PERFORMANCE EVALUATION OF DIFFERENTIAL OFDM-MIMO SYSTEMS

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Abstract: - In this paper we present and study the performance of an OFDM-MIMO system with unitary Space-Time Code for two channel states at the receiver: Perfect Channel State Information (perfect CSI), where we assume that the channel is perfectly known to the receiver. The second state is: Absence of Channel State Information (No CSI), where the channel information is not available at the receiver. In the later case the Differential Unitary Code was implemented. The simulation results show performance degradation in the absence of channel estimation at the decoder. The differential coding method is favorable, in many situations especially when the channel estimation is impossible, or due to rapidly varying channel. An outer Reed Solomon encoding-decoding could be added to improve the error rate performance of the system.

Keywords: – Unitary Code, CSI, OFDM, MIMO.

1 Introduction

High data rate has become an important demand for many applications. In order to achieve high data rate requirements usually more bandwidth is required. However due to limitation in the spectrum, it is often impractical and expensive to increase the bandwidth. In this case using multiple transmit and receive antennas for spectrally efficient transmission can be considered as an excellent solution. This diversity scheme is used to obtain the well known multiple input multiple output MIMO channels.

The transmit diversity based on Space-Time coding has been studies a lot in literature [1-4]. Space-Time coding is necessary in MIMO systems. The code design is selected based on the channel estimation possibility. When the channel parameter estimation is possible at the receiver, there are various space-time codes that could be implemented [2]. On the other hand there are many situations where the channel estimation is impossible. In this case differential Space-Time code is used [1].

In Orthogonal Frequency Division Multiplexing (OFDM) modulation the entire channel is divided into many narrow parallel sub-channels, which in effect will increase the symbol duration and reduce the ISI caused by multipath fading. The use of OFDM is promising in the future high data rate wireless systems. The OFDM could be applied with MIMO systems to enhance the performance of the overall system, by combining together their merits.

In this paper we introduce the OFDM-MIMO based on Unitary Space Time coding. This approach is applied to the case of 2X2 antennas, and maximum likelihood ML decoder. The unitary space-time code is designed as given by [1]. The design could be also generalized for higher numbers of transmitters and receivers that are multiple of 2.

This paper is organized as follows: In section 2 we introduce a full description of the OFDM-MIMO system, showing the unitary space-time code features, and describing the two system structures for the CSI and No CSI channel states at the receivers. Both transmitters and receivers models are given in details. The channel is assumed to be Rayleigh fading channel. The simulations results are shown in section 3, and finally conclusion remarks are given in section 4.

2 System Model

The OFDM-MIMO system models, shown in Fig.1 and Fig.2, are explained in the following sub-sections.

2.1 Unitary Space Time Code

The unitary space-time codes have the following properties:

1- For the code matrices from the group code $\{C_q\}$ where $q = 0,1,\ldots,2^v-1$, (v=1,2,…)

$$C_q C_q^* = I \tag{1}$$

where I is the identity matrix, and * is the complex conjugate.

2- Using Hadamard matrix D (for the case of 2X2 code matrices)

$$D = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \tag{2}$$

a unitary group code is formed by $\{DC_q\}$. The code matrix DC_q is 2X2 unitary matrix, with the mth row transmitted over antenna m (*Space diversity*), and the columns are transmitted at successive time instances (*Time diversity*). And it complies with the following condition:

$$DC_q \cdot DC_q^* = 2I \tag{3}$$

For the quaternary QPSK constellation points {*1, -1, i, -i*} there are 32 code matrices (2X2) that fulfill the above conditions. Also we can add as another condition the distance criterion between two code matrices:

$$\xi = \det\left|C_q - C_{q'}\right| \tag{4}$$

where det| · | is the determinant of the matrix, and $q \neq q'$

The following list of matrices (16 matrices), shows the unitary space-time code matrices that could be used in the OFDM-MIMO system with two transmit antennas (M=2) and two receive antennas (N=2)

$$\begin{aligned}
C_0 &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} & C_1 &= \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix} \\
C_2 &= \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} & C_3 &= \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix} \\
C_4 &= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} & C_5 &= \begin{bmatrix} -i & 0 \\ 0 & i \end{bmatrix} \\
C_6 &= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} & C_7 &= \begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix} \\
C_8 &= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} & C_9 &= \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix} \\
C_{10} &= \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} & C_{11} &= \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix} \\
C_{12} &= \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} & C_{13} &= \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \\
C_{14} &= \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} & C_{15} &= \begin{bmatrix} -i & 0 \\ 0 & -i \end{bmatrix}
\end{aligned} \tag{5}$$

It was found that the code matrices $C_0, C_1, \ldots, C_7$ are optimum over the QPSK constellation points {*1, -1, i, -i*}, and for this reason they will be used (unless otherwise mentioned) in the OFDM-MIMO system design introduced in this paper.

2.2 System Structure

The OFDM-MIMO system structure for the two channel states: with CSI and No CSI are shown in Fig.1 and Fig.2 respectively.

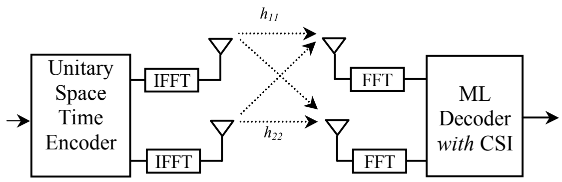

Fig.1 OFDM-MIMO system with CSI at the receiver.

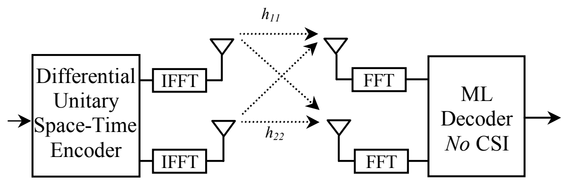

Fig.2 OFDM-MIMO system without CSI at the receiver.

2.3 Transmitter

The input data enters the encoder, and then the coded information is divided into two streams. Each stream leads to an OFDM modulator. The OFDM modulation is performed using the Inverse Fast Fourier Transform (IFFT). A description of the encoding and modulation process is explained next.

2.4 Encoding Process

The encoding process for the two channel states at the receiver is explained here;

2.4.1 Perfect Channel State Information

The mapping process between the input data and the corresponding code is performed in a straightforward manner. For example the input data of 000 110

100… could be mapped to the code matrices: C_0 C_6 C_4 … and so on.
The transmission equation for the perfect CSI case is given by:

$$S_\tau = D\, C_{d_\tau} \quad (6)$$

where $d_\tau \in \{0,1,2\ldots7\}$ is the transmitted data.
The transmitted block matrix before OFDM modulation,

$$S_{b,CSI} = \left[S_1 \;\cdots\; S_{K/2}\right] \quad (7\text{-a})$$

$$S_{b,CSI} = \begin{bmatrix} s_{1,1} & s_{1,2} \cdots & s_{1,K-1} & s_{1,K} \\ s_{2,1} & s_{2,2} \cdots & s_{2,K-1} & s_{2,K} \end{bmatrix} \quad (7\text{-b})$$

Where s_{11}, s_{12}, s_{21}, and s_{22} are the entries of the sub-matrix S_1. And $s_{1,K-1}$, $s_{1,K}$, $s_{2,K-1}$, and $s_{2,K}$ are the entries of the sub-matrix $S_{K/2}$. The transmitted block of the m-th antenna, after OFDM modulation, is expressed by:

$$S_{OFDM}\left[m,b\right] = \sum_{k=1}^{K} S_{b,CSI}\left(m,k\right) e^{i2\pi f_k T_b} \quad (8)$$

where K is the number of sub-channels, $f_k = f_0 + k/T_b$, f_0 is the carrier frequency, and T_b is the block time interval.

2.4.2 Absence of Channel State Information

The transmission equation is expressed by:

$$S_\tau = S_{\tau-1}\, C_{d_\tau} \quad (9)$$

where $d_\tau \in \{0,1,2\ldots7\}$ is the data to be transmitted, and $S_0 = D$, given that D is the initial transmitted matrix at the beginning of every differential OFDM block. The differential transmitted block before OFDM modulation could be expressed in the form,

$$S_{b,CSI} = \left[DC_d \; S_2 \;\cdots\; S_{K/2}\right] \quad (10\text{-a})$$

$$S_{b,noCSI} = \begin{bmatrix} s_{1,1} & s_{1,2} \cdots & s_{1,K-1} & s_{1,K} \\ s_{2,1} & s_{2,2} \cdots & s_{2,K-1} & s_{2,K} \end{bmatrix} \quad (10\text{-b})$$

where s_{11}, s_{12}, s_{21}, and s_{22} are the entries of the sub-matrix DC_d, while $s_{1,K-1}$, $s_{1,K}$, $s_{2,K-1}$, and $s_{2,K}$ are the entries of the sub-matrix $S_{K/2}$.

The transmitted differential OFDM block, over the m-th antenna, for the *No CSI* case is given by:

$$\hat{S}_{OFDM}\left[m,b\right] = \sum_{k=1}^{K} S_{b,noCSI}\left(m,k\right) e^{i2\pi f_k T_b} \quad (11)$$

2.5 OFDM Characteristics

The channel bandwidth is taken as 1.25 MHz, and is divided into 256 sub-channels. To make the tones orthogonal the symbol duration is T_s= 204.8μs. A guard interval T_g of 5.2μs is added in order to provide more protection from ISI due to channel multipath delay spread. The total block length: $T_b = T_s + T_g$= 210 μs. Each OFDM block will be modulated by a maximum of K=128 transmission matrices, as shown in (8) and (11). This OFDM-MIMO system has a bit rate of about 1.83 Mbps, and a bandwidth efficiency of approximately 1.46.

2.6 Channel Model

The MIMO channel matrix H is expressed by:

$$H = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \quad (12)$$

The entries h_{11}, h_{12}, h_{21}, h_{22} are independent Rayleigh distributed variables. The channel is considered constant during one OFDM block interval T_b, so that differential decoding is possible.

2.7 Receiver

The received signal passes through the OFDM demodulator represented by the Fast Fourier Transform (FFT). The received signal matrix after OFDM-demodulation is given below for each channel state.

2.7.1 Perfect Channel State Information

The received signal matrix after the FFT block is represented by:

$$R_{CSI} = \sqrt{\rho}\, H\, S_{CSI} + W \quad (13)$$

where ρ is the signal to noise ratio at each received antenna. H is the MIMO channel matrix with independent Rayleigh distributed entries, and W is an additive complex guassian noise with zero mean and unity variance.
The maximum likelihood ML decoder, for the unitary space-time code, is given by:

$$\hat{C} = \underset{\{C_d\}}{\arg\min} \, \mathbf{Tr}\left\{ \left(R_{CSI} - \sqrt{\rho}\, HC_d\right)\left(R_{CSI} - \sqrt{\rho}\, HC_d\right)^* \right\} \quad (14)$$

where Tr $\{\cdot\}$ is the trace function. The ML-decoder has to perform the search over 8 matrices, when decoding each C_d.

2.7.2 Absence of Channel State Information

The received signal matrix after OFDM demodulation is expressed as;

$$R_{noCSI} = \sqrt{\rho}\, H\, S_{noCSI} + W \quad (15)$$

The maximum likelihood ML-decoder for the differential unitary space-time coding:

$$\hat{C} = \underset{\{C_d\}}{\arg\max} \, \mathrm{Re}\, \mathbf{Tr}\left\{ C_d\, R^*_{\tau,noCSI}\, R_{\tau-1,noCSI} \right\} \quad (16)$$

where Re$(\cdot)$ is the real part, and $R_{\tau,noCSI}$ is the received matrix at time instance τ.

3 Simulation Results

A comparison between the two channel states, CSI and No CSI at the receiver, is shown in Fig. 3. In this figure we observe the degradation (about 3dB, noting that each 3-bits are considered as a word) in the performance of the differential coding-decoding, when compared to perfect channel state information at the receiver. This degradation is acceptable for cases of difficulty or even impossibility of finding the channel parameters at the receiver; in addition to the simplified receiver design for the case of Differential Decoding.

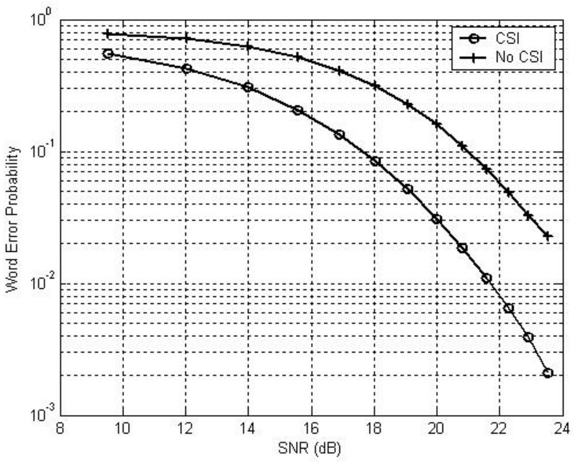

Fig.3 Word Error Probability using ML-decoder.

A further performance evaluation could be performed for the Differential OFDM-MIMO system, by comparing an optimum unitary space-time code of 8 matrices with a sub-optimum unitary space-time code of 16 matrices. The results, given in Fig.4, show the degradation in performance when using space-time code of 16 matrices. The advantage of the 16-code matrices is to increase the bit rate and to improvement the bandwidth efficiency.

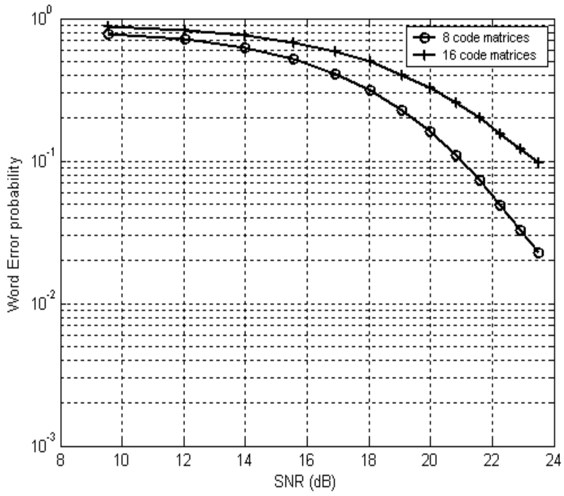

Fig.4 Word Error Probability comparison between 8 and 16 code matrices.

For the Differential OFDM-MIMO system with 8 code matrices, an Outer Reed Solomon encoding (before the Unitary Space-Time Coding) and decoding (after ML-decoder) is added to the system. This will improve the error rate performance as shown in Fig.5. The performance improvement will increase as the SNR increases.

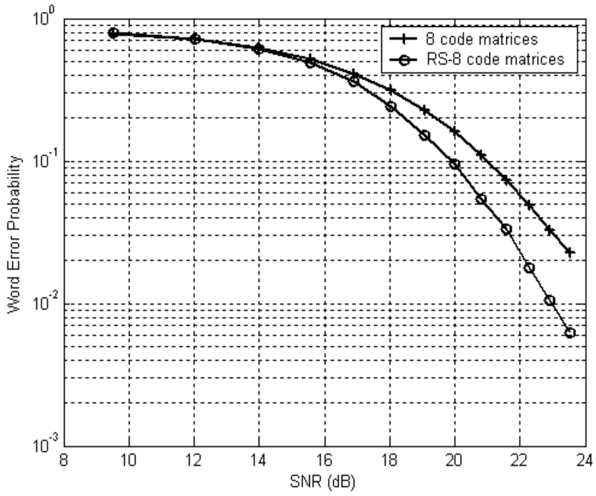

Fig.5 Word Error Probability with and without Reed Solomon outer coding.

4. Conclusions

In this paper we have presented the OFDM-MIMO system using Unitary Space Time Code. The coding-decoding technique was designed for two channel states: The perfect Channel State Information (CSI), and the absence of Channel State Information (No CSI). The results for the CSI case had better performance compared to the No CSI case, due to the assumption of perfect estimation of the channel parameters in the first case while no channel knowledge in the case of differential coding. The ML decoder in this case is simpler, and practical for situations with difficulties in channel estimation or rapidly fading channels. An outer Reed Solomon coding could be used to enhance the performance of the Differential OFDM-MIMO system.

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