tilde{k}_2}{dt} + i_f^2 \hat{\omega}\tilde{\omega}] \quad (22)$$

So, to guarantee the stability of the observer system, the following adaptation laws are defined:

$$\frac{d\hat{k}_1}{dt} = -\gamma_1 \tilde{i}_f [\eta_f - i_f] \quad (23)$$

$$\frac{d\hat{k}_2}{dt} = \gamma_2 i_f^2 \hat{\omega}\tilde{\omega} \quad (24)$$

and hence,

$$\frac{dV}{dt} = -k\tilde{i}_f^2 + k_2 i_f^2 \omega^2 + k_3 \tilde{\omega}^2 \quad (25)$$

Since $k_2 = -k_m^2/(R_a J)$ and $k_3 = -F/J$ are always negative from practical point of view, Eq.(25) is negative definite.

As a result, Eqs.(16), (17), (25), (26) and (27) are the dynamics of the armature and field resistance estimators of the DC-shunt motor.

V. State Feedback Linearization with On-Line Estimator

The controllability and involutivity conditions, Eqs. (11) and (12), of the proposed mathematical models are verified [1], and consequentely, a state transformation $Z = [T_1(x) \;\; T_2(x)]^T$ can be found

$$\begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \end{pmatrix} = \begin{pmatrix} z_2 \\ \psi(z) \end{pmatrix} + \begin{pmatrix} 0 \\ \beta(z) \end{pmatrix} U \quad (26)$$

where $\psi(z)$ represents a function containing the nonlinearity which is fed by the estimated states and values of the system. Hence, by choosing the nonlinear control law

$$U = \frac{(V_{ref} - \psi(z))}{\beta(z)} \quad (27)$$

one completely removes the nonlinearity, and the nonlinear controller can then be written as:

$$U = \frac{V_{ref} - [\alpha_1 f_1 + \alpha_2 f_2 + f_3]}{f_4} \tag{28}$$

where f_1, f_2, f_3 and f_4 are functions of the states and parameters of the system. α_1 and α_2 are design parameters to be adjusted by imposing on the resulting linear system to achieve a certain performance, such as eigenvalue assignment. Applying the above technique to the DC shunt motor, one obtains

$$f_1 = -k_4\omega + 0.5 k_5 i_f^2 \tag{29}$$

$$f_2 = \hat{k}_1 k_5 i_f^2 - \hat{k}_2 k_4 i_f^2 \omega - \hat{k}_2 k_3 \omega \tag{30}$$

$$f_3 = 2\hat{k}_1^2 k_5 i_f^2 - 2\hat{k}_2 k_4 (\hat{k}_1 + k_3) i_f^2 \omega - \hat{k}_2^2 k_4 (1 + i_f^4)\omega \tag{31}$$

$$f_4 = (2\hat{k}_2 k_4 \omega + k_4 k_5 (k_3 - 2\hat{k}_1)) i_f + \hat{k}_2 k_4 k_5 i_f^3 \tag{32}$$

5. Numerical Results and Discussions

VI.1 Parameter Estimator

In the following, the numerical values of the DC series and shunt connected motors are those of practical systems adopted from [3,6]. In a first set of simulation, the estimator was simulated for an initial change in the field and armature resistances of about 50% higher than their nominal values, and the results shown in Fig.1 demonstrate the perfect convergence of the field and armature resistances estimation toward their nominal values. For simulation purposes, k was chosen to be 40, γ_1 and γ_2 were chosen to be 250 and 0.00005, respectively.

VI.2 Closed Loop Responses with and without Estimators when the System is Subjected to Parameter Variations

The nonlinear controller is designed such that the closed loop eigenvalues are placed at –1 and –3, respectively. Moreover, it is desired to demonstrate the effectiveness of the parameter estimation technique proposed in this manuscript and hence, to evaluate the overall performance with on-line parameter estimation. In a first set of simulations, a 20% increase in both armature and field resistances is assumed in the case of shunt connected DC motor, and both the actual output (with the disturbed resistance values) and that of the uncompensated system (without on-line estimation) are plotted and shown in Fig. 2. It should be noted that the deviations in the desired speed and field current are due to the fact that the estimator is off and hence, the feedback controller parameter values are not updated. However, when the on-line value estimator output is fed to the controller, the rotor speed and that of the field current converges toward the desired values. Fig. 3 demonstrates the effectiveness of the on-line parameter estimation proposed earlier in this manuscript.

6. Conclusions

Most Feedback techniques design is based on nominal values of system parameters. While those parameters are assumed constants for all practical purposes, their values might not be exactly known. In particular, DC machines parameters might have values different than those used in the controller design process. Among those parameters are armature and field resistances, friction coefficient, moment of inertia and many others. Differences can, in many cases, be related to component aging, computer truncation, and heat dissipation. The performance of designed feedback controllers might inherently be affected by those undesired variations in system parameters.

The effect of parameter changes and the proposed parameters estimation algorithm that can alleviate this problem are discussed and analyzed. An on-line parameter estimator output can be used to update the parameters values in the controller dynamics and hence, results in better performance. Feedback linearization, a technique is based on exact knowledge of parameters values in order to perfectly eliminate the undesired nonlinearties, is used to demonstrate the superior performance of the on-line updated feedback controllers over currently proposed ones.

Appendix

Numerical Parameters:

The numerical values for the shunt connected DC motors used in this manuscript, for simulation purposes, are adopted from [1]. Those numerical values are cited below for references:

$R_f = 240\,\Omega$, $L_f = 120\,H$, $R_a = 0.6\,\Omega$,
$k_m = 0.98838\,N.m/Amp.Volt$,
$J = 1\,Kg.m^2$,
$F = 0$, $V = 240\,Volt$, $T_l = 29.5\,N.m$.

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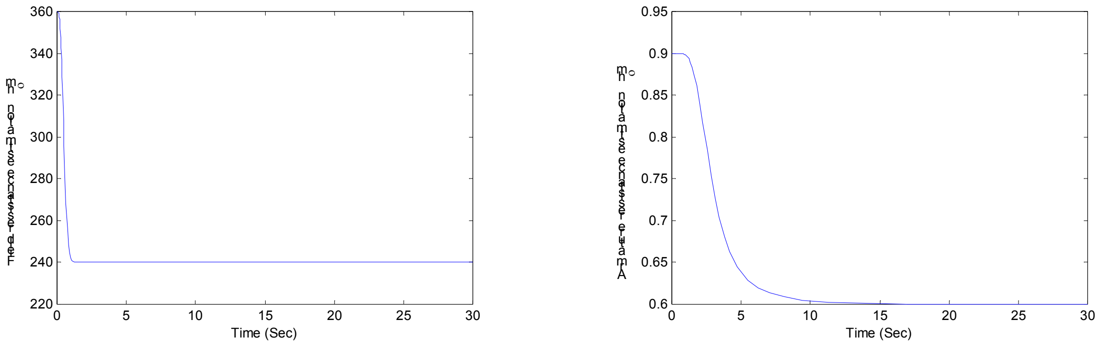


(a) (b)

Figure 1 Shunt motor field and armature resistances estimation
a) Field resistance estimation with initial value 150% above its nominal value
b) Armature resistance estimation with initial value 150% above its nominal value

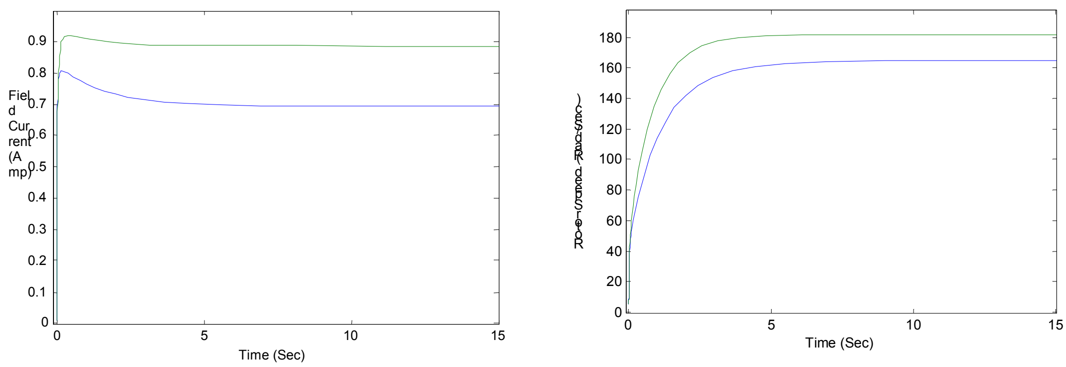
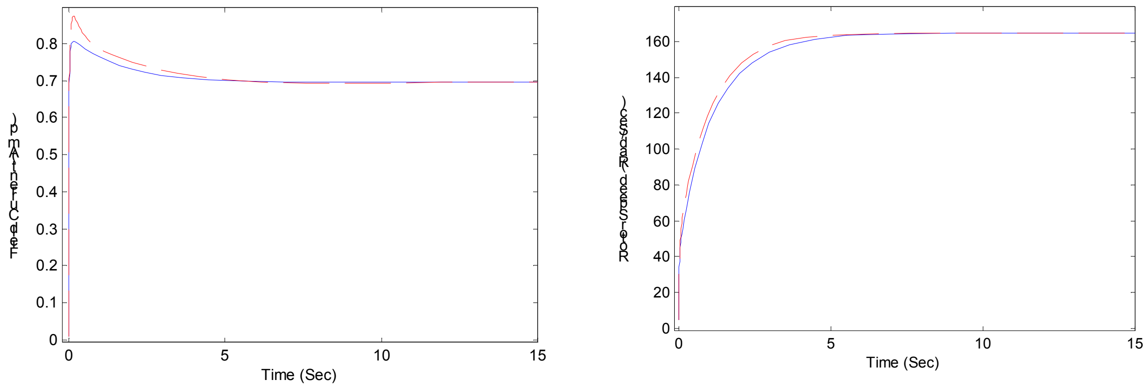


Figure 2 Deviation of the field current and motor angula velocity due to Parameters deviations
a) Field current deviation, from nominal, when the on-line estimation is off

b) Motor angular velocity deviation, from nominal, when the on-line estimation is off



(a) (b)

Figure 3 Feedback linearization with on-line estimation
a) The field current convergence toward the desired one when the on-line estimator is on
b)Rotor angular velocity converges toward the desired one when the on-line estimator is on.