A Generation Method for Constructing Complete Complementary Sequences

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Abstruct: - From the view point of sequence applications, especially spread spectrum mobile communications, sequence sets with nice correlation properties are very important. A sequences set, whose autocorrelation sum is without sidelobe and crosscorrelation sum is zero for every term, is called a set of complete complementary sequences. The complete complementary sequences attract attention in spread spectrum communications because of their nice correlation properties.

In [1], Suehiro has been proposed a class of complete complementary sequences of length N^n. In this paper we propose a new method to generate sequence sets of length MN which compose complete complementary sequences.

Key-Words:- Complete Complementary Sequence, Kronecker Product, Correlation Function, Zero Correlation Zone, CDMA

1 Introduction

It is well-known that the ideal correlation property of finite length sequence can not achieved by single sequence [2]. Suehiro has been proposed a method of constructing a class of complete complementary sequences of length $N^n (N, n \in Z^+; n \geq 2)$, where N is the number of sequences in each subset in the sequence set[1]. Suehiro also proposed the method of coded addition of sequences for designing signals with increased information transmission rate[3][4]. In this method, the channel usage efficiency is increased by using the complete complementary sequences, because they are without co-channnel interference. But in practice, the length of N^n seems long when we make use of the set of sequences in applications, especialy when N is large.

In this paper, we propose a generation method that can construct a class of complete complementary sequences of length $MN (M, N \in Z^+; N \geq M)$. The length of sequence in the set of complete complementary sequence generated by this method is considerably short and abundant compared with the previous method [1].

2 The definition of notation

- Correlation Function

The correlation function of sequence $S_0(n)$ and sequence $S_1(n)$ is defined as

$$R_{S_0,S_1}(n) = \sum_{m=-\infty}^{+\infty} S_0(m)S_1^*(m-n) \qquad (1)$$

where $S_0(n)$ and $S_1(n)$ are complex sequences of length L_0 and L_1

$$S_0(n) = \sum_{l=0}^{L_0-1} s_{0,l}\delta(n-l) \qquad (2)$$

$$S_1(n) = \sum_{l=0}^{L_1-1} s_{1,l}\delta(n-l) \qquad (3)$$

$S^*(n)$ is the complex conjugate of sequence $S(n)$ and $\delta(n)$ is the Dirac function

$$\delta(n) = \begin{cases} 1, & (n=0) \\ 0, & (n \neq 0) \end{cases} \qquad (4)$$

- Unitary Matrix

We also define unitary matrix Z, when matrix Z of size $N \times N$ satisfies

$$Z^T Z^* = Z^* Z^T = NI \qquad (5)$$

Then we call matrix Z unitary matrx. where Z^T is transposed matrix of Z and Z^* is conjugate matrix of Z.

Obviously, if Z is unitary matrix of size $N \times N$, then it satisfies

$$Z_{i,\cdot} Z_{j,\cdot}^{T*} = Z_{\cdot,i}^T Z_{\cdot,j}^* = N\delta(i-j) \qquad (6)$$

Where, $Z_{i,\cdot}$ is ith row vector and , $Z_{\cdot,j}$ is jth column vector of matrix Z.

- Complete Complementary Sequence

The set of complete complementary sequences is defined as follows. There are $M \times N$ sequences of length L

$$S_{i,k}(n) = \sum_{l=0}^{L-1} s_{i,k,l}\delta(n-l) \qquad (7)$$

$$|s_{i,k,l}| = 1 \qquad (8)$$

from a sequence set as

$$\mathcal{S} = [S_{i,k}(n)] =$$

$$\begin{bmatrix} S_{0,0}(n) & S_{0,1}(n) & \cdots & S_{0,N-1}(n) \\ S_{1,0}(n) & S_{1,1}(n) & \cdots & S_{1,N-1}(n) \\ \vdots & \vdots & \cdots & \vdots \\ S_{M-1,0}(n) & S_{M-1,1}(n) & \cdots & S_{M-1,N-1}(n) \end{bmatrix}$$

If the sum of correlation function satiesfies

$$\frac{1}{NL}\sum_{k=0}^{N-1} R_{S_{i,k},S_{j,k}}(n) = \delta(n)\delta(i-j) \qquad (9)$$
$$(0 \le i,j \le M-1; 0 \le k \le N-1)$$

Then, we call $\mathcal{S}$ the set of complete complementary sequences. Sequences $S_{i,k}(n)$ is the kth sequence in ith subset in the set.

- "Diag" Operator

In this paper, the diagonalizing operator "Diag" is defined as
If there is a vector $A = (a_0, a_1, \cdots, a_{N-1})$, Then

$$Diag(A) = \begin{bmatrix} a_0 & & & \\ & a_1 & & \\ & & \ddots & \\ & & & a_{N-1} \end{bmatrix} \qquad (10)$$

Form example, if $A = (1,2,3,4)$, Then

$$Diag(A) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} \qquad (11)$$

3 Method of Generation

We get a set of complete complementary sequences of length MN use matrix A whose derive from M column of unitary matrix of size $N \times N$ and unitary matrics B, C of size $M \times M$ and $N \times N$.

First we generate matrix D use above matrics A, B, C as follow

$$D = (B \otimes C) \cdot Diag\,(Vec\ A) \tag{12}$$

and from row of generated matrix D, construct seqeunces set

$$\mathcal{S} = [S_{i,k}(n)] = \left[\sum_{l=0}^{MN-1} d_{iN+k,l}\delta(n-l) \right] \tag{13}$$
$$(0 \le i \le M-1; 0 \le k \le N-1)$$

Then, this is a set of complete complementary sequences whose including M subsets seqeunces with each subset are N sequences of length MN.

Where

$$Vec\ A = \begin{bmatrix} A_{\cdot,0} \\ A_{\cdot,1} \\ \vdots \\ A_{\cdot,N-1} \end{bmatrix} \tag{14}$$

and $\otimes$ is Kronecker product.

Futhermore, if M will divide into N, we get N/M matrics $A_0, A_1, \cdots, A_{N/M-1}$ of size $M \times N$ from a unitary matrix of size $N \times N$ and generate matrix

$$D = \begin{bmatrix} (B \otimes C) \cdot Diag\,(Vec\ A_0) \\ (B \otimes C) \cdot Diag\,(Vec\ A_1) \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ (B \otimes C) \cdot Diag\left(Vec\ A_{N/M-1}\right) \end{bmatrix} \tag{15}$$

Similarly, construct sequences set

$$\mathcal{S} = [S_{i,k}(n)] = \left[\sum_{l=0}^{MN-1} d_{iN+k,l}\delta(n-l) \right] \tag{16}$$
$$(0 \le i, k \le N-1)$$

Then, this is a set of complete complementary sequences whose including N subsets seqeunces with each subset are N sequences of length MN.

4 Proof

Hence,

$$\begin{aligned} D &= (B \otimes C) \cdot Diag\,(Vec\ A) \\ &= \begin{bmatrix} b_{0,0}C & b_{0,1}C & \cdots & b_{0,M-1}C \\ b_{1,0}C & b_{1,1}C & \cdots & b_{1,M-1}C \\ \vdots & \vdots & \cdots & \vdots \\ b_{M-1,0}C & b_{M-1,1}C & \cdots & b_{M-1,M-1}C \end{bmatrix} \\ &\quad \cdot Diag \begin{bmatrix} A_{\cdot,0} \\ A_{\cdot,1} \\ \vdots \\ A_{\cdot,N-1} \end{bmatrix} \end{aligned}$$

If we set

$$l = pN + q \quad (0 \le p \le M-1; 0 \le q \le N-1) \tag{17}$$

Then

$$d_{iN+j,l} = d_{iN+j,pN+q} = a_{q,p}b_{i,p}c_{j,q} \tag{18}$$

$$S_{i,j}(n) = \sum_{\substack{q=0 \\ p=0}}^{\substack{M-1 \\ N-1}} a_{q,p}b_{i,p}c_{j,q}\delta(n-pN-q) \tag{19}$$

The sum of correlation function is

$$\begin{aligned} & \sum_{k=0}^{N-1} R_{S_{i,k},S_{j,k}}(n) \\ &= \sum_{\substack{m=-\infty \\ k=0}}^{\substack{N-1 \\ +\infty}} S_{i,k}(m)S^*_{j,k}(m-n) \\ &= \sum_{\substack{m=-\infty \\ k=0}}^{\substack{N-1 \\ +\infty}} \left(\sum_{\substack{q=0 \\ p=0}}^{\substack{M-1 \\ N-1}} a_{q,p}b_{i,p}c_{k,q}\delta(m-pN-q) \right) \\ &\quad \cdot \left(\sum_{\substack{v=0 \\ u=0}}^{\substack{M-1 \\ N-1}} a^*_{v,u}b^*_{j,u}c^*_{k,v}\delta[m-n-uN-v] \right) \\ &= \sum_{\substack{q=0 \\ p=0}}^{\substack{M-1 \\ N-1}} \sum_{\substack{v=0 \\ u=0}}^{\substack{M-1 \\ N-1}} a_{q,p}a^*_{v,u}b_{i,p}b^*_{j,u}C^T_{\cdot,q}C^*_{\cdot,v} \\ &\quad \cdot\delta[n-(p-u)N-(q-v)] \\ &= N \sum_{\substack{u=0 \\ p=0}}^{\substack{M-1 \\ M-1}} A^T_{\cdot,p}A^*_{\cdot,u}b_{i,p}b^*_{j,u}\delta[n-(p-u)N] \end{aligned}$$

$$
\begin{aligned}
&= N^2 B_{i,\cdot} B_{j,\cdot}^{T*} \delta[n] \\
&= MN^2 \delta(n)\delta(i-j)
\end{aligned}
$$

$$
\left.\frac{1}{NL}\sum_{k=0}^{N-1} R_{S_{i,k},S_{j,k}}(n)\right|_{L=MN} = \delta(n)\delta(i-j)
$$

5 Example

In this section, we present an example that supports our generation.

Set $M = 2, N = 4$, and use Hadamard matrix as unitary matrix.

$$
A_0 = \begin{bmatrix} ++ \\ +- \\ ++ \\ +- \end{bmatrix} \quad A_1 = \begin{bmatrix} ++ \\ +- \\ -- \\ -+ \end{bmatrix}
$$

$$
B = \begin{bmatrix} ++ \\ +- \end{bmatrix} \quad C = \begin{bmatrix} ++++ \\ +-+- \\ ++-- \\ +--+ \end{bmatrix}
$$

Then

$$
\begin{aligned}
D_0 &= (B \otimes C) \cdot Diag\,(Vec\ A_0) \\
&= \begin{bmatrix} ++++++++ \\ +-+-+-+- \\ ++--++-- \\ +--++--+ \\ ++++---- \\ +-+--+-+ \\ ++----++ \\ +--+-++- \end{bmatrix} \cdot \begin{bmatrix} +0000000 \\ 0+000000 \\ 00+00000 \\ 000+0000 \\ 0000+000 \\ 00000-00 \\ 000000+0 \\ 0000000- \end{bmatrix} \\
&= \begin{bmatrix} +++++-+- \\ +-+-++++ \\ ++--+--+ \\ +--+++-- \\ ++++-+-+ \\ +-+----- \\ ++---++- \\ +--+--++ \end{bmatrix}
\end{aligned}
$$

From rows of matrix D_0, we get the two subsets of complete complementary sequences.

The sequences in 0th subset is

$$
\begin{aligned}
S_{0,0} &= (+++++-+-) \\
S_{0,1} &= (+-+-++++) \\
S_{0,2} &= (++--+--+) \\
S_{0,3} &= (+--+++--)
\end{aligned}
$$

and the sequences in 1st subset is

$$
\begin{aligned}
S_{1,0} &= (++++-+-+) \\
S_{1,1} &= (+-+-----) \\
S_{1,2} &= (++---++-) \\
S_{1,3} &= (+--+--++)
\end{aligned}
$$

Obviously, this is a set of complete complementary sequence consist of two subsets seqeunces with each subset are 4 sequences of length 8.

Similarly, we generate matrix D_1 as follow

$$
\begin{aligned}
D_1 &= (B \otimes C) \cdot Diag\,(Vec\ A_1) \\
&= \begin{bmatrix} ++++++++ \\ +-+-+-+- \\ ++--++-- \\ +--++--+ \\ ++++---- \\ +-+--+-+ \\ ++----++ \\ +--+-++- \end{bmatrix} \cdot \begin{bmatrix} +0000000 \\ 0+000000 \\ 00-00000 \\ 000-0000 \\ 0000+000 \\ 00000-00 \\ 000000-0 \\ 0000000+ \end{bmatrix} \\
&= \begin{bmatrix} ++--+--+ \\ +--+++-- \\ +++++-+- \\ +-+-++++ \\ ++---++- \\ +--+--++ \\ ++++-+-+ \\ +-+----- \end{bmatrix}
\end{aligned}
$$

Then, we can get another two subsets of sequences.

The sequences in 2nd subset is

$$
\begin{aligned}
S_{2,0} &= (++--+--+) \\
S_{2,1} &= (+--+++--) \\
S_{2,2} &= (+++++-+-) \\
S_{2,3} &= (+-+-++++)
\end{aligned}
$$

The sequences in 3rd subset is

$$
\begin{aligned}
S_{3,0} &= (++---++-) \\
S_{3,1} &= (+--+--++) \\
S_{3,2} &= (++++-+-+) \\
S_{3,3} &= (+-+-----)
\end{aligned}
$$

This also is a set of complete complementary sequences consist of two subsets seqeunces.

Combine above tow sets of sequences

$$\mathcal{S}_0 = \begin{bmatrix} S_{0,0}(n)\, S_{0,1}(n)\, S_{0,2}\, S_{0,3}(n) \\ S_{1,0}(n)\, S_{1,1}(n)\, S_{1,2}\, S_{1,3}(n) \end{bmatrix}$$

and

$$\mathcal{S}_1 = \begin{bmatrix} S_{2,0}(n)\, S_{2,1}(n)\, S_{2,2}\, S_{2,3}(n) \\ S_{3,0}(n)\, S_{3,1}(n)\, S_{3,2}\, S_{3,3}(n) \end{bmatrix}$$

construct a set of sequences as follow

$$\mathcal{S} = \begin{bmatrix} S_{0,0}(n)\, S_{0,1}(n)\, S_{0,2}(n)\, S_{0,3}(n) \\ S_{1,0}(n)\, S_{1,1}(n)\, S_{1,2}(n)\, S_{1,3}(n) \\ S_{2,0}(n)\, S_{2,1}(n)\, S_{2,2}(n)\, S_{2,3}(n) \\ S_{3,0}(n)\, S_{3,1}(n)\, S_{3,2}(n)\, S_{3,3}(n) \end{bmatrix}$$

then this set of sequences also is the set of complete complementary sequences whose consist of four subsets seqeunces with each subset are 4 sequences of length 8.

6 Conclusion

In this paper, we propose a generation method to construct the set of complete complementary sequences of length MN. The sequence length generated by the proposed method is short and abundant compare with the previous method proposed by Suehiro[1] which produces sequences of length N^n.

	Previous Method		New Method	
L	N_1	N_1^n	N_2	M
8	2	4	2	4
16	2	8	2	8
			4	4
32	2	16	2	16
			4	8
			8	4
64	2	32	2	32
			4	16
			8	8
			16	4
	4	16	4	16
128	2	64	2	64
			4	32
			8	16
			16	8
			4	32

Table 1

In Table 1 we compared length and number of sequences in the sequences set generated by the proposed method with previous method.

In addition, our method is really simple and easy for the practical use.

References

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