Applying the Lifting Operators in the Study of a Multifrequency Large-Scale System Stability

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Abstract: - The segmented 10m diameter primary mirror of the Gran Telescopio Canarias (GTC) is a large-scale multivariable system. In order to ensure the required quality in the images the telescope provides, it is necessary to carry out the mirror active control. Due to the strong interconnections existing among the different segments composing the GTC's primary mirror, a fully decentralized control does not provide the desired performance. However, it is possible to use a control strategy, we call 'local-global', combining a centralized and a decentralized control actions. When this strategy is used, the closed-loop system results in a multifrequency one, whose stability can not be analitically determined in a easy way. The purpose of this paper is to show how the appropriate inclusion of the lifting operators in the closed-loop system blocks diagram allows to carry out the stability study by computing the closed-loop system matrix eigenvalues.

Key-Words: - Large-scale systems, multivariable control, multifrequency systems, lifting operators, local-global control, pole-placement controller

1 Introduction

As it is well known, the Canary Islands' skies provide excellent conditions for astronomic observations. This is the reason why one of these islands (La Palma) and, concretely, El Roque de los Muchachos Observatory, has been chosen as the best location for the Gran Telescopio Canarias [1]. The 10 meters of diameter of its primary mirror converts the GTC into the largest telescope in the world together with the Keck Telescope in Mauna Kea (Hawaii). Due to the difficulties the construction of such a large monolithic mirror presents, the segmentation of the mirror is advisable. The segmentation into thirty six hexagonal pieces, called segments, was chosen as the best option after carrying out a series of studies. In order the mirror provides images with the required quality, the alignment among the different segments is essential. However, there are some factors that tend to take the segments out of the position in which all of them are aligned. Some of these factors are the effect of the wind inside the dome [2] or the mechanical vibrations in the mirror structure [3]. As a consequence of this, it will be essential to carry out the active control of the GTC's primary mirror [4] and [5] with the purpose of ensuring the appropriate telescope operation in presence of such disturbances.

2 Problem Formulation

From a dynamic point of view, three elements compose the GTC's primary mirror. They are:

1.- The cell or structure [6] supports the other two elements and, after carrying out a finite-elements analysis [7] it is dynamically described through its 30 dominant modes, being the mode in 17*hz* the most important one.

2.- The actuators are the mechanical devices used for moving the segments and are characterized by a 5ms delay and a second order term with a 60*hz* mode.

3.- The segments of the mirror, whose dynamics is described through the three support points with the actuators. A natural frequency of 28*hz* corresponds to each one of these support points.

These elements are schematically represented in Fig. 1.

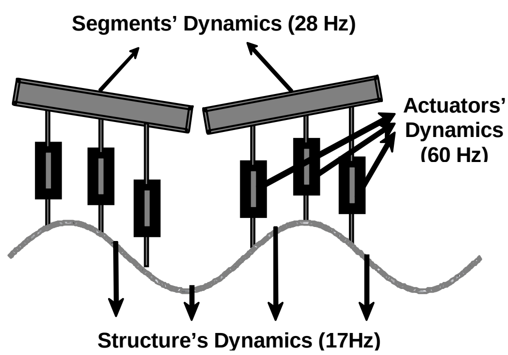


Fig.1. Elements composing the GTC's primary mirror.

The primary mirror open-loop response is characterized by a damped oscillatory response in 17 and 28*hz* and a settling time over two seconds. Both characteristics can be appreciated in Fig. 2 and 3, in which the segments response is shown for two different initial conditions.

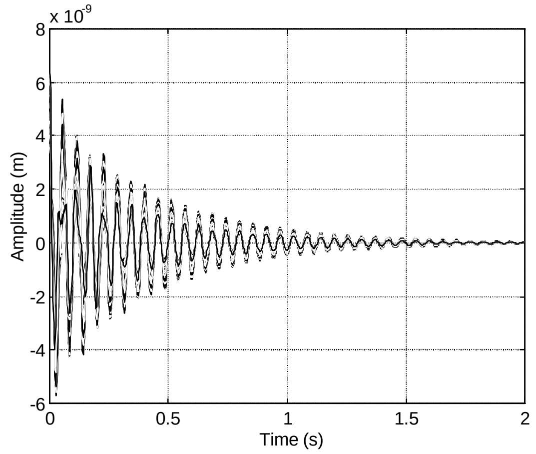


Fig.2. Segments' open-loop response when they are initially aligned.

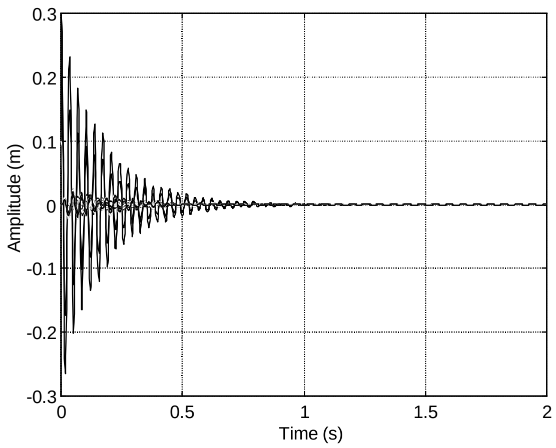


Fig.3. Segments' open-loop response when they are not initially aligned.

In Fig. 2, the segments' starting point corresponds to a position in which all of them are perfectly aligned and behave as a monolithic mirror towards the equilibrium state defined by another plane. As a consequence of this, the segments mode (28*hz*) is not excited and it is the one corresponding to the cell (17*hz*) the predominant mode in the system response. In Fig. 3, all the segments are initially in the equilibrium position except three of them, so in this case the initial condition corresponds to a state in which the segments are non-aligned. This is the reason why the 28*hz* mode is excited in this second case and predominates over the 17*hz* mode.

From the point of view of the mirror interaction with the control system, it must be noticed that the GTC is a large scale system with a huge number of sensors and actuators. On the one hand, in order to move the segments each one of them has three actuators. Since there are 36 segments, the mirror has 108 of such devices. On the other hand, the measurement of the degree of alignment among the segments is carried out by a set of 168 sensors [8]. They are in the common edge of adjacent segments and measure the relative distance between them. A schematic representation of the mirror actuators and sensors is shown in Fig. 4.

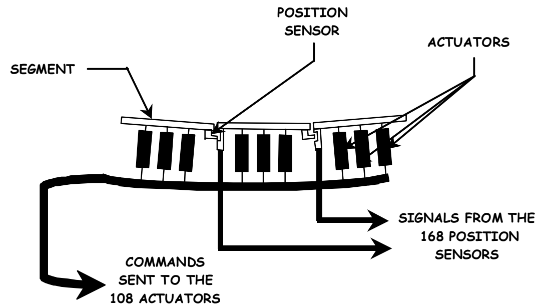


Fig.4. Actuation and sensorial systems of the GTC's primary mirror.

2.1 The local-global control strategy

Due to the high number of inputs and outputs of this large-scale system, the GTC's primary mirror active control [9], [10], [11] will imply the flow of a huge amount of information which can need to be processed at a high frequency. Notice that the segments' control consists of reading the signals coming from the 168 intersegment sensors, processing them and, in function of this information (the mirror degree of alignment), calculating the corresponding 108 commands that will be finally sent to the mirror actuators. This, together with the restrictions imposed by the buses technology used for the telescope design, motivated the study of a control strategy called 'local-global'. It consists of applying to each one of the thirty six primary mirror segments two commands at different levels, as shown in Fig. 5.

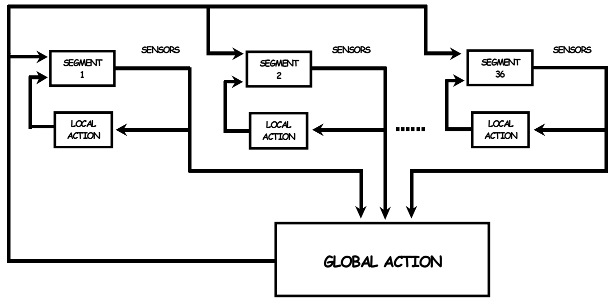


Fig.5. The local-global strategy.

The local control action applied to each segment is calculated using the information provided by three of the segment position sensors. For the calculus of the global control action, the information of all the system sensors is used. Thus, the global control is the one that requires the process of a higher amount of information. The goal of implementing the local-global strategy is to be able to apply the global command at a frequency which is lower than the one that would be necessary in case of disregarding the local control action.

2.2 A pole-placement controller

According to the local-global control strategy, a multivariable pole-placement controller has been designed [12]. Notice that the only design specifications considered when implementing this multivariable controller consists of canceling the oscillations in the system response, which must be also quicker than in the open-loop case (see Fig. 2). In addition to this, the commands sent to the actuation devices of the primary mirror can not go beyond a certain limit value.

The pole-placement controller command is obtained according to the following expression:

$$U = -K_l * S - F_l * \dot{S} - K_g * x - F_g * \dot{x} \qquad (1)$$

being:

$S(168\times1)$: signals of the 168 intersegment sensors,

$\dot{S}(168\times1)$: derivative of the 168 intersegment sensors signals,

$x(708\times1)$: system state vector,

$\dot{x}(708\times1)$: derivative of the system state vector,

$K_l(108\times168)$, $F_l(108\times168)$: parameters of the local controller,

$K_g(108\times708)$, $F_g(108\times708)$: parameters of the global controller,

$U(108\times1)$: commands applied to the 108 actuators of the primary mirror.

Thanks to the symmetries the mirror presents, the pole-placement controller parameters are obtained using only two scalar quantities, called v_K and v_F. Depending on the values these quantities take, the closed-loop system will present one response or another.

2.2.1 The local control action

The local component of the pole-placement controller is the one given by the following expression:

$$u_{local} = -K_l * S - F_l * \dot{S} \qquad (2)$$

Its purpose is to change the segments dynamics using the v_K and v_F parameters in such a way that the closed-loop system response does not present oscillations and is quicker than in the open-loop case. The thirty six segments dynamics changes in the appropriate way with the local control action. However, due to the interactions among them, when the local command is applied to the mirror actuators the system becomes unstable. This is the reason why it is necessary to complement the local action with a global one, whose only objective will be to cancel the interactions among the different segments.

2.2.2 The global control action

The expression corresponding to the global component of the pole-placement controller is the following :

$$u_{global} = -K_g * x - F_g * \dot{x} \qquad (3)$$

The process by means of which the parameters of the global control action are obtained takes place in two different stages:

- in the first one the elements responsible for the closed-loop system instability due to the coupling among the different segments and which appears when applying the local control action are found,
- in the second stage, the parameters K_g and F_g are obtained in such a way that those coupling elements are cancelled.

Thus, the thirty six segments of the GTC's primary mirror behaves as determined by v_K and v_F at the same time as the global action ensures the closed-loop system stability canceling the interactions among segments.

From the diagram shown in Fig. 5, it is clear that the closed-loop system with the local-global controller is a multifrequency plant, whose stability can not be analitically determined in an easy way. In the following section it can be seen how the appropriate use of two operators allows to express the closed-loop system in function of an only frequency. Consequently, the system stability will be determined by the simple inspection of the plant matrix eigenvalues.

3 Using the 'lifting' operators for studying the stability of a multifrequency plant

Chen and Francis define in [13] two operators they call 'lifting' and 'inverse lifting' (L and L^{-1}, respectively). Now consider a discrete-time linear multivariable system provided with m inputs and p outputs. G is the system transfer matrix, that is to say:

$$y = Gu \quad (4)$$

If the same multivariable system is expressed in function of the state variables, the resulting equations are:

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) + Du(k) \end{aligned} \quad (5)$$

Then, a new matrix called $[G]$ can be introduced, being:

$$\begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \\ \vdots \end{bmatrix} = [G] \cdot \begin{bmatrix} u(0) \\ u(1) \\ u(2) \\ u(3) \\ \vdots \end{bmatrix} \quad (6)$$

where each one of the inputs and outputs vectors elements in the k instant are vectors with m and p elements, respectively.

Notice that G and $[G]$ must not be got mixed up.

Then, Chen and Francis define in [7] the L and L^{-1} operators they call 'lifting' and 'inverse lifting', respectively, in such a way that given a set of samples of a given signal v at a frequency f:

$$v = \{v(0), v(1), v(2), \ldots\} \quad (7)$$

when the lifting operator is applied to v, it results in a new signal w, whose samples are spaced out with a frequency f/n, being n a positive integer number:

$$w = L * v = \left\{ \begin{bmatrix} v(0) \\ v(1) \\ \vdots \\ v(n-1) \end{bmatrix}, \begin{bmatrix} v(n) \\ v(n+1) \\ \vdots \\ v(2n-1) \end{bmatrix}, \ldots \right\} \quad (8)$$

As it can be seen, each sample of w is a vector of n components, which are the samples of the original signal, v, grouped together.

In an analogous way, given another signal r at a frequency f/n:

$$r = \left\{ \begin{bmatrix} r_1(0) \\ r_2(0) \\ \vdots \\ r_n(0) \end{bmatrix}, \begin{bmatrix} r_1(1) \\ r_2(1) \\ \vdots \\ r_n(1) \end{bmatrix}, \ldots \right\} \quad (9)$$

the inverse lifting operator is defined in such a way that when it is applied to r the resulting signal, s, is the following:

$$s = L^{-1} * r = \{r_1(0), \ldots r_n(0), r_1(1), \ldots r_n(1), \cdots\} \quad (10)$$

whose samples are spaced out with a frequency f.

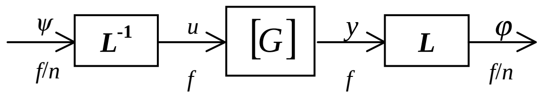
In the light of the expressions above, the matrices corresponding to the lifting and the inverse lifting operators are, particularizing for n=2, the following ones:

$$L = \left(\begin{array}{c|c|c|c|c|c|c} 1 & 0 & 0 & 0 & 0 & 0 & \cdots \\ 0 & 1 & 0 & 0 & 0 & 0 & \cdots \\ \hline 0 & 0 & 1 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & 1 & 0 & 0 & \cdots \\ \hline 0 & 0 & 0 & 0 & 1 & 0 & \cdots \\ 0 & 0 & 0 & 0 & 0 & 1 & \cdots \\ \hline \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \end{array} \right)$$

and

$$L^{-1} = \left(\begin{array}{cc|cc|cc|c} 1 & 0 & 0 & 0 & 0 & 0 & \cdots \\ \hline 0 & 1 & 0 & 0 & 0 & 0 & \cdots \\ \hline 0 & 0 & 1 & 0 & 0 & 0 & \cdots \\ \hline 0 & 0 & 0 & 1 & 0 & 0 & \cdots \\ \hline 0 & 0 & 0 & 0 & 1 & 0 & \cdots \\ \hline 0 & 0 & 0 & 0 & 0 & 1 & \cdots \\ \hline \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \end{array} \right)$$

Chen and Francis prove that when applying the L and L^{-1} operators to the $[G]$ system in the way shown below:



the $[G_L]$ matrix of the resulting system is:

$$\varphi = L[G]L^{-1}\psi = [G_L]\psi \quad (11)$$

If the g packed notation is used:

$$g = \left(\begin{array}{c|c} A & B \\ \hline C & D \end{array} \right)$$

it can be written that, for an arbitrary n, the g_L matrix is the following:

$$g_L = \left(\begin{array}{c|c} A_L & B_L \\ \hline C_L & D_L \end{array} \right) = \left(\begin{array}{c|cccc} A^n & A^{n-1}B & A^{n-2}B & \cdots & B \\ \hline C & D & 0 & \cdots & 0 \\ CA & CB & D & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ CA^{n-1} & CA^{n-2}B & CA^{n-3}B & \cdots & D \end{array} \right)$$

3.1 Applying the lifting operators to the GTC's primary mirror

The formalism of the lifting and the inverse lifting operators has been applied to the GTC's primary mirror in order to carry out an analitic study of the closed-loop system stability with the local-global controller [13], [14], [15]. The idea, as it was said at the end of the previous section, is to express this multifrequency system in function of only one sampling period. The only restriction that will be imposed is that the local frequency is a multiple of the global one.

Concretely, this methodology has been applied with the local-global pole-placement controller described in section 2, taking v_K=-1500 and v_F=-0.044. The most general situation is that one in which the system sampling frequency is *f*, while the local control is applied at a frequency f/n_1 and the global one is applied at a frequency f/n_2, being both n_1 and n_2 integer numbers. However, two assumptions have been made in order to simplify the development:

1. On the one hand, the system is sampled at a frequency *f*, which is the same frequency at which the local component of the pole-placement controller is applied. The global component of the controller is applied at a frequency equal to *f*/*n*, being *n* an integer. Consequently, this is a multisampling system including two different frequencies: *f* and *f*/*n*.
2. The expression of the command is simplified in such a way that the derivative terms are removed from the control law:

$$U = -K_l * S - K_g * x$$

Notice that the same stability study can be carried out including the derivative terms.

Using the lifting and the inverse lifting operators, the closed-loop system can be expressed in function of the lowest frequency (*f*/*n*). The corresponding blocks diagram is the one shown in Fig. 6.

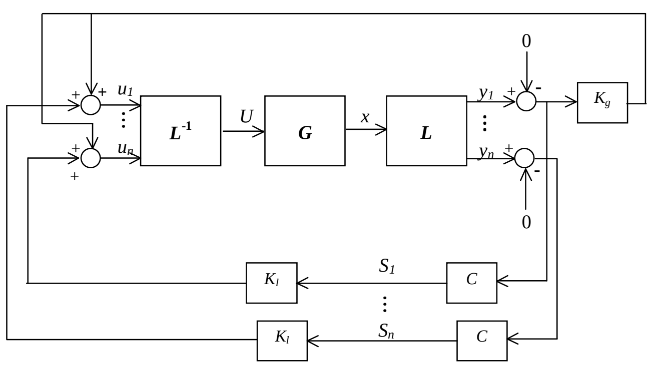


Fig.6. Closed-loop system blocks diagram including the lifting operators.

The global command is applied at a frequency equal to *f*/*n*. This fact is taken into account in the blocks diagram of Fig. 6 taking the value of the system output in the *k* stage for obtaining the global component of the command in the *k*,*k*+1,...*k*+*n*-1 stages. Notice also that for the calculus of the local command in the *k* stage, the information provided by the mirror sensors in the same instant is used. In other words, its value is updated every $1/f$ seconds.

From the blocks diagram in Fig. 6, it is easy to obtain an analytic expression for the closed-loop system state matrix, A_{lc}. This is the following:

$$A_{lc} = A_L + B_L M C_L \quad (12)$$

where *M* is the matrix that relates the commands $(u_1, u_2, ...u_n)$ and the outputs of the G_L system: $(y_1, y_2, ...y_n)$.

3.2 Results

Calculating the closed-loop system poles, that is to say, the A_{lc} matrix eigenvalues, for different values of *n*, the following results are found when *f*=2000*hz*:

1. for *n*=1,2,...10, that is to say, global frequencies equal to 2000, 1000,...200*hz*, the closed-loop system is stable,
2. for *n*>10, which means global frequencies lower than 200*hz*, the system is unstable.

If this procedure is repeated for other values of the system sampling frequency, which goes on coinciding with the local command one, a graphic representation of the results found with respect to the closed-loop system stability applying the local-global pole-placement controller can be carried out. Such a representation is shown in Fig. 7.

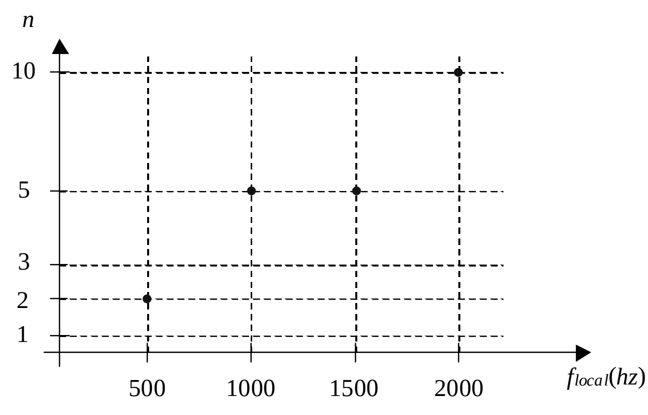


Fig. 7. Results obtained in the study of the closed-loop system stability.

Concretely, the values considered for the system sampling frequency are, in addition to 2000*hz*, 1500, 1000 and 500*hz*. Notice that in the horizontal axis in Fig. 7 the frequency at which the local command is

applied is represented, while the vertical axis corresponds to the different values for n, being $f_{global}=f_{local}/n$. The spots represent the highest value of n, that is to say, the lowest value of the global frequency, for which the closed-loop system is stable for each value of the local one.
The representation above shows that, whatever the local frequency value, the global command must be applied at least at 200*hz* in order the closed-loop system is stable. Notice that these results coincide with the ones found in simulation with the local-global pole-placement controller.

4 Conclusion

The primary mirror of the Gran Telescopio Canarias is a large scale multivariable system whose active control, motivated by the mirror segmentation, implies the management of a huge amount of information. In order to cope with the desired design specifications, the control frequency can result prohibitive. The local-global control strategy presented in this paper consists of applying to the mirror segments two commands at different frequencies: a 'fast' local command, which is obtained using the information of three sensors, and a 'slower' global one for whose calculus the 168 sensors are used. A local-global multivariable pole-placement controller has been designed and implemented according to this strategy. The implementation of a local-global controller implies to work with a multifrequency system, since the local command is updated at a frequency f and the global one is updated at f/n, being n an integer number. However, the study of the closed-loop plant stability in function of the local and global frequencies is much simpler if the system is expressed in function of only one sampling period. This goal is achieved by using the 'lifting' and the 'inverse lifting' operators. It is found that the minimum global frequency for which the closed-loop system is stable is 200*hz*. The closed-loop system simulation corroborate the results of the stability study using the lifting and the inverse lifting operators.

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