Neuro-Fuzzy Systems Based On Weighted Triangular Norms

KRZYSZTOF CPAŁKA[1, 2], LESZEK RUTKOWSKI[1, 2]
(1) Department of Computer Engineering
Technical University of Częstochowa
and
(2) Department of Artificial Intelligence
WSHE University in Łódź
POLAND

Abstract:-In the paper we present new neuro-fuzzy systems. They are called the AND–type neuro-fuzzy inference systems (NFIS). Based on the input-output data we learn not only parameters of membership functions but also a type of the systems and aggregating parameters. We propose the weighted t-norm and s-norm to neuro-fuzzy inference systems.

Key-Words:-Fuzzy systems, Compromise implications, Weighted triangular norms, Neuro-fuzzy inference systems.

1. Introduction

In the literature two approaches have been proposed to design neuro-fuzzy systems with linguistic consequences. The first approach, called Mamdani method, uses "engineering implications" for inference (see Mendel [4]) and disjunction to aggregate individual rules. In the Mamdani approach the most widely used operators measuring the truth of the relation between input and output are the following:

$$I(a,b) = T\{a,b\} \overset{e.g.}{=} a \cdot b \tag{1}$$

The aggregation is performed by an application of s-norm:

$$\begin{aligned} S\{a_1,a_2,\ldots,a_n\} &= S\{\mathbf{a}\} \\ &= a_1 \overset{S}{*} a_2 \overset{S}{*} \ldots \overset{S}{*} a_n \\ &= \underset{i=1}{\overset{n}{S}}\{a_i\} \\ &\overset{e.g.}{=} 1-\prod_{i=1}^{n}(1-a_i) \end{aligned} \tag{2}$$

The other paradigm applies fuzzy implications for inference and conjunction for aggregation. For example we use an S-implication:

$$I(a,b) = S\{1-a,b\} \overset{e.g.}{=} 1\text{-}a + a \cdot b \tag{3}$$

For NFIS with a logical implication the aggregation is realized by a t-norm:

$$\begin{aligned} T\{a_1,a_2,\ldots,a_n\} &= T\{\mathbf{a}\} \\ &= a_1 \overset{T}{*} a_2 \overset{T}{*} \ldots \overset{T}{*} a_n \\ &= \underset{i=1}{\overset{n}{T}}\{a_i\} \\ &\overset{e.g.}{=} \prod_{i=1}^{n} a_i \end{aligned} \tag{4}$$

It should be noted that the aggregation of antecedents in each rule is performed by the same formula (4) for both Mamdani and logical-type systems. Various structures of NFIS are discussed in [1-9].

In this paper we propose a new class of neuro-fuzzy inference systems characterized by automatic determination of a fuzzy inference in the process of learning. Consequently, the structure of the system is determined in the process of learning. We refer to this class as to AND-type fuzzy systems. The performance of neuro-fuzzy structures is tested on typical classification and approximation problems.

2. Flexibility in NFIS

2.1. NFIS realized by compromise implication

Following by Yager and Filev [12] we propose a compromise fuzzy implication given by:

$$I(a,b) = N(\lambda)T\{a,b\} + \lambda S\{N(a),b\} \tag{5}$$

where $\lambda \in [0,1]$, $N(\lambda)=1-\lambda$, and based on implication (5) we derive a compromise neuro-fuzzy system. It includes Mamdani-type, logical-type, more Mamdani-type than logical-type and more logical-type than Mamdani-type fuzzy inference systems. It should be noted that parameter λ, determining a type of the system, can be found in the process of learning.

2.2. NFIS realized by t-norms and s-norms with weighted arguments

We propose the weighted t-norm:

$$T^*\{a_1,a_2;w_1,w_2\} = T\begin{Bmatrix} 1-w_1(1-a_1), \\ 1-w_2(1-a_2) \end{Bmatrix} \tag{6}$$

For a moment we interpret parameters a_1 and a_2 as antecedents of a rule. The weights w_1 and w_2 are corresponding credibilities of the both antecedents. Observe that:

$$T^*\{a_1,a_2;1,1\} = T\{a_1,a_2\} \tag{7}$$

and

$$\begin{aligned} T^*\{a_1,a_2;0,w_2\} &= T\{1,1-w_2(1-a_2)\} \\ &= 1-w_2(1-a_2) \end{aligned} \tag{8}$$

The s-norm corresponding to the t-norm (6) is defined as follows:

$$S^*\{a_1,a_2;w_1,w_2\} = S\{w_1a_1,w_2a_2\} \tag{9}$$

The weights w_1 and w_2 can be found in the process of learning subject to the constraint $w_1, w_2 \in [0,1]$. One may apply the weighted t-norm for selection of significant inputs.

3. Compromise weighted NFIS

In this paper, we consider multi-input, single-output NFIS mapping $\mathbf{X} \to \mathbf{Y}$, where $\mathbf{X} \subset \mathbf{R}^n$ and $\mathbf{Y} \subset \mathbf{R}$.

The fuzzifier performs a mapping from the observed crisp input space $\mathbf{X} \subset \mathbf{R}^n$ to the fuzzy sets defined in $\mathbf{X}$. The most commonly used fuzzifier is the singleton fuzzifier which maps $\bar{\mathbf{x}} = [\bar{x}_1,\ldots,\bar{x}_n] \in \mathbf{X}$ into a fuzzy set $A' \subseteq \mathbf{X}$ characterized by the membership function:

$$\mu_{A'}(\mathbf{x}) = \begin{cases} 1 & \text{if} \quad \mathbf{x} = \bar{\mathbf{x}} \\ 0 & \text{if} \quad \mathbf{x} \neq \bar{\mathbf{x}} \end{cases} \tag{10}$$

The fuzzy rule base consists of a collection of N fuzzy IF-THEN rules in the form:

$$R^{(k)}: \text{IF } \mathbf{x} \text{ is } A^k \text{ THEN } y \text{ is } B^k \tag{11}$$

where $\mathbf{x} = [x_1,\ldots,x_n] \in \mathbf{X}$, $y \in \mathbf{Y}$, $A_1^k, A_2^k, \ldots, A_n^k$ are fuzzy sets characterized by membership functions $\mu_{A_i^k}(x_i)$, whereas B^k are fuzzy sets characterized by membership functions $\mu_{B^k}(y)$, respectively, $k = 1,\ldots,N$.

The fuzzy inference determines a mapping from the fuzzy sets in the input space $\mathbf{X}$ to the fuzzy sets in the output space $\mathbf{Y}$. Each of N rules (11) determines a fuzzy set $\bar{B}^k \subset \mathbf{Y}$ given by the compositional rule of inference:

$$\bar{B}^k = A' \circ \left(A^k \to B^k\right) \tag{12}$$

where $A^k = A_1^k \times A_2^k \times \ldots \times A_n^k$. Fuzzy sets $\bar{B}^k$, according to the formula (12), are characterized by membership functions expressed by the *sup-star* composition:

$$\mu_{\bar{B}^k}(y) = \sup_{\mathbf{x} \in \mathbf{X}} \left\{ \mu_{A'}(\mathbf{x}) \overset{T}{*} \mu_{A_1^k \times \ldots \times A_n^k \to B^k}(\mathbf{x}, y) \right\} \tag{13}$$

where $\overset{T}{*}$ can be any operator in the class of t-norms. It is easily seen that for a crisp input $\bar{\mathbf{x}} \in \mathbf{X}$, i.e. a singleton fuzzifier (10), formula (13) becomes:

$$\begin{aligned} \mu_{\bar{B}^k}(y) &= \mu_{A_1^k \times \ldots \times A_n^k \to B^k}(\bar{\mathbf{x}}, y) \\ &= \mu_{A^k \to B^k}(\bar{\mathbf{x}}, y) \\ &= I\left(\mu_{A^k}(\bar{\mathbf{x}}), \mu_{B^k}(y)\right) \end{aligned} \tag{14}$$

where $I(\cdot)$ is an "engineering implication" or fuzzy implication. The aggregation operator, applied in order to obtain the fuzzy set B' based on fuzzy sets $\bar{B}^k$, is the t-norm or s-norm operator, depending on the type of fuzzy implication.

The defuzzifier performs a mapping from a fuzzy set B' to a crisp point $\bar{y}$ in $\mathbf{Y} \subset \mathbf{R}$. The COA (centre of area) method is defined by following formula:

$$\bar{y} = \frac{\int_{\mathbf{Y}} y \mu_{B'}(y) dy}{\int_{\mathbf{Y}} \mu_{B'}(y) dy} \quad \text{or by} \quad \bar{y} = \frac{\sum_{r=1}^{N} \bar{y}^r \cdot \mu_{B'}(\bar{y}^r)}{\sum_{r=1}^{N} \mu_{B'}(\bar{y}^r)} \tag{15}$$

in the discrete form, where $\bar{y}^r$ denotes centres of the membership functions $\mu_{B^r}(y)$, i.e. for $r = 1,\ldots,N$:

$$\mu_{B^r}(\bar{y}^r) = \max_{y \in \mathbf{Y}} \left\{ \mu_{B^r}(y) \right\} \tag{16}$$

Now we propose new structure of NFIS. The novel system is characterized by:

- weights in antecedents of the rules $w_{i,k}^{\tau} \in [0,1]$, $i = 1,\ldots,n$, $k = 1,\ldots,N$,
- weights in aggregation of the rules $w_k^{\mathrm{agr}} \in [0,1]$, $k = 1,\ldots,N$.

The weighted soft NFIS of the AND-type is presented below:

$$\tau_k(\bar{\mathbf{x}}) = T^* \left\{ \begin{array}{c} \mu_{A_1^k}(\bar{x}_1),\ldots,\mu_{A_n^k}(\bar{x}_n)\ ; \\ w_{1,k}^{\tau},\ldots,w_{n,k}^{\tau} \end{array} \right\} \tag{17}$$

$$I_{k,r}(\bar{\mathbf{x}}, \bar{y}^r) = \left(\begin{array}{c} (1-\lambda) T\{\tau_k(\bar{\mathbf{x}}), \mu_{B^k}(\bar{y}^r)\} + \\ + \lambda S\{1-\tau_k(\bar{\mathbf{x}}), \mu_{B^k}(\bar{y}^r)\} \end{array} \right) \tag{18}$$

$$\mathrm{agr}_r(\bar{\mathbf{x}}, \bar{y}^r) = \left(\begin{array}{c} N(\lambda)\, S^* \left\{ \begin{array}{c} I_{1,r}(\bar{\mathbf{x}}, \bar{y}^r),\ldots,I_{N,r}(\bar{\mathbf{x}}, \bar{y}^r); \\ w_1^{\mathrm{agr}},\ldots,w_N^{\mathrm{agr}} \end{array} \right\} + \\ + \lambda T^* \left\{ \begin{array}{c} I_{1,r}(\bar{\mathbf{x}}, \bar{y}^r),\ldots,I_{N,r}(\bar{\mathbf{x}}, \bar{y}^r); \\ w_1^{\mathrm{agr}},\ldots,w_N^{\mathrm{agr}} \end{array} \right\} \end{array} \right) \tag{19}$$

$$\bar{y} = \frac{\sum_{r=1}^{N} \bar{y}^r \cdot \mathrm{agr}_r(\bar{\mathbf{x}}, \bar{y}^r)}{\sum_{r=1}^{N} \mathrm{agr}_r(\bar{\mathbf{x}}, \bar{y}^r)} \tag{20}$$

Compromise operator in formula (18) is defined as follows:

$$\begin{aligned} \tilde{N}_\nu(a) &= (1-\nu)N(a) + \nu N(N(a)) \\ &= (1-\nu)N(a) + \nu a \end{aligned} \tag{21}$$

where $\nu \in [0,1]$.

4. Simulation results

Compromise Weighted NFIS, described by formulas (17)-(20), is simulated on *Iris classification* [11] and *Modelling of Static Nonlinear Function (HANG)* [10] problems.

4.1. Iris classification problem

The Iris data is a common benchmark in classification and pattern classification studies. It contains 50 measurements of four features (sepal length in cm, sepal width in cm, petal length in cm, petal width in cm) from each of the following three species: iris setosa, iris versicolor, and iris virginica. In our experiments, all sets are divided into a learning sequence (105 sets) and a testing sequence (45 sets). The results are given in Table 1 and Fig. 1.

Table 1. Experimental results
(Iris classification problem)

Name of flexibility parameter	Initial/final values	Mistakes [%] (learning sequence)		Mistakes [%] (testing sequence)	
		Zadeh tr. norms	Product tr. norms	Zadeh tr. norms	Product tr. norms
λ	0.5/ 1.0000	0,95%	0,95%	4,44%	4,44%
λ	0/ -	0,95%	0,95%	6,67%	6,67%
λ $\mathbf{w}^{\tau}$ $\mathbf{w}^{\mathrm{agr}}$	0.5/ 1.0000 **1**/Fig. 1 **1**/Fig. 1	0,00%	0,00%	4,44%	4,44%

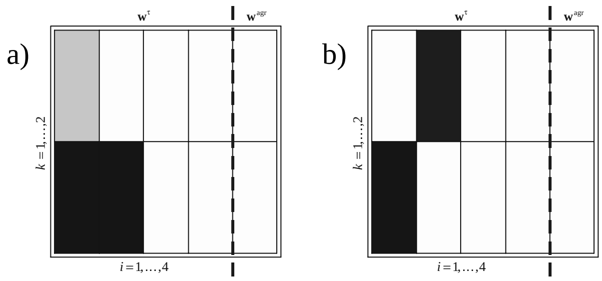

Fig. 1. Weights representation in the Iris problem for AND-type system and a) Zadeh triangular norms, b) product triangular norms

4.2. Modelling of Static Nonlinear Function (HANG) problem

The problem is to approximate a nonlinear function given by

$$y(x_1, x_2) = \left(1 + x_1^{-2} + x_2^{-1.5}\right)^2 \tag{22}$$

We obtained 50 input-output data by sampling the input range $x_1, x_2 \in [1,5]$. The results are given in Table 2 and Fig. 2.

5. Final remarks

In this paper we studied a new class of neuro-fuzzy systems characterized by a compromise fuzzy implication. The final fuzzy reasoning has been established in the process of learning of parameter λ. Due to the incorporation of weighted triangular norms into the construction of neuro-fuzzy systems, we have achieved a high accuracy in simulation problems given in Section IV.

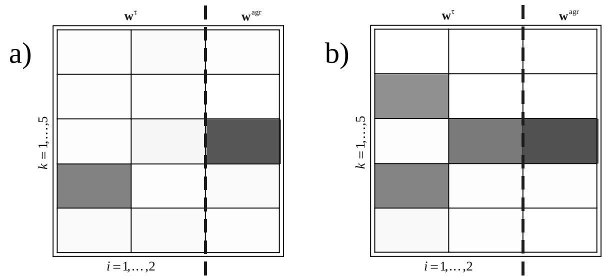

Fig. 2. Weights representation in the HANG problem for AND-type system and a) Zadeh triangular norms, b) product triangular norms

Table 2. Experimental results
(Iris classification problem)

Name of flexibility parameter	**Initial/final values**	**RMSE (learning sequence)**	
		Zadeh tr. norms	**Product tr. norms**
λ	0.5/ 0.0000	0,1258	0,1098
λ	1.0/ -	0,1516	0,1205
λ $\mathbf{w}^{\tau}$ $\mathbf{w}^{agr}$	0.5/ 0.0000 **1**/Fig. 2 **1**/Fig. 2	0,0964	0,0529

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