Design of a Robust Controller for a Heat Exchanger System with ℓ_∞-Performance Objectives and Repeated Perturbations

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Abstract: - This paper presents a novel approach to robust performance controller design achieving a pre-specified ℓ_∞-norm-bounded performance objective. The class of systems considered is subject to structured, *repeated*, linear time-varying, induced ℓ_∞-norm-bounded perturbations. The efficiency of the approach is assessed in application to a heat exchanger system.

Key-Words: - Robust Controller Synthesis, ℓ_∞-Stability, Repeated Perturbations, Optimal Control.

1 Introduction

Robust ℓ_p-stability problems are usually re-stated as satisficing problems involving the computation of a structured norm (SN) (see §2 for definition). However, the computation of SN is generally difficult. It is hence customary to substitute SN by its upper bound approximation as derived from the scaled small gain theorem, see [12].

In the particular case when the perturbations are structured, independent (i.e., not repeated), linear time-varying, and induced ℓ_∞-norm-bounded, it was shown by Khammash, [5], that the abovementioned upper bound is indeed equal to the SN value itself. A few years later, similar results were proved by Shamma, [10], for the induced ℓ_2-norm-bounded problem and by Young&Dahleh, [11], for the general induced ℓ_p-norm-bounded problem.

It is well known that the SN upper bound can be cast in the form of a scaled ℓ_1-norm minimization problem. In the case of repeated perturbations, the scaling matrices are block diagonal. In [1], it is shown how the optimization domain (i.e., the set of admissible scaling matrices) of the ℓ_1 problem involved can be significantly reduced while preserving global optimality. In [2], the work of Khammash (see [5] and [6]) is further generalize by the development of a novel lower bound for the associated SN. This lower bound allows to estimate the conservativeness of the scaled small gain theorem in applications to problem with repeated perturbations. A necessary robust ℓ_∞-stability condition follows directly from this lower bound.

In this paper, it is shown how the results first presented in [1] and [2] can be applied to solve an ℓ_∞ robust performance problem for the linearized model of a heat exchanger system. The simulation results demonstrate the usefulness of the theory presented in [1] and [2].

2 Notation, Problem Statement, and Methodology

2.1 Notation

Let $\mathbb{Z}^+$ and $\mathbb{Z}^*$ denotes the set of positive and non-negative integers, respectively.

Let ℓ_n^p denotes the space of all sequences of vectors of lenght n $\{s(k)\}_{k=0}^\infty$, $s(k) \in \mathbb{R}^n$, equipped with the norm $\|s\|_p \triangleq \sum_{k=0}^{\infty} |s(k)|^p < \infty$ (note that $\|s\|_\infty \triangleq \sup_{k\geq 0} |s(k)|$). Given a bounded operator $S : \ell_n^p \mapsto \ell_m^p$, let $\|S\|_{p-ind} \triangleq \frac{\|S(s)\|_p}{\|s\|_p}$ be the induced-p norm of S. Furthermore, if S is linear and causal, then $S(s)$ is characterized by the convolution $(S * s)(k) \triangleq \sum_{l=0}^{k} S(k,l)s(l)$, where $S(k,l)$ denotes the kernel of S.

Let S_1 and S_2 be two systems, then $F_l(S_1, S_2)$ and $F_u(S_1, S_2)$ denotes a lower and a upper linear fractional

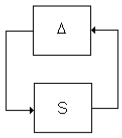

Figure 1: $S\Delta$-loop

transformation between these two systems, respectively.

Suppose that a discrete-time linear time-invariant (LTI) system S with p inputs and q outputs is characterized by the impulse response $\{S(k)\}_{k=0}^{\infty}$, where $k \in \mathbb{Z}^*$ is a discrete time instant. If S is stable, then $\{S(k)\}_{k=0}^{\infty} \in \ell_1^{q\times p}$ and the ℓ_1 norm of S is given by

$$\|S\|_1 \triangleq \max_{i\in\{1,\ldots,q\}} \sum_{j=1}^{p} \sum_{k=0}^{\infty} |S_{ij}(k)|.$$

Let the set $\mathbf{\Delta}$ denote a given arbitrary class of admissible perturbations. This set is assumed to carry all the important information relevant to the nature and structure of the perturbations. Assume that $\Delta \in \mathbf{\Delta}$ and that S is an LTI system of dimension compatible with Δ, as illustrated by Fig.1. The *structured norm* of S is then defined as:

$$SN_{\mathbf{\Delta},p}(S) \triangleq \frac{1}{\inf\limits_{\Delta\in\mathbf{\Delta}} \{\|\Delta\|_{p-ind} : (I - S\Delta)^{-1} \text{not } \ell_p\text{-stable}\}}.$$

If for every $\Delta \in \mathbf{\Delta}$, $(I - S\Delta)^{-1}$ remains ℓ_p-stable, then $SN_{\mathbf{\Delta},p}(S) = 0$. Recall that, the structured norm is not a norm, see [3]. Also, assuming that $\|\Delta\|_{p-ind} < 1$, it is straightforward to show that robust ℓ_p-stability of the $S\Delta$-loop is equivalent to the condition: $SN_{\mathbf{\Delta},p}(S) \leq 1$.

More specifically, given a $n \in \mathbb{Z}^+$, a $\{r_l\}_{l=1}^{n}$, $r_l \in \mathbb{Z}^+$, and a $n_P \in \mathbb{Z}^+$, define the following classes of linear time-varying (LTV) perturbations:

$$\begin{aligned}
\mathbf{\Delta}_{LTV}^{p\times q} &\triangleq \{\Delta : \Delta \text{ is causal, LTV, has } p \text{ outputs and } q \text{ inputs}\},\\
\mathbf{\Delta}_S &\triangleq \{\text{diag}(\delta_{r_1} I_1, \ldots, \delta_{r_n} I_n) : \delta_l \in \mathbf{\Delta}_{LTV}^{1\times 1},\\
&\qquad l \in \{1, \ldots, n\}\}, \qquad (1)\\
\mathbf{\Delta}_C &\triangleq \{\text{diag}(\Delta_S, \Delta_P) : \Delta_S \in \mathbf{\Delta}_S, \Delta_P \in \mathbf{\Delta}_{LTV}^{n_P\times n_P}\}.
\end{aligned}$$

2.2 General Design Problem Statement

Consider Fig.2a, where G is an augmented plant, K is a controller, Δ_S is a perturbation, d is a disturbance input, z is a performance output, u is a command input, y is a measured output, u_Δ is a perturbation input, and y_Δ is a perturbation output such that

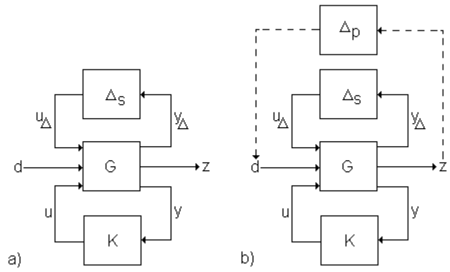

Figure 2: Stability and Performance

$$\begin{bmatrix} y_\Delta \\ z \\ y \end{bmatrix} = G \begin{bmatrix} u_\Delta \\ d \\ u \end{bmatrix}, \qquad (2)$$

with $u = Ky$ and $u_\Delta = \Delta_S\, y_\Delta$. For simplicity, G, K, and Δ_S are assumed square of dimensions $(n_\Delta + n_P + n_K) \times (n_\Delta + n_P + n_K)$, $n_K \times n_K$, and $n_\Delta \times n_\Delta$ respectively, where $n_\Delta, n_P, n_K \in \mathbb{Z}^+$. Such assumptions are not very limiting as it is always possible to introduce redundant inputs and outputs to augment the plant G and obtain the desired dimensions. Moreover, define

$$\begin{aligned}
M &\triangleq M(K) \triangleq F_l(G, K) \qquad (3)\\
T &\triangleq T(K, \Delta_S) \triangleq F_u(M, \Delta_S).
\end{aligned}$$

Assume that G, K (and thus M) are discrete-time LTI systems, $\Delta_S \in \mathbf{\Delta}_S$, and that $\|\Delta_S\|_{\infty-ind} < 1$ and $\|d\|_\infty \leq 1$. The objective of the controller is then to achieve $\|z\|_\infty \leq \nu$ for a desired treshold value $\nu \in \mathbb{R}^+$. Then, the associated robust disturbance rejection problem consists in finding a stabilizing controller K satisfying

$$\min_K \sup_{\Delta_S} \|T(K, \Delta_S)\|_{\infty-ind} \leq \nu. \qquad (4)$$

Without loss of generality, ν is set to 1. Note that, by virtue of (1), $n_\Delta = \sum\limits_{l=1}^{n} r_l$.

Consider Fig.2b, where a fictitious performance perturbation block Δ_P is added to the original system framework presented in Fig.2a. By slightly extending the result presented in [5], it follows that

$$\sup_{\Delta_S} \|T(K, \Delta_S)\|_{\infty-ind} = SN_{\mathbf{\Delta}_C,\infty}(M(K)). \qquad (5)$$

However, in general, it is not possible to compute (5) exactly due to the complexity of such a task. Practical approaches hence rely on the introduction of upper and lower bounds for $SN_{\mathbf{\Delta}_C,\infty}(M)$ that can be computed with relative ease, but at the cost of some conservativess.

2.3 An Upper Bound for the Structured Norm, [1]

Given the same $n \in \mathbb{Z}^+$, $\{r_l\}_{l=1}^n$, $r_l \in \mathbb{Z}^+$, and $n_P \in \mathbb{Z}^+$, as in (1), define

$$\mathbf{D} \triangleq \{D = \mathrm{diag}(D^1, ..., D^n, I_{n_P}) : D^l \in \mathbb{R}^{r_l \times r_l}, \\ D^l_{11} \geq D^l_{12} \geq ... \geq D^l_{1r_l} \geq 0, l \in \{1, ..., n\}\}.$$

It was shown in [1] that an upper bound for $SN_{\mathbf{\Delta}_C,\infty}(M)$ follows from the scaled small gain theorem, see [12], in the form of

$$\overline{SN}_{\mathbf{D},\infty}(M) \triangleq \inf_{D \in \mathbf{D}} \|D^{-1}MD\|_1, \tag{6}$$

where $\overline{SN}_{\mathbf{D},\infty}(M) \geq SN_{\mathbf{\Delta}_C,\infty}(M)$. The optimization problem (6) is nonconvex and nondifferentiable with respect to D.

2.4 A Lower Bound for the Structured Norm, [2]

Suppose that $\{M(k)\}_{k=0}^\infty$ is the impulse response of M (for a given controller K). Again, given the same $n \in \mathbb{Z}^+$, $\{r_l\}_{l=1}^n$, $r_l \in \mathbb{Z}^+$, and $n_P \in \mathbb{Z}^+$ as in (1), $\{M(k)\}_{k=0}^\infty$ is partitioned as follows

$$M(k) \triangleq \left[M^{IJ}(k)\right]_{\substack{I \in \{1,...,n+1\} \\ J \in \{1,...,n+1\}}} \in \mathbb{R}^{(n_\Delta + n_P)\times(n_\Delta + n_P)},$$

where $M^{IJ}(k) \triangleq \left[M^{IJ}_{ij}(k)\right]_{\substack{i \in \{1,...,r_I\} \\ j \in \{1,...,r_J\}}} \in \mathbb{R}^{r_I \times r_J}$ and $r_{n+1} = n_P$.

For a given $\tau \in \mathbb{Z}^+$, define the following class of admissible collections of subsets

$$\mathbf{\Upsilon} \triangleq \left\{ \Upsilon : \Upsilon = \{\mathbf{\Gamma}(\kappa)\}_{\kappa=0}^{\upsilon-1}, \mathbf{\Gamma}(\kappa) \subseteq \{0, ..., \tau - 1\}, \right. \\ \left. \mathbf{\Gamma}(\kappa) \neq \emptyset, \bigcap_{\kappa=0}^{\upsilon-1} \mathbf{\Gamma}(\kappa) = \emptyset, \upsilon \in \mathbb{Z}^* \right\}.$$

Note that $\upsilon \leq \tau$, as each $\Upsilon \in \mathbf{\Upsilon}$ is a collection of distinct subsets of $\{0, ..., \tau - 1\}$.

Define

$$\mathbf{Y} \triangleq \{\Upsilon^{IJ}\}_{\substack{I \in \{1,...,n+1\} \\ J \in \{1,...,n+1\}}} \text{ and } \mathbf{N} \triangleq \{N_I\}_{I \in \{1,...,n+1\}},$$

where $\Upsilon^{IJ} = \{\mathbf{\Gamma}^{IJ}(\kappa)\}_{\kappa=0}^{\upsilon_{IJ}-1} \in \mathbf{\Upsilon}$, $N_I \in \mathbb{Z}^+$ and $n \in \mathbb{Z}^+$ is implicitely defined by (1).

For the above fixed values of $n \in \mathbb{Z}^+$, $\upsilon_{IJ} \in \mathbb{Z}^+$, and $N_I \in \mathbb{Z}^+$, where $I, J \in \{1, ..., n+1\}$, define the set of indices $\mathbf{x} \triangleq$

$$\{(\kappa, I, J, \imath, \jmath) : \kappa \in \{0, ..., \upsilon_{IJ} - 1\}, I \in \{1, ..., n+1\}, \\ J \in \{1, ..., n+1\}, \imath \in \{1, ..., N_I\}, \jmath \in \{1, ..., N_J\}\}$$

and the class of admissible sets of real numbers

$$\mathbf{d} \triangleq \{d : d = \{d^{IJ}_{\imath\jmath}(\kappa)\}_{(\kappa,I,J,\imath,\jmath)\in\mathbf{x}}, d^{IJ}_{\imath\jmath}(\kappa) \in \mathbb{R}\}.$$

It was shown in [2] that, given any $\mathbf{Y}$ and any $\mathbf{N}$, a lower bound for $SN_{\mathbf{\Delta}_C,\infty}(M)$ is given by

$$\underline{SN}_{\mathbf{Y},\mathbf{N},\infty}(M) \triangleq \max_{\substack{d \in \mathbf{d} \\ \max_{d\in\mathbf{d}} \sum_{\jmath=1}^{N_J} |d^{IJ}_{\imath\jmath}(\kappa)| < 1}} \rho\left(\Xi(d)\right), \tag{7}$$

where $\underline{SN}_{\mathbf{Y},\mathbf{N},\infty}(M) \leq SN_{\mathbf{\Delta}_C,\infty}(M)$ and $\Xi(d) \triangleq \left[\xi^{IJ}\right]_{\substack{I \in \{1,...,n+1\} \\ J \in \{1,...,n+1\}}}$, $\xi^{IJ} \triangleq \left[\xi^{IJ}_{\imath\jmath}\right]_{\substack{\imath \in \{1,...,N_I\} \\ \jmath \in \{1,...,N_J\}}}$, $\xi^{IJ}_{\imath\jmath} \triangleq \sum_{\kappa=0}^{\upsilon_{IJ}-1}\left(d^{IJ}_{\imath\jmath}(\kappa) \sum_{k \in \mathbf{\Gamma}^{IJ}(\kappa)} M^{IJ}(k)\right)$. The optimization problem in (7) is nonconvex in d. Guidelines for the selection of $\mathbf{Y}$ and $\mathbf{N}$ are provided in [2].

2.5 Solution of the General Design Problem

From (6) and (5), if one finds a stabilizing controller K satisfying

$$\min_K \inf_{D \in \mathbf{D}} \|D^{-1}M(K)D\|_1 \leq 1, \tag{8}$$

then such a controller K will also satisfy (4). However, if the left hand side (LHS) of (8) is greater than one, it does not necessarily imply that K does not fulfill the performance objective. Still, by virtue of (7), it is sometimes possible to demonstrate that such a K indeed does not satisfy (4).

While it is obvious from (6) that (8) is not convex in D, it is, however, possible to simplify the optimization process (8) (with respect to K) as demonstrated below. By incorporating the Youla parameterization of all stabilizing controllers, see [12], it is possible to redefine $M(K)$ in the form of

$$M(K) \triangleq M(Q) \triangleq H + U * Q * V, \tag{9}$$

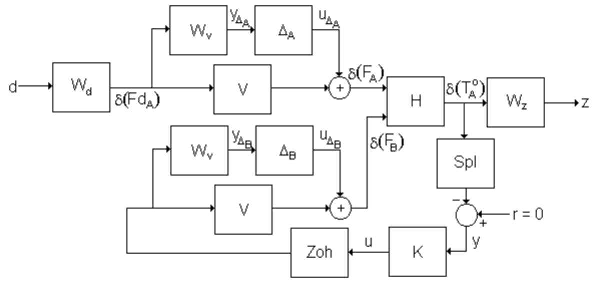

Figure 3: Experimental Setup

where Q is a stable discrete-time LTI system of dimension $n_K \times n_K$ and H, U, and V are also stable discrete-time LTI systems of dimensions compatible with Q and M. Moreover, if the transfer function of Q is of the form

$$Q \triangleq (q_0 + q_1 z^{-1} + ... + q_{n_Q} z^{-n_Q})/z^{-n_Q},$$

where $q_0, ..., q_{n_Q} \in \mathbb{R}^{n_K \times n_K}$, $n_Q \in \mathbb{Z}^+$, then

$$\min_Q \inf_{D \in \mathbf{D}} \|D^{-1}HD + D^{-1}U * Q * VD\|_1, \qquad (10)$$

is convex in the Q parameters $\{q_0, ..., q_{n_Q}\}$. Observe that (10) equals the LHS of (8). The problem in (10) is usually solved by the DQ-algorithm, see [3] and [12]. As shown in [4], [9], and [7], for a fixed D, the optimization problem in Q can be rewritten as a linear programming problem. On the other hand, for a fixed Q, the optimization problem in D can be solved by non-smooth optimization techniques. For improved efficiency, the DQ-algorithm may be employed within a global optimization framework that performs a systematic gridding of the D parameter space.

3 A case Study: the Heat Exchanger System

3.1 Presentation of the Experimental Setup

Let $\delta(v)$ denote a small variation of a variable v away from its operating point.

Fig.3 illustrates the block diagram of the linearized experimental setup. This setup involves a heat exchanger (H), two valves (V) with uncertain dynamics, a controller (K) with a zero-order hold (Zoh) and a sampler (Spl), and several filters. Models for each subsystems are described below.

A heat exchanger comprises two pipelines circulating fluids. Inside the heat exchanger, the pipeline containing the warmer fluid transfers some heat to the pipeline containing the colder fluid. Here those pipelines are labeled A and B. Let F_A and F_B denote the flow rates inside pipeline A and B, respectively. Let T_A^i, T_A^m, and T_A^o denote the temperatures of the fluid in pipeline A at the entry, in the middle, and at the exit of the heat exchanger, respectively. The temperatures T_B^i, T_B^m and T_B^o are defined similarly for pipeline B. The heat exchanger model is linearized around the following operating point: $F_A = 50cm^3/s$, $F_B = 7.8cm^3/s$, $T_A^m = 22.60^oC$, $T_A^o = 26.59^oC$, $T_B^m = 26.87^oC$, and $T_B^o = 22.64^oC$. Note that T_A^i and T_B^i are always set to 22^oC and 55^oC, respectively. The linearized model of the heat exchanger H, see [8], is then given by

$$\delta(T_A^o) = C(sI - A)^{-1}B \begin{bmatrix} \delta(F_A) \\ \delta(F_B) \end{bmatrix} \triangleq [H_{11} H_{12}] \begin{bmatrix} \delta(F_A) \\ \delta(F_B) \end{bmatrix}$$

where $A = \begin{bmatrix} -30.54 & 0 & 0 & 28.55 \\ 1.99 & -30.54 & 28.55 & 0 \\ 0 & 16.16 & -16.32 & 0 \\ 16.16 & 0 & 0.160 & -16.32 \end{bmatrix}$,

$$B = \begin{bmatrix} -0.024 & 0 \\ -0.159 & 0 \\ 0 & 0.570 \\ 0 & 0.086 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}.$$

The states are defined as follows: $x(1) = \delta(T_A^m)$, $x(2) = \delta(T_A^o)$, $x(3) = \delta(T_B^m)$, and $x(4) = \delta(T_B^o)$.

A simplified linear model of a valve was proposed in the form of

$$\delta(F) = (V + W_v \Delta)\, \delta(Fd)$$

where $V = \frac{2}{s+2}$, $W_v = \frac{0.12s+0.012}{s+0.2}$, Fd is the desired flow rate, F is the flow rate, and $\Delta \in \mathbf{\Delta}_{LTV}^{1\times1}$, $\|\Delta\|_{\infty-ind} < 1$, is a perturbation. The filter W_v together with the perturbation Δ embody all possible neglected high frequency dynamics inherent to the saturation in the valve opening (and closing) rate. The two valves in the above setup are identical and are assumed to operate under similar conditions, thus they share the same model.

The signal Fd_A is the desired flow rate in pipeline A. It is assumed that Fd_A is determined by another process, hence it is seen as a disturbance to the system considered. Morever, $\delta(Fd_A)$ is bounded in magnitude by $\pm 10cm^3/s$, hence $\delta(Fd_A) \in \ell_\infty$. Without loss of generality, let $d\ W_d = \delta(Fd_A)$, where $W_d = 10\frac{0.6}{s+0.6}$ and $\|d\|_\infty \leq 1$. The nominal low pass filter $\frac{0.6}{s+0.6}$ in W_d captures the fact that $\delta(Fd_A)$ has a limited rate of variation.

The performance objective is to maintain $\delta(T_A^o)$ within $\pm 0.2^o C$ and hence attenuate the influence of $\delta(Fd_A)$ on T_A^o. This requirement is equivalent to the satisfaction of the condition $\|z\|_\infty \leq 1$, where $z = W_z \delta(T_A^o)$, $W_z = 5$.

The subsystems in Fig.3 are rearranged to agree with (2), namely

$$\begin{bmatrix} y_{\Delta_A} \\ y_{\Delta_B} \\ z \\ y \end{bmatrix} = G_{exp} \begin{bmatrix} u_{\Delta_A} \\ u_{\Delta_B} \\ d \\ u \end{bmatrix} \text{ with} \qquad (11)$$

$$G_{exp} = \begin{bmatrix} 0 & 0 & W_d W_v & 0 \\ 0 & 0 & 0 & W_v Zoh \\ W_z H_{11} & W_z H_{12} & W_d W_z V H_{11} & W_z V H_{12} Zoh \\ -H_{11} & -H_{12} & -W_d V H_{11} & -V H_{12} Zoh \end{bmatrix}.$$

Note that $n = 1$, $r_1 = 2$, $n_\Delta = 2$, $n_P = 1$, and $n_K = 1$.

3.2 Simulation Results

In an attempt to fulfill the above performance objective, two controllers are synthesized below according to the design strategy presented in §2. The core of this methodology consists of the following equations: (2), (3), (9), (10), and (11). It is important to note that the discrete-time form of (11) is required for the controller synthesis. Here a sampled time of $1s$ is employed.

Note that the dimension of Q is set to $n_Q = 9$. Furthermore, to achieve a finite computational scheme, the original domain $\mathbf{\Delta}$ is restricted to the following compact set $\mathbf{D} \bigcap [-1.4, 1.4]^{3\times 3}$. A global optimization approach which combines a gridding of the reduced scaling domain with the application of the DQ-algorithm is then employed to solve (10). For each Q found, the corresponding controller K is recovered as in [3], hence Q will itself be refered to as the controller.

The first synthesis attempt yields a finite impulse response transfer function Q_1 and a scaling matrix D_1 (their numerical values are given in the Appendix). The corresponding structured norm upper bound is

$$\overline{SN}_{D_1,\infty}(M(Q_1)) = 1.38. \qquad (12)$$

Since $\overline{SN}_{D_1,\infty}(M(Q_1)) > 1$, it does not ensure the satisfaction of the performence objective. The methodology proposed in (7) now allows to compute a structured norm lower bound. Using $\mathbf{Y}$ and $\mathbf{N}$, as given in the Appendix,

$$\underline{SN}_{\mathbf{Y},\mathbf{N},\infty}(M(Q_1)) = 1.25 > 1. \qquad (13)$$

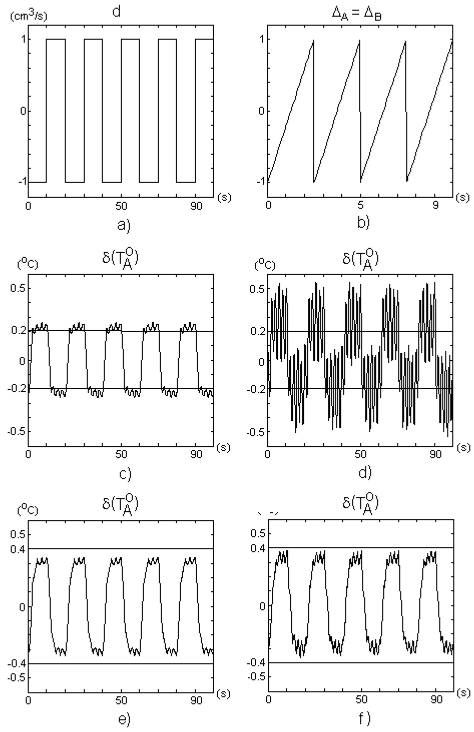

Figure 4: Simulation Results

Clearly, this indicates that it is impossible to achieve the desired robust performance level.

Therefore, the performance objective is next relaxed by setting $W_z = 2.5$ which implies that the condition $\delta(T_A^o) \in [-0.4^o C, 0.4^o C]$ must hold. Using Q_1 and D_1 as initial values, the DQ-algorithm is applied and returns Q_2 and D_2 (see Appendix) such that

$$\overline{SN}_{D_2,\infty}(M(Q_2)) = 0.97 < 1. \qquad (14)$$

Thus, in this case, it is guaranteed that Q_2 achieves the new performance objective.

The controllers K_1 and K_2 (see Appendix) are further tested in the context of both the linearized model and the original nonlinear model. The efficiency of each controller is assessed in terms of its ability to attenuate the influence of the disturbance signal d on the temperature expressed in terms of $\delta(T_A^o)$. In all the simulations presented here, the disturbance is the signal d shown in Fig.4a. Moreover, all the simulations involving the linearized system use the memoryless perturbation $\Delta_A (= \Delta_B)$ shown in Fig.4b.

The simulation results are presented in Fig.4 whose sub-plots represent the following. Fig.4c and Fig.4d show the signal $\delta(T_A^o)$ when K_1 is applied on the linearized and nonlinear system models, respectively. It is seen that K_1 is indeed unable to achieve the desired performance level, i.e., $|\delta(T_A^o)| \leq 0.2^oC$. The deficiency of K_1 is particularly pronounced in its application to the nonlinear model. Similarly, Fig.4e and Fig.4f show the signal $\delta(T_A^o)$ when K_2 is applied to the linearized and nonlinear system models, respectively. Note that K_2 achieves the relaxed performance objective in both cases, i.e., $|\delta(T_A^o)| \leq 0.4^oC$.

4 Conclusion

An ℓ_∞ robust controller was designed for the linearized model of a heat exchanger system. This model involves a pair of structured, *repeated*, linear time-varying, and induced ℓ_∞-norm-bounded perturbations. The efficiency of the controller was further tested on the original nonlinear model of the heat exchanger system. The simulation results confirmed the efficiency of the adopted design strategy. The controller developed in this paper will next be applied and tested on an existing laboratory prototype of a real heat exchanger system.

5 Appendix

5.1 Numerical value of Q_1, D_1, K_1, Q_2, D_2, and K_2

$Q_1 = \frac{numQ_1}{z^{-9}}, numQ_1 = 6.7152 - 8.8361z^{-1} + 4.8873z^{-2} - 1.7857z^{-3} + 0.5544z^{-4} - 0.2306z^{-5} + 0.1495z^{-6} - 0.0933z^{-7} + 0.0403z^{-8} - 0.0087z^{-9}$,

$Q_2 = \frac{numQ_2}{z^{-9}}, numQ_1 = 3.7733 - 3.3959z^{-1} + 0.8952z^{-2} - 0.2199z^{-3} + 0.2316z^{-4} - 0.2082z^{-5} + 0.0869z^{-6} - 0.0171z^{-7} + 0.0033z^{-8} - 0.0008z^{-9}$,

$K_1 = \frac{numK_1}{denK_1}, numK_1 = 6.715 - 15.960z^{-1} + 16.442z^{-2} - 10.021z^{-3} + 4.275z^{-4} - 1.531z^{-5} + 0.622z^{-6} - 0.342z^{-7} + 0.194z^{-8} - 0.086z^{-9} + 0.025z^{-10} - 0.004z^{-11}$,

$denK_1 = 1.000 - 2.089z^{-1} + 1.628z^{-2} - 0.525z^{-3} - 0.006z^{-4} + 0.063z^{-5} - 0.018z^{-6} - 0.006z^{-7} + 0.009z^{-8} - 0.001z^{-9} - 0.001z^{-10} + 0.001z^{-11}$,

$K_2 = \frac{numK_2}{denK_2}, numK_2 = 3.773 - 7.399z^{-1} + 5.723z^{-2} - 2.374z^{-3} + 0.847z^{-4} - 0.549z^{-5} + 0.389z^{-6} - 0.183z^{-7} + 0.055z^{-8} - 0.012z^{-9} + 0.002z^{-10}$, $denK_2 = 1.000 - 1.639z^{-1} + 0.817z^{-2} - 0.035z^{-3} - 0.066z^{-4} - 0.009z^{-5} + 0.024z^{-6} - 0.005z^{-7} - 0.004z^{-8} + 0.002z^{-9}$,

$$D_1 = \begin{bmatrix} 1.17 & 0.33 & 0 \\ 0.21 & -0.17 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \text{ and } D_2 = \begin{bmatrix} 1.39 & 0.38 & 0 \\ 0.21 & -0.12 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

5.2 Numerical value of $\mathbf{Y}$ and $\mathbf{N}$

$\mathbf{N} = \{2, 1\}$ and $\mathbf{Y} = \{\Upsilon^{11}, \Upsilon^{12}, \Upsilon^{21}, \Upsilon^{22}\}$, where $\Upsilon^{11} = \{\{7, 9, 11, ..., 47\}, \{0, 3, 5\}, \{1, 2, 4\}, \{6, 8, 10, ..., 46\}\}$, $\Upsilon^{12} = \{\{0, 1\}, \{2, 19, 21, 23, ..., 47\}, \{3, 4, 5, ..., 18, 20, 22, ..., 46\}\}$, $\Upsilon^{21} = \{\{7, 9, 11, ..., 47\}, \{1, 2, 4\}, \{0, 3, 5\}, \{6, 8, 10, ..., 46\}\}$, and $\Upsilon^{22} = \{\{4, 7, 10, 13, 14, 15, 28, 30, 32, ..., 46\}, \{0, 1, 2, 3, 5, 6, 8, 9, 11, 12, 16, 17, 18, ..., 27, 29, 31, ..., 47\}\}$. Note that, by applying the guidelines proposed in [2], it is possible to construct a $\mathbf{Y}$, as efficient as the one above, that leads to a much simpler problem of the form (7) .

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