# Minimization of operational amplifiers finite gain effects in switched-capacitor biquads

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*Abstract:* - An combined approach for reducing the errors in the pole frequency $f_p$, in the pole $Q$-factor $Q_p$ and in the amplitude at the pole frequency $H_p$ of switched-capacitor biquads is presented. At first, the conventional integrators in the biquads are replaced by gain-and offset-compensated integrators. Subsequently, the errors $\Delta f_p/f_p$, $\Delta Q_p/Q_p$ and $\Delta H_p/H_p$ are minimized by modifying three capacitances: the two integrating capacitances and an appropriately chosen capacitance. The effectiveness of this approach is demonstrated by designing a bandpass biquad.

*Key-Words:* - Filters, gain-and offset-compensation, operational amplifiers, switched-capacitor integrators.

## 1 Introduction

Switched-capacitor (SC) integrators are the basic building blocks of SC filters. A key requirement of these circuits is that their performances remain insensitive to variations in operational-amplifiers (op amps) offset voltage, gain and bandwidth. This is especially important in high-frequency SC filters, where low amplifier gain is often the result of wide bandwidth design. It has been shown that in SC filters the effect of op amps finite gain $A$ is more serious than that of the finite bandwidth [1]. This has led to the development of gain – and offset-compensated (GOC) integrators where the phase error is proportional to $1/A^2$ [2-6]. In a conventional integrator this is a simple inverse dependence $1/A$. In the most of the GOC integrators, reported in the literature, the reduction in the phase error $\theta(\omega)$ was obtained at the expense of increased gain error $m(\omega)$. According to the authors knowledge, the Betts-91 [5] and the Shafeeu-91 [6] circuits are the two GOC integrators that have the same sample correction property, which results in simultaneous reduction of gain and phase errors. The Betts-91 integrator is however quite complex. The Shafeeu-91 integrator uses fewer components but requires a four-phase clock.

A second-order filter section, known as a biquad, is commonly realized using a feedback loop containing one inverting and one noninverting integrators. The gain errors $m(\omega)$ of the integrators affect the pole frequency $f_p$ of the biquad, while the phase errors $\theta(\omega)$ affect the pole quality factor $Q_p$ and the magnitude of the biquad transfer function at the pole frequency $H_p$.

In this paper a combined approach for minimization of the errors in the pole frequency $f_p$, in the pole Q-factor $Q_p$ and in the amplitude at the pole frequency $H_p$ of SC biquads with low but precisely known and stable op amps dc gain is proposed. The effectiveness of this approach is demonstrated by designing a bandpass SC biquad.

## 2 Proposed combined approach

The z-domain biquadratic transfer function has the general form

$$H(z) = K\frac{1 + a_1 z^{-1} + a_2 z^{-2}}{1 + b_1 z^{-1} + b_2 z^{-2}} = \frac{N(z)}{D(z)} \qquad (1)$$

where $z = exp(\ j2\pi f/f_s\ )$, with $f_s$ denoting the sampling frequency.

For any pair of complex conjugate poles in the z-domain, one can write the denominator as

$$1 + b_1 z^{-1} + b_2 z^{-2} = 1 - 2R\cos\theta . z^{-1} + R^2 . z^{-2}\,, \qquad (2)$$

where $R$ is the radius and $\theta$ is the angle to the pole.

From (2) the following relationships for the pole frequency and the pole Q-factor can be derived:

$$f_p = \frac{f_s}{2\pi}\sqrt{\theta^2 + (\ln R)^2} \qquad (3)$$

and

$$Q_p = -\frac{\pi . f_p / f_s}{\ln R}. \qquad (4)$$

For small ratio $f_p/f_s$ and high Q-factor the pole frequency $f_p$ and the pole $Q_p$ are approximately given by

$$f_p \cong \frac{f_s}{2\pi}\sqrt{1+b_1+b_2} \qquad (5)$$

and

$$Q_p \cong \frac{\sqrt{1+b_1+b_2}}{1-b_2}. \qquad (6)$$

For a given SC structure, in standard design the op amp gain value is assumed to be infinite. Then the coefficients in (1) are a function only of the capacitances. In this case, from (5), (6) and (1) the logarithmic sensitivities of $f_p$, $Q_p$ and of the amplitude at the pole frequency $H_p$ to the capacitances $C_q$ can be obtained.

Using the simple classical definition

$$S_x^y = (\partial y / \partial x)(x / y)$$

the results are

$$S_{Cq}^{fp} = \frac{C_q}{2(1+b_1+b_2)}\left(\frac{\partial b_1}{\partial C_q} + \frac{\partial b_2}{\partial C_q}\right), \qquad (7)$$

$$S_{Cq}^{Qp} = \frac{C_q}{2(1+b_1+b_2)} \left[\frac{\partial b_1}{\partial C_q} + \left(\frac{3+2b_1+b_2}{1-b_2}\right)\frac{\partial b_1}{\partial C_q}\right] \qquad (8)$$

and

$$S_{Cq}^{Hp} = \frac{C_q\left[a_1 + (1+a_2)\cos(\omega_p T_s)\right]}{N_p}\frac{\partial a_1}{\partial C_q} + \frac{C_q\left[a_2 + a_1\cos(\omega_p T_s) + \cos(2\omega_p T_s)\right]}{N_p}\frac{\partial a_2}{\partial C_q} - \frac{C_q\left[b_1 + (1+b_2)\cos(\omega_p T_s)\right]}{D_p}\frac{\partial b_1}{\partial C_q} - \frac{C_q\left[b_2 + b_1\cos(\omega_p T_s) + \cos(2\omega_p T_s)\right]}{D_p}\frac{\partial b_2}{\partial C_q} \qquad (9)$$

where

$$N_p = 1 + a_1^2 + a_2^2 + 2a_1(1+a_2)\cos(\omega_p T_s) + 2a_2\cos(2\omega_p T_s)$$

$$D_p = 1 + b_1^2 + b_2^2 + 2b_1(1+b_2)\cos(\omega_p T_s) + 2b_2\cos(2\omega_p T_s).$$

The proposed combined approach for minimization of the errors $\Delta f_p/f_p$, $\Delta Q_p/Q_p$ and $\Delta H_p/H_p$ in SC biquads consists in the following consecutive steps:

Step 1. At first, for reducing the effect of op amp imperfections (dc gain $A$ and offset voltage $V_{OS}$) the conventional integrators in the biquad considered are replace by Nagaraj-86 [2] and Ki-89 [3] GOC SC integrators. These simple bi-phase integrators form an excellent GOC integrator-pair without the use of extra clock phases or holding circuits to satisfy the sampling conditions. The reduced phase errors of the GOC integrators provide a reduction in the errors $\Delta Q_p/Q_p$ and $\Delta H_p/H_p$.

Step 2. The gain error of the integrators is equivalent to an element value variation $\Delta C_i$ of the integrating capacitance $C_i$. If the finite dc gain $A$ is known, the value of $C_i$ can be replaced by $C_i^{/} = C_i(1+m)$ in the two integrators of the GOC biquad, thereby essentially reducing the gain errors $m(\omega)$ [7]. This prewarping technique automatically provides minimization of the pole frequency error $\Delta f_p/f_p$ of the biquad considered.

The prewarped capacitance values $C_{iN}^{/}$ and $C_{iK}^{/}$ for the Nagaraj-86 and Ki-89 integrators are calculated on the basic of the expressions

$$\mathrm{C}_{\mathrm{iN}}^{/} = \left(C_i - \frac{C_h + \sum_{q=1}^{N} C_q}{A_0}\right)\left(1 + \frac{1}{A_0}\right)^{-1} \qquad (10)$$

and

$$\mathrm{C}_{\mathrm{iK}}^{/} = \left(C_i - \frac{\sum_{q=1}^{N} C_q}{A_0}\right)\left(1 + \frac{1}{A_0}\right)^{-1} \qquad (11)$$

where

● The terms in the sums are all the capacitances (excepting the holding capacitance $C_h$) connected to the "super-virtual ground" node of the integrators;

● $A_0$ is the nominal value of the op amp dc gain $A$;

● For the Nagaraj-86 integrator the holding capacitance $C_h$ is equal to the smallest biquad capacitance;

● For the Ki-89 integrator $C_h$=$C^{/}_{iK}$ .

<u>Step 3.</u> The errors $\Delta Q_p/Q_p$ and $\Delta H_p/H_p$ can be further minimized by modifying one capacitance. This capacitance $C_q$ is chosen such that the following relations hold:

$$S^{Qp}_{Cq} \cong -1\,,\; S^{Hp}_{Cq} \cong -1\,,\; \left|S^{fp}_{Cq}\right| << 1\,.$$

The sensitivities $S^{fp}_{Cq}$, $S^{Qp}_{Cq}$ and $S^{Hp}_{Cq}$, defined by (7), (8) and (9), are calculated for the standard synthesis values of the capacitances computed assuming the op amp gain $A$ to be infinite. The relative deviation in pole-Q factor due to a change in the capacitance $C_q$ around its nominal value is approximately given by

$$\frac{\Delta Q_p}{Q_p} \approx S^{Qp}_{Cq}\,\frac{\Delta C_q}{C_q}\,. \qquad (12)$$

The calculated relative error $\Delta Q_p/Q_p$ of the resulting GOC biquad with prewarped integrating capacitances is substituted with opposite sign for the relative deviation into (12).

The new value $C^{/}_{q}$ of the capacitance $C_q$ is the solution of the equation

$$S^{Qp}_{Cq}\left(\frac{C^{/}_{q}}{C_q}-1\right) = -\left(\frac{\Delta Q_p}{Q_p}\right). \qquad (13)$$

The relationships (12) and (13) are valid for small variations of the capacitance around its standard synthesis value. That is why the preliminary GOC approach is indispensable for the subsequent compensation by modifying the capacitance $C_q$.

## 3 Application of the proposed approach

The proposed approach is illustrated by means of the Fleischer and Laker's E-type bandpass BP01 SC biquad [8,9], shown in Fig.1.

The circuit has a pole frequency $f_p$=500$kHz$, a quality factor $Q_p$=16, a peak gain of 10$dB$ at $f_p$ and sampling frequency $f_s$=10$MHz$. The component values are $C_A$=11.67$pF$, $C_B$=37.22$pF$, $C_C$=11.38$pF$, $C_D$=36.81$pF$, $C_E$=2.28$pF$, and $C_J$=2.26$pF$ [8].

It was found that for op amp gain $A_1$=$A_2$=100 the deviation of $f_p$, $Q_p$ and $H_p$ from the ideal case are

$\Delta f_p/f_p$ = -1.202%, $\Delta Q_p/Q_p$= -25.37%, $\Delta H_p/H_p$ = -25.38%

According to the proposed approach the first integrator in the conventional biquad (Fig.1) is replaced by the Nagaraj-86 inverting integrator and the second integrator – by the Ki-89 noninverting integrator. The

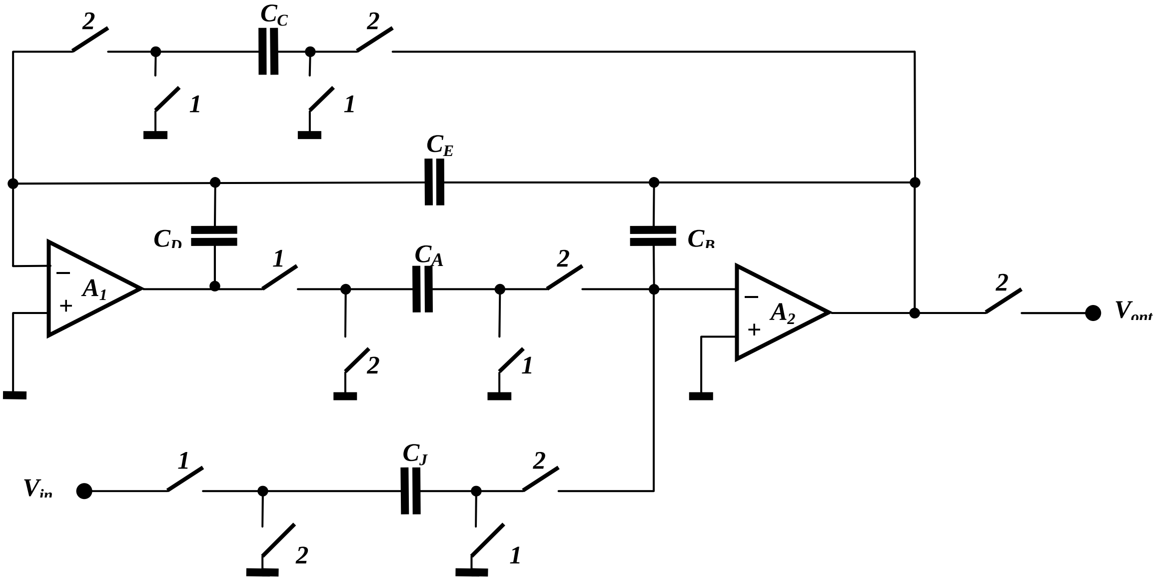

Fig.1. Bandpass SC biquad with conventional integrators

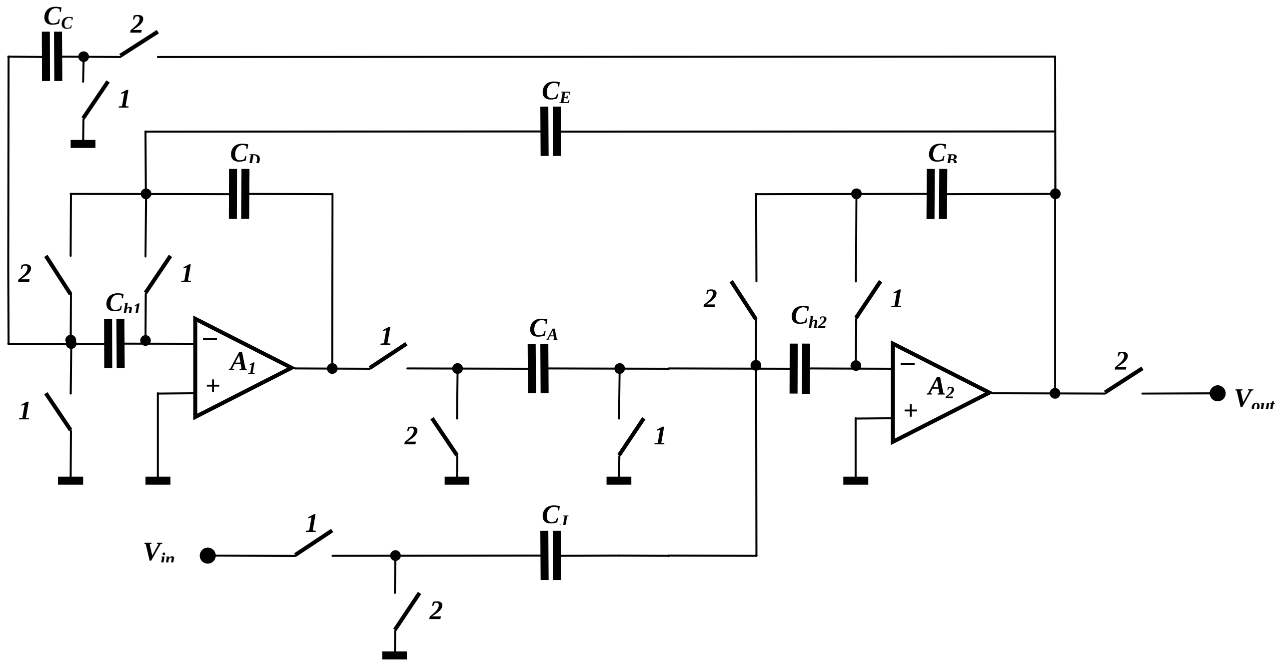

Fig.2. Bandpass biquad with GOC integrators

resulting filter is shown in Fig.2, where $C_{h1}=C_J$ and $C_{h2}=C_B$.

The performance parameters of the GOC biquad for $A_1=A_2=100$ are summarized in Table 1.

| GOC biquad with: | $\Delta f_p/f_p$ [%] | $\Delta Q_p/Q_p$ [%] | $\Delta H_p/H_p$ [%] |
|---|---|---|---|
| $C_B$ and $C_D$ | -1.41199 | 2.0395 | 2.026 |
| $C_B^{/}$ and $C_D^{/}$ | $-3.4764.10^{-3}$ | 0.63116 | 0.62675 |

Table 1: Performance parameters of the GOC biquad

Subsequently, for reducing the pole frequency error the integrating capacitances $C_D$ and $C_B$ are modified according to the relation (10) and (11). The prewarped capacitances $C_D^{/}$ and $C_B^{/}$ are given by the expressions

$$C_D^{/}=\left(C_D-\frac{C_C+C_E+C_J}{A_{01}}\right)\left(1+\frac{1}{A_{01}}\right)^{-1}, \quad (14)$$

$$C_B^{/}=\left(C_B-\frac{C_A+C_J}{A_{02}}\right)\left(1+\frac{1}{A_{02}}\right)^{-1}. \quad (15)$$

One obtains $C_D^{/}$=36.28792 $pF$ and $C_B^{/}$=36.71356 $pF$ for $A_{01}=A_{02}=100$.

The corresponding performance parameters of the biquad are also given in Table 1.

The ideal z-domain transfer function is

$$H_{id}^{21}(z)=\frac{\frac{C_J}{C_B}\left(1-z^{-1}\right)z^{-1/2}}{\left(1-\frac{C_AC_E}{C_BC_D}\right)z^{-2}-\left[2-\frac{C_A(C_E+C_C)}{C_BC_D}\right]z^{-1}+1}. \quad (16)$$

The pole frequency $f_p$ and the quality factor $Q_p$ are approximately given by

$$f_p\approx\frac{f_s}{2\pi}\sqrt{\frac{C_AC_C}{C_BC_D}}, \quad (17)$$

$$Q_p\approx\frac{1}{C_E}\sqrt{\frac{C_CC_BC_D}{C_A}}. \quad (18)$$

The corresponding logarithmic sensitivities to the capacitance $C_E$ are

$$S_{C_E}^{f_p}=0,\ S_{C_E}^{Q_p}=-1,\ S_{C_E}^{H_p}=-0.9999297.$$

Therefore, the errors $\Delta Q_p/Q_p$ and $\Delta H_p/H_p$ can be further minimized by modifying the capacitance $C_E$ according to the expression

$$C_E^{/}=C_E\left(1+\frac{\Delta Q_p}{Q_p}\right) \quad (19)$$

where $\Delta Q_p/Q_p$=6.3116.10$^{-3}$.

This gives $C_E^{/}$=2.29439 $pF$.

The performance parameters or the GOC biquad with modified capacitances $C_B$, $C_D$, $C_E$ and with gain variation $A_1$=$A_2$=$A$=100$\pm$8 are summarized in Table 2.

| $A$ | $\Delta f_p/f_p$ [%] | $\Delta Q_p/Q_p$ [%] | $\Delta H_p/H_p$ [%] |
|---|---|---|---|
| 92 | -0.1236 | 0.1405 | 0.1383 |
| 100 | -5.98.10$^{-4}$ | -9.1.10$^{-4}$ | -1.92.10$^{-3}$ |
| 108 | 0.1044 | -0.125 | -0.1252 |

Table 2: Performance parameters of the GOC biquad with modified capacitances $C_B$, $C_D$, $C_E$

# 4 Conclusion

A combined approach for minimization the effects of op amps finite gain in switched-capacitor biquads has been presented. At first, the conventional integrators in the biquads have been replaced by gain-and offset-compensated integrators. Subsequently, the errors in the pole frequency $f_p$, in the pole $Q$-factor $Q_p$ and in the amplitude $H_p$ at the pole frequency have been minimized by modifying three capacitances: the two integrating capacitances and an appropriately chosen capacitance. The proposed approach has been illustrated by designing a bandpass biquad.

The obtained results demonstrate the possibility of considerable performance improvement by using the proposed approach.

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