Robust PID Control Based on Partial Knowledge about Time-delay Systems

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Abstract: - This paper presents a design method of robust PID controller with two degrees of freedom (2DOF PID controller) for SISO plants with a time-delay and parametric uncertainty based on the partial knowledge about the plant models. It is assumed that the adjustable parameters of the 2DOF PID controller are chosen so as to minimize the two performance indices for step reference and disturbance responses which maximized by the plant parameters belonging to a given bounded set. Thus the design problem is formulated as a multi-objective minimax optimization problem, which is solved by a optimaization tool. A novel feature of the present paper lies in the exact robust stability check for time-delay systems without using approximant. Numerical examples show the effectiveness of the present approach.

Key-Words: - Partial knowledge about the plant models, 2DOF(2 degrees of freedom) PID controller, Minimax optimization, Time-delay systems, Robust stability, Zero exclusion principal, Edge theorem

1 Introduction

There has been a renewed interest in proportional-integral-derivative (PID) controllers, since it has become easier to implement PID-based algorithms due to the recent development of digital process controllers. In fact, over 90% of industrial control problems are solved by PID controllers (or by their variants) in spite of the simple structures [1]. Many tuning methods of PID controller have been developed consequently [1, 2]. Since industrial plants are too complex to obtain precise dynamics, the PID controllers must be robust for a wide range of operating conditions. There have been developed some works for robust PID tunings methods taking into account robust closed-loop stability and robust performance in the presence of plant uncertainty. For example, the IMC based method of designing PID controller is considered in [3], where a low-pass filter is introduced to achieve robust stability and robust performance under nonparametric uncertainty. In [4], the mixed H_2/H_∞ optimization is employed to design a robust PID controller in the presence of nonparametric uncertainty, where the problem of minimizing the integral of squared error (ISE) subject to robust stability constraint is solved by using a genetic algorithm (GA).

I have also developed a GA based method of designing robust I-PD controller, a variant of PID controller, for time-delay systems with parametric uncertainty, where the time-delay is replaced by its Padé approximant [5, 6]. More specifically, the I-PD parameters are adjusted so as to minimize performance criterion maximized by plant parameters with bounded uncertainty. It is shown in [?] that a minimax design based on the ISE tends to lead the oscillating response with lage overshoot in the optimal closed-loop response. In order to circumvent this difficulty, we have applied the minimax design technique to a generalized ISE that includes a penalty on the derivative of the control variable [5, 6]. It is shown that the use of the generalized ISE cost function, which has been used in the LQ regulator based design of servomechanism, is very effective in shaping the closed-loop responses by adjusting a weighting parameter. This method, however, is unable to apply the multiple specification design problem, such as the case required the optimization of reference response and disturbance response at the same time [2]. And if we tried to apply the method proposed in [5] to the PID controller with two degrees of freedom, that will be failure, since the generalized ISE diverge in this case. Moreover, the use of Padé approximants is also another drawback of this method, since the stability criterion with Padé approximants for a time-delay element may yield erroneous robust stability conditions and the evaluation of the (generalized) ISE is not exact [7, 8].

In this paper, to overcome these weakpoints, a new design method of robust PID controller with two degrees of freedom (2DOF PID controller) for time-delay systems based on the partial knowledge about plant models [9, 10] will be proposed. It's the design method based on the low order moment of the transfer function of referenced model. It's a kind of the approximate pole placement design method and can apply to reference and disturbance responses simultaneously. To the best of authors' knowledge, there does not exist the method extended this technique to the robust controller design. We propose, therefore, the new method of robust 2DOF PID controller design based on the partical knowledge about plant models. Furthermore, a robust stability criterion based on zero exclusion principle [11] for a polytope of quasi-polynomials[1] will be given. This criterion gives a necessary and sufficient condition for stability of the polytope of quasi-polynomials, and is conveniently used in the present problem as well as in problems of adjusting parameters of controllers with a fixed configuration.

2 Problem Formulation

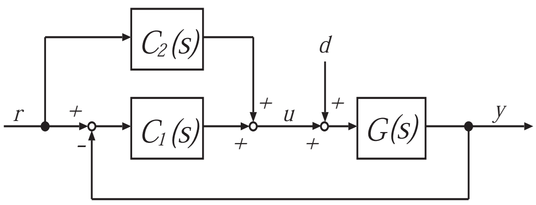

Fig. 1 Block diagram of control system

Consider the SISO control system shown in Fig. 1, where r is the step input, u the manipulated variable, y the measured variable and d the step disturbance. We assume that the plant model is described by $G(s) := P(s)e^{-Ls}$, where $P(s)$ is strictly proper rational function with parametric uncertainties and L is a delay. Let θ be the parameter vector of $P(s)$ and belong to a bounded set $\Theta = \{\theta \,|\, \theta_l \le \theta \le \theta_u\}$. The 2DOF PID controller is consist of feedback part, $C_1(s)$, and feedforward part $C_2(s)$ as:

$$\begin{aligned} C_1(s) &= \frac{K_I + K_P s + K_D s^2}{s}, \\ C_2(s) &= -\alpha K_P - \beta K_D s \end{aligned}$$

where K_P, K_I, $K_D \in \mathbf{R}$ and $0 \le \alpha, \beta \le 1$ are parameters of the 2DOF PID controller. Now, if $\alpha = \beta = 0$ then this 2DOF PID controller is the standard 1DOF PID controller. On the other hand, in the case of $\alpha = \beta = 1$, it is the so-called 1DOF I-PD controller.

Let $q_1 := (K_P, K_I, K_D)^T$ and $q_2 := (\alpha, \beta)^T$ be the 2DOF PID controller parameter vectors belonging to bounded sets $Q_1 = \left\{q_1 \,|\, q_1^l \le q_1 \le q_1^u\right\} \subset \mathbf{R}^3$ and $Q_2 = \left\{q_2 \,|\, q_2^l \le q_2 \le q_2^u\right\} \subset \mathbf{R}^2$, which are the ranges of adjustable 2DOF PID parameters. It is assumed that Θ, Q_1 and Q_2 are given a priori.

From fig. 1, H_{yr} which is the transfer function from r to y and H_{yd}, the transfer function from d to y, are given by

$$H_{yr}(s) = \frac{[C_1(s) + C_2(s)]P(s)e^{-Ls}}{1 + C_1(s)P(s)e^{-Ls}} \tag{1}$$

$$H_{yd}(s) = \frac{P(s)e^{-Ls}}{1 + C_1(s)P(s)e^{-Ls}}. \tag{2}$$

[1]The quasi-polynomial is a polynomial in one complex variable and exponential powers of the variable.

Let $P(s) = N(s)/D(s)$, and define $A(s)$, $B(s)$, $D_d(s)$ and $D_r(s)$ as

$$\begin{aligned} A(s) &= sD(s), \\ B(s) &= [K_P + K_P s + K_D s^2]N(s), \\ D_r(s) &= [K_I + (1-\alpha)K_P s + (1-\beta)K_D s^2]N(s), \\ D_d(s) &= sN(s). \end{aligned} \tag{3}$$

Then $H_{yd}(s)$ and $H_{yr}(s)$ are described as

$$H_{yd}(s) = \frac{D_d(s)e^{-Ls}}{A(s) + B(s)e^{-Ls}} \tag{4}$$

$$H_{yr}(s) = \frac{D_r(s)e^{-Ls}}{A(s) + B(s)e^{-Ls}} \tag{5}$$

respectively. The characteristic equation of the closed-loop system is given by

$$f(s) := A(s) + B(s)\,e^{-Ls} = 0. \tag{6}$$

Since the plant is strictly proper, the degree of $A(s)$ is always greater than that of $B(s)$, so that this system is a retarded system.

The design technique based on the partial knowledge about plant models is also called as PMM (Partial Model Matching) approach, which is the design method by using the low order moment of the transfer function of referenced model. The novel idea of PMM approach has been proposed by Kitamori [9] and Emre et al.[10]. It's a kind of the approximate pole placement design method. In this method, firstly the standard model transfer function $M(s)$ which indicated the ideal response is prepared as :

$$M(s) = \frac{1}{1 + \gamma_1 \tau s + \gamma_2 (\tau s)^2 + \cdots + \gamma_n (\tau s)^n}. \tag{7}$$

Then, we can design the controller by closing the form of actual closed-loop transfer function to $M(s)$. To the best of authors' knowledge, there does not exist the method extended the PMM approach to the robust controller design. We propose, therefore, the new method of robust 2DOF PID controller design based on the PMM appraoch.

Let the standard model transfer function for disturbance response M_d and the model transfer function for reference response M_r are defined as

$$M_d(s) := \frac{K_I s}{1 + \gamma_1 \tau s + \gamma_2 \tau^2 s^2 + \cdots + \gamma_n (\tau s)^n} \tag{8}$$

$$M_r(s) := \frac{1}{1 + \gamma_1 \tau s + \gamma_2 (\tau s)^2 + \cdots + \gamma_n (\tau s)^n} \tag{9}$$

Then, the aim of PMM approach is described as

$$H_{yd} \simeq M_d(s) \text{ and } H_{yr} \simeq M_r(s).$$

For this aim, we set the low order coefficients of Maclaurin expansion of $1/H_{yd}$ and $1/H_{yr}$ to be consistent with the corresponding coefficients of M_d and M_r respectively. Hence the performance indices are defined as follows.

$$J_d = e_{d1}^2 + e_{d2}^2 + e_{d3}^2 , \quad (10)$$
$$J_r = e_{r1}^2 + e_{r2}^2 \quad (11)$$

where

$$e_{d1} = \gamma_1 \tau - \frac{a_0 + K_P}{K_I}$$
$$e_{d2} = \gamma_2 \tau^2 - \frac{a_1 + K_D}{K_I}$$
$$e_{d3} = \gamma_3 \tau^3 - \frac{a_2}{K_I}$$
$$e_{r1} = \gamma_1 \tau - \frac{a_0 + \alpha K_P}{K_I}$$
$$e_{r2} = \gamma_2 \tau^2 - \left[\frac{a_1 - \beta K_D}{K_I} - \frac{K_P(1-\alpha)(K_P\alpha - a_0)}{K_I^2} \right].$$

a_k $(k = 0, 1, 2, 3)$ are the low order coefficients of Maclaurin expansion of $\frac{1}{G(s)} = \frac{1}{P(s)e^{-Ls}}$ as ;

$$\frac{1}{G(s)} = a_0 + a_1 s + a_2 s^2 + \cdots + a_k s^k + \cdots \quad (12)$$
$$a_k = \frac{1}{k} \left[\frac{d^k}{ds^k} \frac{1}{G(s)} \right]_{s=0} , \quad (13)$$

and $\tau := \frac{\gamma_3 a_3}{\gamma_4 a_2}$. J_d is the performance index for the disturbance response and J_r is the index for the reference response.

Now the design of robust 2DOF PID controller is reduced to a minimax optimization problem:

Design problem :

$$\min_{q_1 \in Q_1} \max_{\theta \in \Theta} J_d \quad (14)$$
$$\min_{q_1 \in Q_1, q_2 \in Q_2} \max_{\theta \in \Theta} J_r \quad (15)$$

s.t. closed-loop system is stable for $\forall\, \theta \in \Theta$

We note that this design problem is a multiobjective optimization problem with a significant constraint, that is the closed-loop system should be robustly stable for all plant parameters in the set Θ. We therefore propose a new robust stability criterion[8] for the dime-delay systems with parametric uncertainties in the next chapter.

3 Robust Stability Criterion for Time-Delay Systems

3.1 Stability Criterion

We first consider the stability criterion of the characteristic equation of eq. (6) when the coefficients of $A(s)$ and $B(s)$ are fixed. Let us regard L as a variable and write eq. (6) as

$$f(s, L) := A(s) + B(s)e^{-Ls} = 0. \quad (16)$$

Since a common factor of $A(s)$ and $B(s)$ is a root of (16) for any $L \geq 0$, we assume that $A(s)$ and $B(s)$ have no common factors. A procedure of finding the stability regions for h consists of four steps [7].

Step 1: Examine the stability of (16) at $L = 0$.

Step 2: Consider infinitesimally small positive L. For retarded systems, all the new roots appear in the left half-plane.

Step 3: Determine the positive zeros[2] of the polynomial

$$W(\omega^2) := A(j\omega)A(-j\omega) - B(j\omega)B(-j\omega), \quad (17)$$

and compute the corresponding L. Then analyze the behavior of the roots in the neighborhood of L.

Step 4: Check the stability region of L by sorting destabilizing and stabilizing critical value of h. If a given time-delay L is in the stability region, the closed-loop system is stable. Otherwise, the system is unstable.

3.2 Robust Stability of Time-delay Systems

Next we consider the robust stability criterion against parametric uncertainties. Namely, we assume that $N(s)$ and $D(s)$ in eq. (3) are the (real) interval polynomials. Let N denote the number of vertices of the hyperrectangle Θ in the parameter space and

$$f^i(s) = A^i(s) + B^i(s)e^{-hs}, i = 1, \cdots, N, \quad (18)$$

be the quasi-polynomials corresponding to each vertex. Then the characteristic quasi-polynomial is generated by the convex combination of these quasi-polynomials:

$$f(s, k) := \sum_{i=1}^{r} k_i\, f^i(s), \quad (19)$$

$$s.t. \sum_{i=1}^{N} k_i = 1,\ 0 \leq k_i \leq 1,\ i = 1, \cdots, N.$$

Let $k := (k_1, \cdots, k_N)^T$ and define the convex polyhedron K, the quasi-polynomial family S, and the value set S_ω as

$$\begin{aligned} K &:= \left\{ k \;\middle|\; \sum_{i=1}^{N} k_i = 1,\ 0 \leq k_i \leq 1 \right\}, \\ S &:= \{ f(s,k) \mid k \in K \}, \\ S_\omega &:= \{ f(j\omega, k) \mid k \in K \}. \end{aligned} \quad (20)$$

[2]Each zero indicates the point where the root loci against L touch or cross the imaginary axis. As is obvious from (17), such locations are independent of delay L. Another key fact is that the number of zeros is finite, although (16) has infinitely many solutions.

The shape of S_ω is a polygonal region and each quasi-polynomial segment corresponding to the boundary of S_ω is called edge. From the edge theorem [12] and the zero exclusion principle [11], we see that the quasi-polynomial family S is stable if and only if the boundary of S_ω does not contain or pass through the origin for all $\omega \geq 0$. Since the boundary of S_ω is the value set of the segment quasi-polynomial corresponding to two generating points of S, we consider an edge connecting $f^1(s)$ and $f^2(s)$. Define the quasi-polynomial segment

$$f(s,\lambda) := (1-\lambda)f^1(s) + \lambda f^2(s), \tag{21}$$

with $\lambda \in [0,1]$. From (18) we can express (21) as

$$f(s,\lambda) = A(s,\lambda) + B(s,\lambda)\, e^{-hs}, \tag{22}$$

where $A(s,\lambda)$ and $B(s,\lambda)$ are defined as

$$A(s,\lambda) := (1-\lambda)A^1(s) + \lambda A^2(s),$$

$$B(s,\lambda) := (1-\lambda)B^1(s) + \lambda B^2(s).$$

A stability criterion of the quasi-polynomial segment (21) is given as follows:

Theorem 1

Given $\lambda \in [0,1]$, let ω_λ be

$$\omega_\lambda = \sup\{\omega \mid A(j\omega,\lambda)A(-j\omega,\lambda) - B(j\omega,\lambda)B(-j\omega,\lambda) = 0\}.$$

Also let $\bar{\omega}$ be

$$\bar{\omega} = \sup\{\omega_\lambda \mid \lambda \in [0,1]\}.$$

Then, the quasi-polynomial segment (21) contains or passes through the origin for $\omega \geq 0$ if and only if there exist $\omega \in [0, \bar{\omega}]$ satisfying the following condition[3]

$$\begin{array}{l} \mathrm{Re}[f^1(j\omega)]\,\mathrm{Im}[f^2(j\omega)] - \mathrm{Re}[f^2(j\omega)]\,\mathrm{Im}[f^1(j\omega)] = 0, \\ \mathrm{Re}[f^1(j\omega)]\,\mathrm{Re}[f^2(j\omega)] \leq 0, \\ \mathrm{Im}[f^1(j\omega)]\,\mathrm{Im}[f^2(j\omega)] \leq 0. \end{array} \tag{23}$$

One should note that this theorem holds the necessary and sufficient conditions. We, therefore, can be fairly certain that the closed-loop response designed by using this theorem is much better than the one designed by pre-existing other methods. The improvement of the performance is due to the exact treatment of time-delay element via this theorem.

We can summarize the procedure examining stability of the quasi-polynomial family S as follows:

Step 1: Examine the stability of one quasi-polynomial in S. If the quasi-polynomial is stable, go to Step 2. If not, S is not stable.

Step 2: Check whether $0 \notin S_{\omega_0}$ for one $\omega_0 \geq 0$. If $0 \notin S_{\omega_0}$, go to Step 3. If not, S is not stable.

Step 3: For each edge, check the existence of $\omega \geq 0$ such that (23) holds. If there exist such an ω on at least one edge, S is not stable. If not, S is stable.

4 Design Algorithm

For computational purpose, let $Q_{1d} := \{q_1^1, \cdots, q_1^N\}$ and $Q_{2d} := \{q_2^1, \cdots, q_2^N\}$ be discrete approximations of the sets Q_1 and Q_2. Then the design algorithm of robust 2DOF PID parameters is summarized as follows.

Step 1: Check robust stability of q_1^i for all $\theta \in \Theta$, where q_1^i and q_2^j are generated by an optimization tool.

Step 2: If the closed-loop system with q_1^i is robustly stable, compute

$$Jm_{id} = \max_{\theta\in\Theta} J_d(q_1^i, \theta), \tag{24}$$

$$Jm_{ir} = \max_{\theta\in\Theta} J_r(q_1^i, q_2^j, \theta). \tag{25}$$

If the system is not robustly stable, set $Jm_{id} = \infty$ and $Jm_{ir} = \infty$.

Step 3: If an algorithm of optimaization tool stops, go to Step 4. Otherwise, go to Step 1.

Step 4: Let the minimum of Jm_{id} and Jm_{ir} be Jm_{id}^o and Jm_{ir}^o respectively. Then the corresponding q_1^{io} and q_2^{io} yields the minimax robust 2DOF PID controller.

5 Design Examples

We consider these plants with transfer functions

$$G_1(s) = \frac{K_p}{1+T_p s}e^{-Ls} \tag{26}$$

$$G_2(s) = \frac{K_p}{(1+T_{p1}s)(1+T_{p2}s)}e^{-Ls} \tag{27}$$

and assume that the set Q_1 and Q_2 for controller are given by

$$Q_1 = \{q_1 \mid \underbrace{\begin{bmatrix} 0.10 \\ 0.01 \\ 0.01 \end{bmatrix}}_{q_1^l} \leq \underbrace{\begin{bmatrix} K_P \\ K_I \\ K_D \end{bmatrix}}_{q_1} \leq \underbrace{\begin{bmatrix} 15.00 \\ 120.00 \\ 120.00 \end{bmatrix}}_{q_1^u}\} \tag{28}$$

$$Q_2 = \{q_2 \mid \underbrace{\begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix}}_{q_2^l} \leq \underbrace{\begin{bmatrix} \alpha \\ \beta \end{bmatrix}}_{q_2} \leq \underbrace{\begin{bmatrix} 1.00 \\ 1.00 \end{bmatrix}}_{q_2^u}\} \tag{29}$$

[3] In contrast to the polynomial family case, the first equation in (23) is not a polynomial but a nonlinear function of ω. Therefore we must perform a line search instead of solving polynomial roots. However, since the interval to be searched is finite, such a computation is tractable.

The values of γs in the coefficients of standard model transfer fnctions $M_d(s)$ and $M_r(s)$ are set as

$$\gamma_1 := 1.00 \ , \ \gamma_2 := 0.50 \ , \ \gamma_3 := 0.15 \ , \ \gamma_4 := 0.03.$$

These values are obtained from the good form of step responses of standard model transfer functions by simulations.

In the experiments, the genetic algorithm (GA) is used as a optimization tool in the design algorithm, since the formulated design problem is a multi-objective minimax optimization problem and GA is known as one of a effective tool of this type-optimization problems.

5.1 Numerical result of $G_1(s)$

Let the uncertainty set of the plant parameters be given by

$$\Theta = \{\theta \mid \underbrace{\begin{bmatrix} 0.8 \\ 5.6 \end{bmatrix}}_{\theta_l} \leq \underbrace{\begin{bmatrix} K_p \\ T_p \end{bmatrix}}_{\theta} \leq \underbrace{\begin{bmatrix} 1.2 \\ 8.4 \end{bmatrix}}_{\theta_u} \} \qquad (30)$$

The delay-time of e^{-Ls} is $L = 2.0$. The design result is given in table 1 and values of J_d and J_r are 0.9749 and 0.4457 respectively.

Table 1 Design result ($G_1(s)$)

K_P	K_I	K_D	α	β
6.70	1.971	1.193	0.224	0.0276

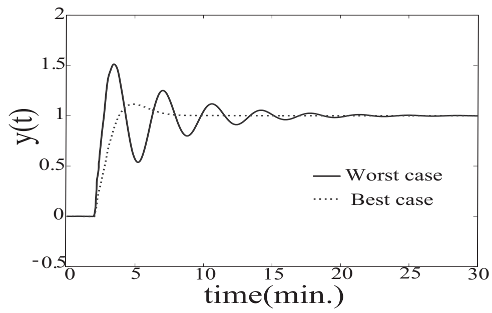

Fig. 2 Step reference responses of closed-loop system

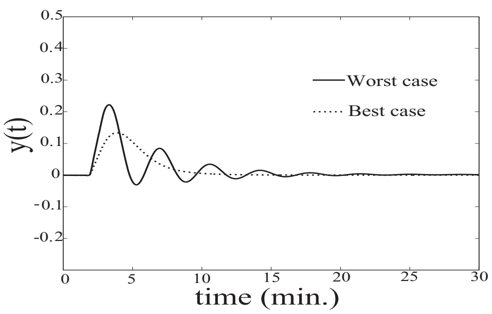

Fig. 3 Step disturbance responses of closed-loop system

Figs. 2 and 3 show the step responses of closed-loop system by using the values in table 1. Solid line shows the worst case response and dashed lines shows the best response in the Θ. We observe the good robust performance.

5.2 Numerical result of $G_2(s)$

Let

$$\Theta = \{\theta \mid \underbrace{\begin{bmatrix} 0.8 \\ 4.57 \\ 1.16 \end{bmatrix}}_{\theta_l} \leq \underbrace{\begin{bmatrix} K_p \\ T_{p1} \\ T_{p2} \end{bmatrix}}_{\theta} \leq \underbrace{\begin{bmatrix} 1.2 \\ 6.85 \\ 1.74 \end{bmatrix}}_{\theta_u} \} \qquad (31)$$

The delay-time is $L = 1.0$. The numerical result is shown in table 2 and values of J_d and J_r are 10.672 and 1.0041 respectively. Closed-loop responses are depicted in figs. 7 and 8.

Table 2 Design result ($G_2(s)$)

K_P	K_I	K_D	α	β
8.25	1.624	8.993	0.184	0.0126

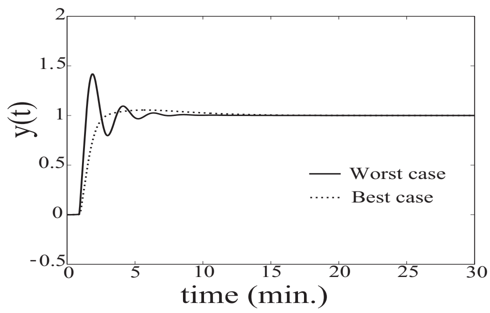

Fig. 4 Step reference responses of closed-loop system

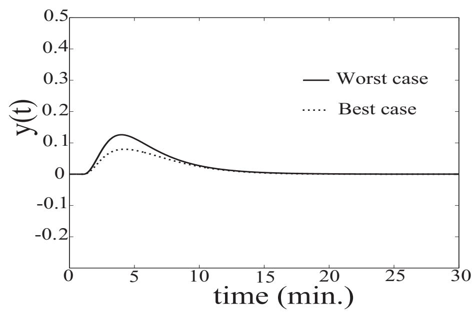

Fig. 5 Step disturbance responses of closed-loop system

Figs. 4 and 5 show the step responses of closed-loop system with the values in table 2. We also observe the good robust preformance.

6 Conclusion

In this paper, a new design method of robust 2DOF PID controller based on the partial knowledge about plant models has been proposed. The robust stability criterion for time-delay systems with parametric uncertainties have been also proposed. The 2DOF PID controller designed by proposed method guarantees the robust stability and the best performance under worst case scenario. And the effectiveness of proposed method ale able to be recognized from simulation results.

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