Power Transformer Analysis based on 3D Finite Elements and Mixed Numerical Techniques

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Abstract: - Power transformer analysis and design focusing on the equivalent circuit parameter evaluation by magnetic field numerical calculation is presented. Two kinds of numerical techniques are examined: the three-dimensional (3D) finite element method (FEM), adopting a particular reduced scalar potential formulation and the 3D mixed finite element-boundary element (FEM-BEM) method. The methods are validated through comparison to the theory of images and analytical solutions and are afterwards applied to a wound core distribution transformer. The results are used for the calculation of the short-circuit impedance and the resulting values are compared to the measured ones. The methodology is very promising for investigation concerning losses and short circuit voltage variations with the main geometrical parameters.

Key-Words: - power transformers, 3D numerical methods, theory of images, analytical solution, short-circuit impedance.

1 Introduction

Transformers are electric machines that enable the transmission and distribution of electric energy in a simple and cost-effective way, since their efficiency overcomes 95%.

The modern industry requirements necessitate the construction of a great variety of transformers that do not fit in to standardized large-scale constructions. In such cases, experiential ways of electric characteristics calculation do not afford satisfying accuracy, as they concern particular geometries. Moreover, the limited delivery time does not allow the experimental verification of the predicted transformer characteristics [1, 2].

Numerical modeling techniques are now-a-days well established for power transformer analysis and enable representation of all important features of these devices. The application of these methods to the transformer magnetic field simulation can afford accurate equivalent circuit parameters evaluation as well as contribute to the prediction of the transformer operating characteristics.

In the present paper, 3D numerical techniques are used for the magnetic field analysis of power transformers under short-circuit. The methods are validated through comparison to the theory of images and analytical solutions. The results of their application to a wound core distribution transformer are compared to the measured ones, proving their ability to constitute a powerful tool for power transformer modeling and design.

2 Finite Element Method

Techniques based on finite elements present interesting advantages for magnetic field calculation of electrical machines and nonlinear characteristics simulation. The leakage inductance evaluation has been extensively analyzed [3, 4], as well as eddy current loss in transformer tank walls, iron lamination characteristics and design considerations. The systematic increase of computer efficiency along with the evolution of numerical methods of magnetic field simulation enable the detailed transformer magnetic field analysis with the use of low cost and widely popular computational systems. The finite element method is one of the numerical methods that have prevailed in the field analysis of three-dimensional configurations that comprise materials with non-linear characteristics, like transformers, and may be applied within reasonable time in an appropriate personal computer.

2.1 Description of the method

In the present paper a particular scalar potential formulation has been developed, enabling the 3D magnetostatic field analysis. According to this method, the magnetic field strength **H** is conveniently partitioned to a rotational and an irrotational part as follows [5]:

$$\mathbf{H}=\mathbf{K}-\nabla\Phi \qquad (1)$$

where Φ is a scalar potential extended all over the solution domain while **K** is a vector quantity (fictitious field distribution), defined in a simply connected subdomain comprising the conductor, that satisfies Ampere's law and is perpendicular on the subdomain boundary.

2.2 Validation of the proposed 3D FEM formulation by the Theory of Images

In order to evaluate the accuracy of the proposed 3D finite element model, field calculations for simplified configurations that approximate the transformer geometry have been conducted with the use of the theory of images, [6].

Fig.1 shows the geometry of the model considered: It consists of a circular coil segment near an infinitely permeable material. According to the theory of images, the magnetic field can be calculated by considering that the coil segment inside the material carries an opposite current of the same magnitude as the one in the air (Fig. 2). This assumption approximates the boundary condition (Ht=0) along the boundary between the air and the highly permeable material. The magnetic field calculation is conducted using the integral equation method, [7]. Fig.3 shows the change of the Bz component along y-axis, while Fig. 4 contains the same results as they were produced by the 3D finite element modeling of the problem.

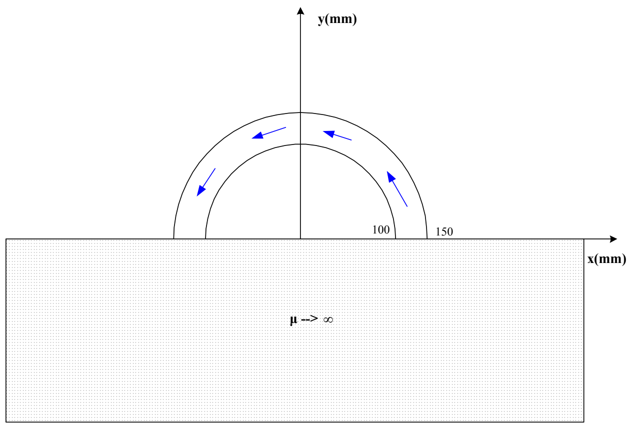


Fig.1 Geometry of the simplified problem considered for the comparison between 3D FEM and the theory of images.

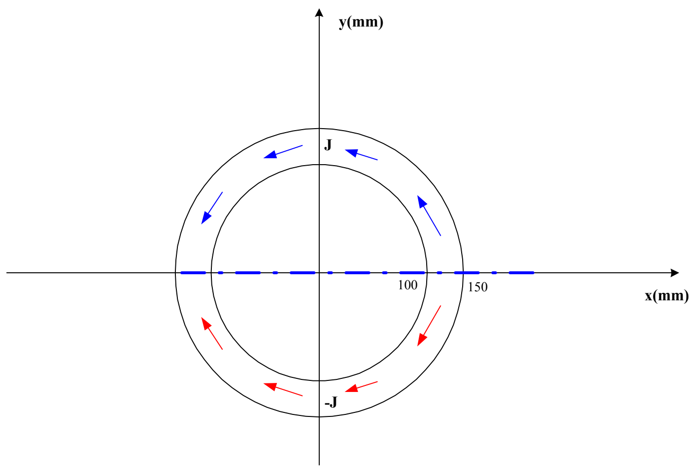


Fig.2 Representation of the geometry of Fig. 1 with the theory of images.

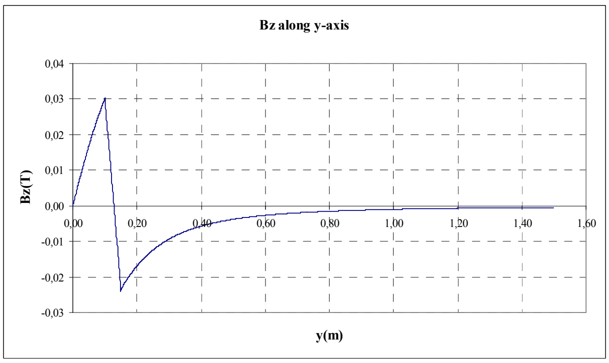


Fig.3 Results of the integral equation calculation method.

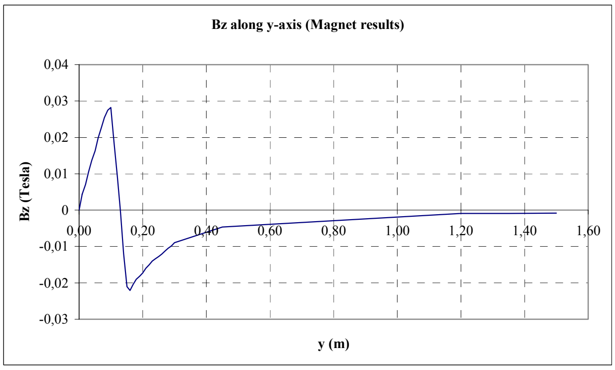


Fig.4 Results of the proposed 3D finite element method.

3 Mixed 3D Finite Element-Boundary Element Method

The boundary element method is a numerical field analysis technique that uses the integral form of magnetic field equations and discretizes only the boundaries of the considered areas (in comparison to the finite element method which discretizes the whole field). Therefore, this method is suitable for open-boundary problems as well as geometries with extensive parts of air. Moreover, the combination of boundary and finite elements is widely used for electromagnetic problems since the electromagnetic field is not only confined to the conductors but it expands over extensive parts of air, where the use of a boundary element representation can significantly decrease the computational effort [8,9].

3.1 Description of the method

3.1.1 Boundary Element method equations

The boundary element method is derived through discretization of an integral equation that is mathematically equivalent to the original partial differential equation. The boundary integral equation corresponding to Laplace equation is of the form:

$$c(s)\Phi(s) + \oint_{\Gamma}\left[\Phi(s)\frac{\partial G(s',s)}{\partial n} - G(s',s)\frac{\partial \Phi(s')}{\partial n'}\right]ds' = 0 \qquad (2)$$

where s is the observation point, s' is the boundary Γ coordinate, n' is the unit normal and G the fundamental solution of Laplace equation in free space. The re-formulation of the PDE that underlies the BEM consists of an integral equation that is defined on the boundary of the domain and an integral that relates the boundary solution to the solution at the points in the domain. Therefore, while in the finite element method an entire domain mesh is required, in the BEM formulation a mesh of the boundary only is required, resulting to the significant reduction of the problem size.

3.1.2 Coupling the FE and BE techniques

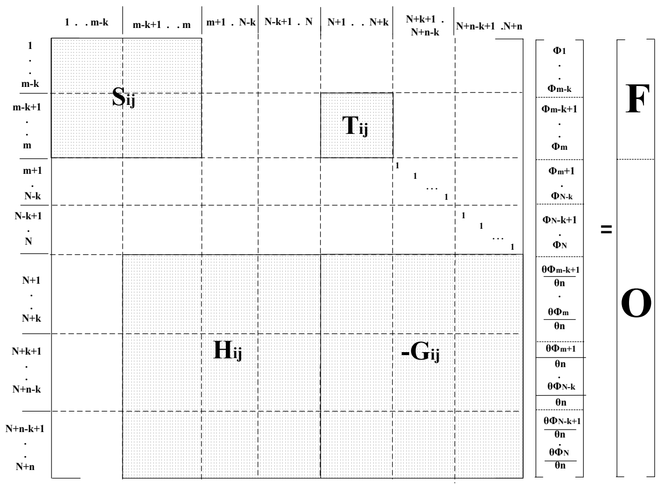
Let us consider a coupled finite element/boundary element solution domain, comprising m FE nodes, n BE nodes and r common nodes in the interface boundary. The matrices for the BE region can be written as follows:

$$HU=GQ \qquad (3)$$

where U is a vector of the BE nodal potential values, Q a vector of the BE nodal values, while Hij and Gij correspond to the Eq. (2) integrals $\oint_{\Gamma_j}\frac{\partial G_i(s',s)}{\partial n}ds'$ and $\oint_{\Gamma_j} G_i(s',s)ds'$ respectively, [6].

The matrices of the FE region can be written as SU=F, where S is the stiffness matrix and F the source vector.

The global system matrix has the form:



where Tij is the term used to link the finite element region to the boundary element region (involving the potential and normal derivative values of the FE-BE interface boundary nodes).

3.2 Validation of the mixed 3D FEM-BEM by the use of analytical solution

The validation of the proposed mixed FEM-BEM method was conducted by comparison to the results obtained by an analytical solution for a simplified configuration approximating the transformer geometry. The geometry of the considered problem, consisting of one coil surrounded by air, is shown in Fig. 5. The field distribution obtained by the mixed FEM-BEM method was juxtaposed to the one of a coil with rectangular cross-section composed of four circular and four straight segments (Fig. 6). The solution of this problem can be derived by using Biot-Savart's law [7]. Fig. 7 shows the respective results for the magnetic induction magnitude along the line AB of Fig. 5.

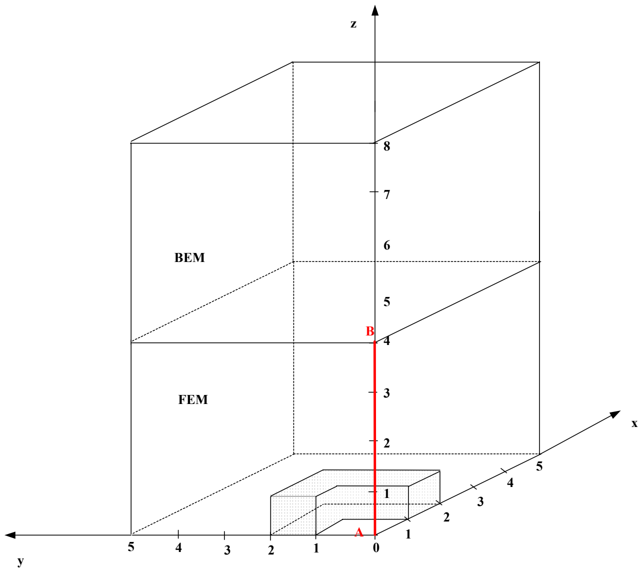


Fig.5 Geometry of the problem considered for comparison of the FEM-BEM method results to the ones obtained by an analytical solution.

4 Results and discussion

The proposed method has been applied to the 1000 kVA wound core distribution transformer of Fig. 8. Fig. 9 illustrates the perspective view of the one-phase transformer part modeled, comprising the iron core, low and high voltage windings.

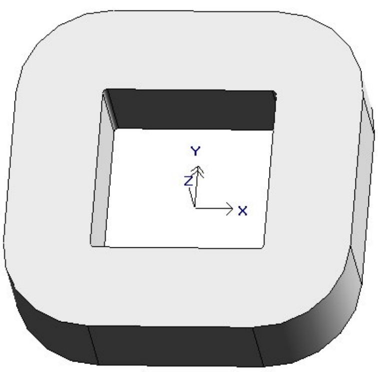


Fig. 6 Geometry of the coil treated by Biot-Savart's law.

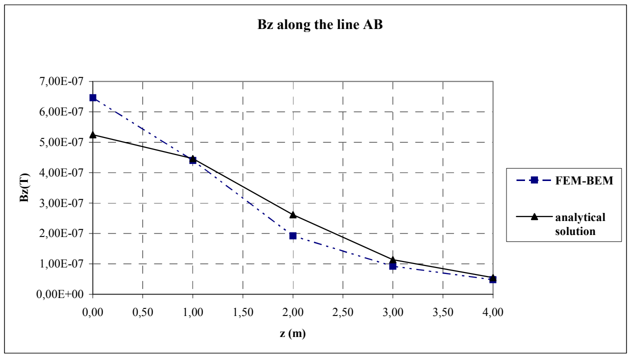


Fig.7 Comparison of mixed FEM-BEM results to the analytical solution results

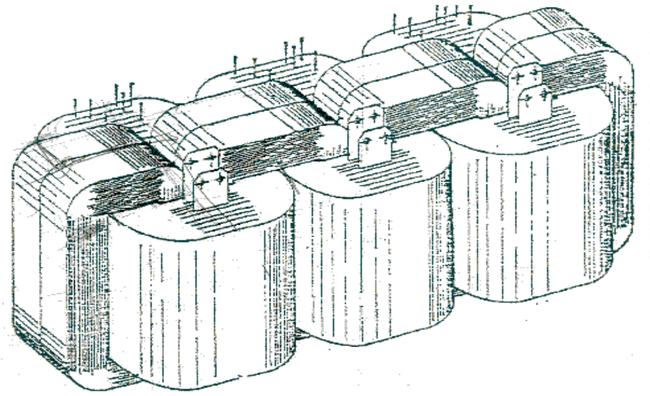


Fig. 8. Active part configuration of the three phase shell type distribution transformer considered.

The use of this one phase model instead of the whole three phase transformer model was conducted for the following reasons:

i) The smaller model size enables the construction of more dense tetrahedral finite element mesh without great computational cost (given that the exact representation of the transformer magnetic field requires great accuracy which is dependent on the mesh density and the total execution time of the finite element calculations).

ii) The representation of one phase of the active part does not affect the accuracy of the equivalent circuit parameters calculation.

Due to the symmetries of the problem, the solution domain was reduced to one fourth of the device (although there is a slight disymmetry due to the terminal connections in one side). These symmetries were taken into account by the imposition of Dirichlet boundary condition (Φ=0) along xy-plane and Neumann boundary condition ($\frac{\partial \Phi}{\partial n}$=0) along the other three faces of the air box that surrounds the transformer active part.

Figure 10 shows the magnetic flux density magnitude distribution during short-circuit test, as it was calculated by the proposed 3D finite element method with the use of a mesh of approximately 90.000 nodes.

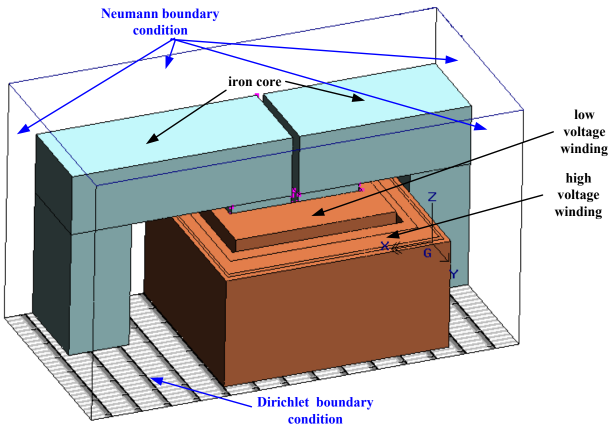


Fig. 9. Perspective view of the one phase transformer part modeled.

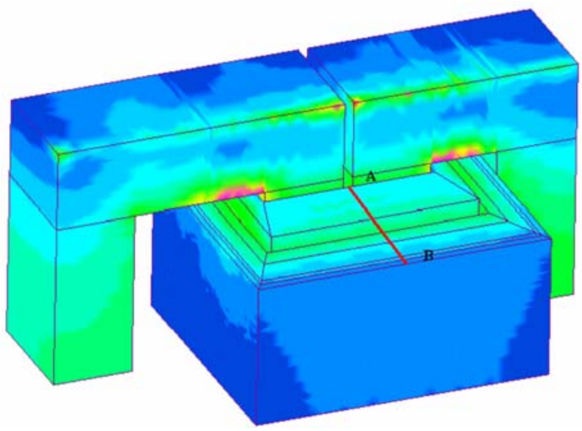


Fig. 10. Magnetic flux density magnitude distribution during short-circuit test.

Table 1 compares the measured short-circuit voltage value deduced by the short-circuit test and the calculated one with the use of different mesh densities, in order to evaluate the respective variation of the error. The error of the short-circuit voltage calculation reduces significantly as the mesh density increases, while it appears to be similar for the two high voltage levels.

4 Conclusions

The paper examined various numerical methods for power transformer analysis and design involving 3D magnetostatics. A formulation based on a particular reduced scalar potential technique necessitating no source field calculation, has been adopted. A mixed finite element - boundary element method presents important advantages for this class of problems. This methodology has been checked in short circuit cases and is very promising for design office applications.

number of nodes	calculated short-circuit voltage	measured short-circuit voltage	error (%)	High Voltage Level
14491	**6,38**	6,13	**4,00**	**20 kV**
49047	**6,30**		**2,84**	
86903	**6,19**		**0,98**	
14491	**6,13**	5,95	**3,01**	**15 kV**
49047	**6,09**		**2,30**	
86903	**5,97**		**0,34**	

Table 1. Comparison of calculated and measured short-circuit voltage values for the 1000 kVA transformer modelled.

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