Nonlinear four wheel slip-ratio control of Anti-lock brake systems

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Abstract- In this paper a sliding mode controller is applied to a four wheel vehicle system to obtain a satisfactory longitudinal slip for each wheel. A seven degree of freedom model with all of the system nonlinearities is used to evaluate the proposed controller. Simulation results reveal the effectiveness of the proposed controller.

Key-Words: - sliding mode control, four wheel system and longitudinal slip.

1 Introduction

Nowadays the Antilock Braking System (ABS) is an important part of a complex system for the modern car. The ABS controls the slip of each wheel to prevent it from locking such that a high friction is achieved and steer ability is maintained. ABS provides significant stopping advantages on slippery roads. With the increase of the speed and density of road vehicles, the safety performance is obviously demanded and Anti-lock Braking Systems (ABS) have been developed and now are used in more and more vehicles in recent decades[1-6]. Generally, for an ABS, the control target is to maintain friction coefficients within maximum range in corresponding operating condition in order to obtain the maximum braking force from road surface, and meanwhile ensuring vehicle stability. When braking is present during a maneuver, the tire friction coefficient decreases with increasing values of tire slip, indicating that braking slip significantly reduces the friction coefficient. But the optimum slip occurs at each tire peak friction. The optimal target slip ratio for ABS controller deigned should be on-line calculated considering the fact of complex road-tire characteristics and changeable braking conditions. Sliding mode control has successfully been applied to a lot of applications in previous years [7-12]. In this work a sliding mode controller is applied to a complex seven degree of freedom model without any simplification. The organization of the paper is as follows: In Section 2 the perfect vehicle model is presented. In Section 3 the design procedure of sliding mode control is considered. The simulation results on the slip-ratio control of an ABS are presented in section 4. Finally, Section 5 concludes the paper.

2 Vehicle model

The seven degrees of freedom model to be used in the controller design is depicted in Fig. 1. In this figure, the longitudinal tire friction forces for front left, front right, rear left and rear right wheels are represented as F_{xfl}, F_{xfr}, F_{xrl} and F_{xrr} respectively. Also the lateral tire friction forces for front left, front right, rear left and rear right wheels are represented as F_{yfl}, F_{yfr}, F_{yrl} and F_{yrr} respectively.

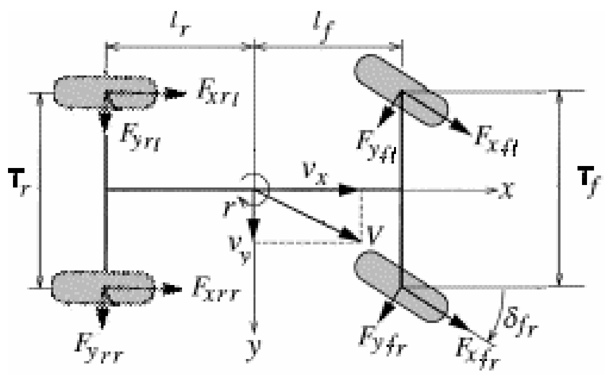

Fig. 1. Planar vehicle model

The basic equations of motion for this model can be derived as follows:

$$\dot{v}_x = v_y r + \frac{1}{m}\begin{pmatrix} F_{xfl}\cos(\delta_{fl}) + F_{xfr}\cos(\delta_{fr}) \\ -F_{yfl}\sin(\delta_{fl}) + F_{yfr}\sin(\delta_{fr}) \\ +F_{xrl} + F_{xrr} - c_x {v_x}^2 \end{pmatrix} \quad (1)$$

$$\dot{v}_y = -v_x r + \frac{1}{m}\begin{pmatrix} F_{xfl}\sin(\delta_{fl}) + F_{xfr}\sin(\delta_{fr}) \\ +F_{yfl}\cos(\delta_{fl}) + F_{yfr}\cos(\delta_{fr}) \\ +F_{xrl} + F_{xrr} - c_y {v_y}^2 \end{pmatrix} \quad (2)$$

$$\dot{r} = \frac{1}{I_z}\begin{pmatrix} L_f\begin{pmatrix} F_{xfl}\sin(\delta_{fl}) + F_{xfr}\sin(\delta_{fr}) \\ +F_{yfl}\cos(\delta_{fl}) + F_{yfr}\cos(\delta_{fr}) \end{pmatrix} \\ -L_r\left(F_{yrl} + F_{yrr}\right) \\ +\frac{T_f}{2}\left(F_{xfl}\cos(\delta_{fl}) + F_{xfr}\cos(\delta_{fr})\right) \\ -\frac{T_f}{2}\left(F_{yfl}\sin(\delta_{fl}) + F_{yfr}\sin(\delta_{fr})\right) \\ +\frac{T_r}{2}\left(F_{xrl} - F_{xrr}\right) + M_z \end{pmatrix} \quad (3)$$

$$\begin{aligned} \dot{\omega}_{fl} &= \frac{1}{I_{wfl}}\left(T_{wfl} - R_w F_{xfl}\right) \\ \dot{\omega}_{fr} &= \frac{1}{I_{wfr}}\left(T_{wfr} - R_w F_{xfr}\right) \\ \dot{\omega}_{rl} &= \frac{1}{I_{wrl}}\left(T_{wrl} - R_w F_{xrl}\right) \\ \dot{\omega}_{rr} &= \frac{1}{I_{wrr}}\left(T_{wrr} - R_w F_{xrr}\right) \end{aligned} \quad (4)$$

where v_x and v_y are the longitudinal and lateral velocity respectively, r is the yaw rate, ω_{fl}, ω_{fr}, ω_{rl} and ω_{rr} are the angular velocities for front left, front right, rear left and rear right wheels respectively, δ_{fl} and δ_{fr} are the steering angles for front left and right wheels, T_i is the torque applied on each wheel, I_{wi} is the wheel inertia, R_w is the wheel radius, l_f and l_r are distance from CoG (Cente of Gravity) to front and rear axles respectively, T_f and T_r are distances between front wheels and rear wheels respectively, I_z is Mass moment of Inertia , M_z is self aligning moment and m is vehicle mass. The longitudinal speed of each wheel with respect to Fig. 1 would be obtained as:

$$\begin{aligned} v_{fl} &= \sqrt{\left(v_x - \frac{T_f}{2}r\right)^2 + \left(v_y + L_f r\right)^2} \\ v_{fr} &= \sqrt{\left(v_x + \frac{T_f}{2}r\right)^2 + \left(v_y + L_f r\right)^2} \\ v_{rl} &= \sqrt{\left(v_x - \frac{T_r}{2}r\right)^2 + \left(v_y - L_r r\right)^2} \\ v_{rr} &= \sqrt{\left(v_x + \frac{T_r}{2}r\right)^2 + \left(v_y - L_r r\right)^2} \end{aligned} \quad (5)$$

from (5) the slip angles are expressed by:

$$\begin{aligned} \alpha_{fl} &= \delta_{fl} - tg^{-1}\left(\frac{\left(v_y + L_f r\right)}{\left(v_x - \frac{T_f}{2}r\right)}\right) \\ \alpha_{fr} &= \delta_{fr} - tg^{-1}\left(\frac{\left(v_y + L_f r\right)}{\left(v_x + \frac{T_f}{2}r\right)}\right) \\ \alpha_{rl} &= -tg^{-1}\left(\frac{\left(v_y - L_r r\right)}{\left(v_x - \frac{T_r}{2}r\right)}\right) \\ \alpha_{rr} &= -tg^{-1}\left(\frac{\left(v_y - L_r r\right)}{\left(v_x + \frac{T_r}{2}r\right)}\right) \end{aligned} \quad (6)$$

In the braking state, the longitudinal slip for each wheel is defined as:

$$\lambda_i = 1 - \frac{R_w \omega_i}{v_i} \quad (7)$$

The vertical load on each wheel F_z is a function of both the vehicle's weight and the dynamic weight transfer associated with longitudinal and lateral acceleration. The longitudinal acceleration affects the normal loading through the vehicle pitch mode while the lateral acceleration affects the normal loading through the vehicle roll motion. Although the pitch and roll motions are not included in the vehicle model, their effects on the normal tire forces are accounted for:

$$
\begin{aligned}
F_{zfl} &= \frac{mgL_r}{2(L_f+L_r)} - ma_x\frac{h}{2(L_f+L_r)} + ma_y\frac{L_rh}{T_f(L_f+L_r)}\\
F_{zfr} &= \frac{mgL_r}{2(L_f+L_r)} - ma_x\frac{h}{2(L_f+L_r)} - ma_y\frac{L_rh}{T_f(L_f+L_r)}\\
F_{zrl} &= \frac{mgL_f}{2(L_f+L_r)} + ma_x\frac{h}{2(L_f+L_r)} + ma_y\frac{L_rh}{T_f(L_f+L_r)}\\
F_{zrr} &= \frac{mgL_f}{2(L_f+L_r)} + ma_x\frac{h}{2(L_f+L_r)} - ma_y\frac{L_rh}{T_f(L_f+L_r)}
\end{aligned} \tag{8}
$$

where a_x and a_y are longitudinal and lateral acceleration respectively and will be calculated as:

$$
\begin{aligned}
a_x &= \dot{v}_x - v_y r\\
a_y &= \dot{v}_y + v_x r
\end{aligned} \tag{9}
$$

2-1 Tire modeling

In this work a Pacejka's tire model [13] is used for controller design. In this model the longitudinal friction force is written as:

$$
\begin{aligned}
&bx = 22 + \frac{f_z - 1940}{430}\\
&cx = 1.35 - \frac{f_z - 1940}{16125}\\
&dx = 1750 + \frac{f_z - 1940}{0.965}\\
&ex = 0.1 \quad shx = 0 \quad svx = 0\\
&\Phi x = (1 - ex)(\lambda + shx) + \frac{ex}{bx} tg^{-1}(bx(\lambda + shx))\\
&f_x = dx \sin(cx)\, tg^{-1}(bx\, \Phi x) + svx
\end{aligned} \tag{10}
$$

Also the lateral friction force is written as:

$$
\begin{aligned}
&by = 0.22 + \frac{5200 - f_z}{40000}\\
&cy = 1.26 - \frac{f_z - 5200}{22750}\\
&dy = -0.00003 f_z^2 + 1.0096\, f_z - 22.73\\
&ey = -1.6 \quad shy = 0 \quad svy = 0\\
&\Phi y = (1 - ey)(\alpha + shy) + \frac{ey}{by} tg^{-1}(by(\alpha + shy))\\
&f_y = dy \sin(cy)\, tg^{-1}(by\, \Phi y) + svy
\end{aligned} \tag{11}
$$

the self aligning moment will be attained as:

$$
\begin{aligned}
&M_z = t_{mf}\alpha_f + t_{mr}\alpha_r\\
&t_{mf} = -1280 \quad t_{mr} = -1120
\end{aligned} \tag{12}
$$

3 Sliding mode Controller Design

In this section the control design procedure is presented. The controller should be designed while the longitudinal slip of each wheel tracks the reference value and also the vehicle stops soon. In sliding mode control, first the sliding surface should be selected, so the following sliding surfaces are defined as:

$$
\begin{aligned}
s_{fl} &= \lambda_{fl} - \lambda_{ref}\\
s_{fr} &= \lambda_{fr} - \lambda_{ref}\\
s_{rl} &= \lambda_{rl} - \lambda_{ref}\\
s_{rr} &= \lambda_{rr} - \lambda_{ref}
\end{aligned} \tag{13}
$$

to satisfy reaching law in the sliding mode approach, the derivative of sliding surface is considered as[14]:

$$
\dot{s} = -k\,sign(s) \tag{14}
$$

where k is a positive constant. From (13) it is concluded:

$$
\dot{s} = \dot{\lambda} = -k\,sign(s) \tag{15}
$$

then

$$
\dot{\lambda} = -R_w \frac{\omega\dot{v} - v\dot{\omega}}{v^2} \tag{16}
$$

by substituting (4) in (16) the brake torque will be calculated.

$$
\begin{aligned}
T_{wfl} &= R_w F_{xfl} + 50\frac{I_{wfl} v_{fl}}{R_w} sign(s_{fl}) + \frac{I_{wfl}}{v_{fl}} \dot{v}_{fl}\\
T_{wfr} &= R_w F_{xfr} + 50\frac{I_{wfr} v_{fr}}{R_w} sign(s_{fr}) + \frac{I_{wfr}}{v_{fr}} \dot{v}_{fr}\\
T_{wrl} &= R_w F_{xrl} + 50\frac{I_{wrl} v_{rl}}{R_w} sign(s_{rl}) + \frac{I_{wrl}}{v_{rl}} \dot{v}_{rl}\\
T_{wrr} &= R_w F_{xrr} + 50\frac{I_{wrr} v_{rr}}{R_w} sign(s_{rr}) + \frac{I_{wrr}}{v_{rr}} \dot{v}_{rt}
\end{aligned} \tag{17}
$$

4 Simulation Results

In this section a sliding mode controller is designed to be applied to a four wheel vehicle system with the shown vehicle parameters in the table 1. Here the steering angles are assumed to be zero during braking state.

Table Vehicle Parameters

m	*1208 kg*
I_z	*1600 kg.m²*
L_f	*1.249 m*
L_r	*1.251 m*
I_w	*2.11 kg.m²*
T_f & T_r	*1.321 m*
h	*0.55 m*
c_x	$0.30\ m^{-1}$
c_y	$0.15\ m^{-1}$
R_w	*0.30 m*

It is assumed in the zero time that the longitudinal and lateral velocities of CoG are *30m/s* and *5m/s* respectively and the yaw rate is zero. In Fig. 2 the longitudinal slips for all wheels are shown. It is obvious that the reference value is very well tracked in about one second. The longitudinal and lateral velocities of CoG are depicted in Fig. 3. As it is seen the vehicle is stopped after about 4 seconds. In Fig. 4 angular velocities for front left and rear right wheels are shown. Also longitudinal velocities for front left and rear right wheels are depicted in Fig. 5.

5 Conclusion

In this paper, a seven degree of freedom model for a four wheel system was considered and a sliding mode controller was designed to control the vehicle during braking. As it was illustrated the reference longitudinal slip of each wheel was tracked well. Also the longitudinal and lateral velocities were decreased very smoothly and vehicle was stopped very soon.

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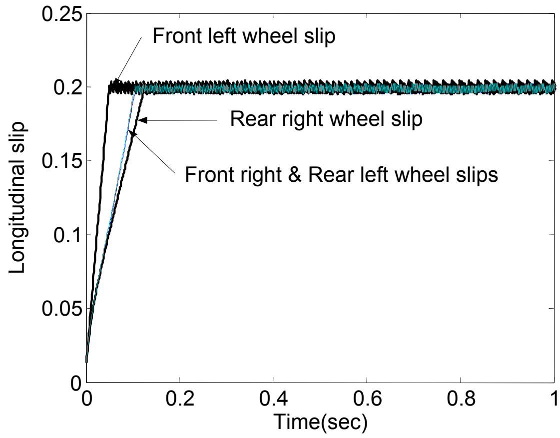

Fig. 2. Longitudinal slip for all wheels

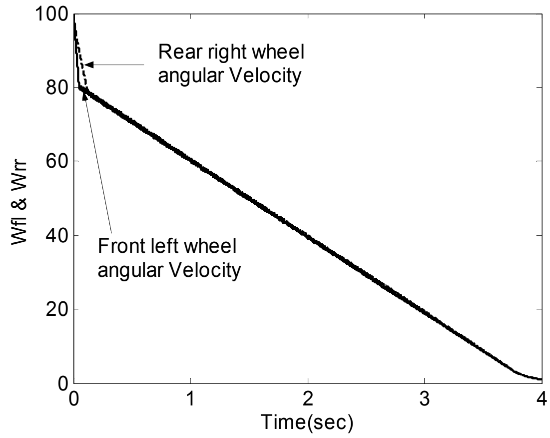

Fig. 4. Angular velocities for front left and rear right wheels

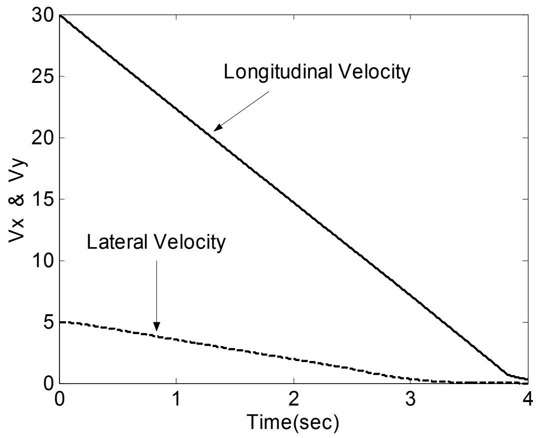

Fig. 3. Longitudinal and lateral velocities of the COG

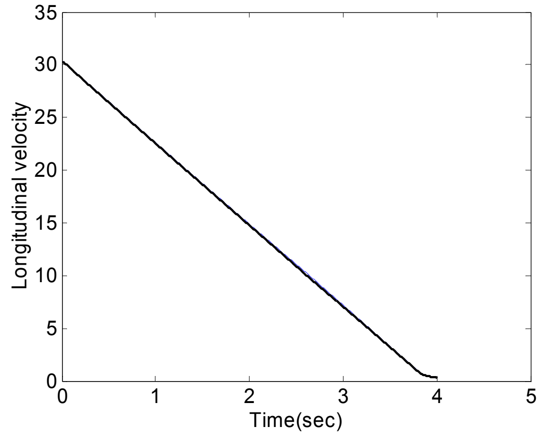

Fig. 5. Longitudinal velocities for front left and rear right wheels