An Optimal Four Wheel Steering Vehicles Control Based on Pole Placement Method

M.R. MOSTAVI[1], M. SHARIATPANAHI[1] and R. KAZEMI[2]
[1]Department of Mechanical Engineering, University of Tehran, Iran
[2] Department of Mechanical Engineering,
K.N. Toosi University of Technology, Tehran, Iran

Abstract: - Four wheel steering vehicles are being used increasingly due to high performance and stability that they bring to the vehicles. In this paper a novel high performance four wheel steered vehicle model is considered which is optimally controlled during a lane change manoeuvre in high speeds. The specific configuration of this work is a novel technique for predetermination of system's stability based on pole placement method. Simulation results reveal the effectiveness of the proposed model and controller.

Key-Words: - Four wheel steering, Pole Placement, Optimal control, Body side slip angle, Yaw rate.

1 Introduction

There is an increasing trend toward sophisticated chassis control systems in vehicle design. The three main systems of chassis control are: lateral control, vertical control, and longitudinal control. These systems were developed independently to improve vehicle handling, ride comfort, and traction/braking performance as well as to relieve driver's workload. Among them, active four wheel steering (4WS) systems enhance vehicle cornering ability by steering the front and rear wheels in accordance with vehicle states. With such steering control systems, it becomes possible to improve the lateral stability and handling performance in a range where the vehicle dynamics could be described by linear and nonlinear models [1]. A great number of studies have been made on various control strategies for 4WS vehicles since the first 4WS system was reported. In standard two wheel steering vehicles, the rear set of wheels are always directed forward therefore and do not play an active role in controlling the steering. In four wheel steering systems, the rear wheels can turn left and right. To keep the driving controls as simple as possible, a computer is used to control the rear wheels. The nonlinear behavior, the stability of the vehicle and the nonlinear effects on vehicle dynamics, hence, require a systematical analysis.

In the present work, a new methodology of mathematically modelling for the 4WS vehicle–driver system (which was previously developed [2,3]) is enhanced so that the stability of the closed loop control system is guaranteed.

In this paper by optimizing a cost function regards to state variables, the control law is determined. The optimal control used in this paper is based on the Riccatti differential equations [4,5].

The paper is organized as follows: Section 2 describes a complete model of vehicle. In Section 3 the design procedure of optimal control is considered. Simulation results of the system are provided in Section 4. Finally, the paper is concluded in Section 5.

2 Vehicle modelling

The vehicle model considered here (the reduced nonlinear two track model) considers much more nonlinearities and hence gives a more precise result. In this model, each tyre has forces in direction of the wheel plane and perpendicular to it which are called F_L and F_S respectively. The three degrees of freedom model to be used in the controller design is depicted in Fig. 1. In this figure, the longitudinal and lateral tyre friction forces are represented as F_{lij} and F_{sij} (i,j=1, …, 4), respectively. The external force along the vehicle x and y axes F_x and F_y are assumed to result exclusively from the friction forces between tyres and road.

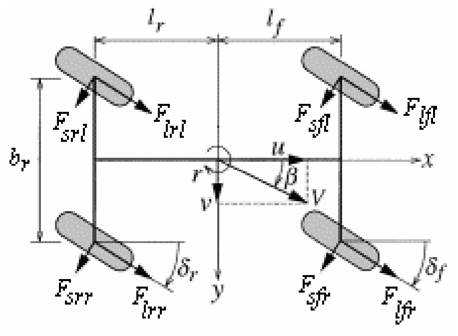

Fig. 1 The four wheel steering vehicle model

We may introduce two coordinate systems as:

- *"CoG"* (Center of Gravity) for the chassis coordinate system
- *"In"* for the fixed inertial system

The reduced model should contain only those state variables which are essential for vehicle dynamic control. These are the vehicle speed V_{cog}, the vehicle body side slip angle β, and yaw rate $\dot{\psi}$. Now the vehicle speed can be transformed to fixed inertial coordinate system:

$$\begin{bmatrix} \dot{x}_{in} \\ \dot{y}_{in} \end{bmatrix} = V_{cog} \begin{bmatrix} \cos(\beta+\psi) \\ \sin(\beta+\psi) \end{bmatrix} \tag{1}$$

By differentiating the equation (1) we will have:

$$\begin{bmatrix} \ddot{x}_{in} \\ \ddot{y}_{in} \end{bmatrix} = V_{cog}(\dot{\beta}+\dot{\psi}) \begin{bmatrix} -\sin(\beta+\psi) \\ \cos(\beta+\psi) \end{bmatrix} + \dot{V}_{cog} \begin{bmatrix} \cos(\beta+\psi) \\ \sin(\beta+\psi) \end{bmatrix} \tag{2}$$

These accelerations are now transformed from the inertial into the *CoG* coordinate system.

$$\begin{bmatrix} \ddot{x}_{cog} \\ \ddot{y}_{cog} \end{bmatrix} = \begin{bmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{bmatrix} \begin{bmatrix} \ddot{x}_{in} \\ \ddot{y}_{in} \end{bmatrix} = V_{cog}(\dot{\beta}+\dot{\psi}) \begin{bmatrix} -\sin\beta \\ \cos\beta \end{bmatrix} + \dot{V}_{cog} \begin{bmatrix} \cos\beta \\ \sin\beta \end{bmatrix} \tag{3}$$

By neglecting gravitational forces F_{gx} and F_{gy}, rolling resistance F_r, lateral wind force and the wind velocity V_{windy}, the complete equation for horizontal translatory motion are then given by:

$$\mathrm{V}_{cog}.(\dot{\beta}+\dot{\psi}) \begin{bmatrix} -\sin\beta \\ \cos\beta \end{bmatrix} + \dot{V}_{cog} \begin{bmatrix} \cos\beta \\ \sin\beta \end{bmatrix} = \frac{1}{M_{cog}} \begin{bmatrix} F_{xfl}+F_{xfr}+F_{xrl}+F_{xrr}+F_{windx} \\ F_{yfl}+F_{yfr}+F_{yrl}+F_{yrr} \end{bmatrix} \tag{4}$$

which yields:

$$\dot{V}_{cog} = \frac{\cos\beta}{M_{cog}}(F_{xfl}+F_{xfr}+F_{xrl}+F_{xrr} - C_{aerx}A_l\frac{\rho}{2}V^2{}_{cog}) + \frac{1}{M_{cog}}(F_{yfl}+F_{yfr}+F_{yrl}+F_{yrr})\sin\beta \tag{5}$$

$$\dot{\beta} = \frac{1}{M_{cog}V_{cog}\cos\beta}(F_{yfl}+F_{yfr}+F_{yrl} + F_{yrr} - M_{cog}\dot{V}_{cog}\sin\beta) - \dot{\psi} \tag{6}$$

$$\begin{aligned} J_z\ddot{\psi} &= (F_{yfr}+F_{yfl}).(L_f - n_{lf}) - (F_{yrr}+F_{yrl}).(L_r+n_{lr}) + (F_{xrr}-F_{xrl})\frac{b_r}{2} \\ &+ F_{xfr}(\frac{b_f}{2} - n_{sfr}\sin\delta_{wf}) - F_{xfl}(\frac{b_f}{2} + n_{sfl}\sin\delta_{wf}) \\ &+ F_{xrr}(\frac{b_r}{2} - n_{srr}\sin\delta_{wr}) - F_{xrl}(\frac{b_r}{2} + n_{srl}\sin\delta_{wr}) \end{aligned} \tag{7}$$

by substitution of :

$$\begin{aligned} F_{xij} &= F_{lij}\cos\delta_{wi} - F_{sij}\sin\delta_{wi} \\ F_{yij} &= F_{sij}\cos\delta_{wi} + F_{lij}\sin\delta_{wi} \end{aligned} \tag{8}$$

in eq. 5 to 7 we will have the state space variables in terms of longitudinal and lateral forces and other vehicle parameters.

The longitudinal forces F_{lij} are regarded as control inputs (by assuming a vehicle with four electrical driving motors on four wheels). The wheel lateral forces F_{sij} are now approximated to be proportional to the tyre side slip angle α_{ij} [6,7].

$$F_{sfl} = C_{fl}.\alpha_{fl} = C_{fl}(\delta_{wf} - \beta - \frac{l_f\dot{\psi}}{V_{cog}}) \tag{9}$$

$$F_{sfr} = C_{fr}.\alpha_{fr} = C_{fr}(\delta_{wf} - \beta - \frac{l_f\dot{\psi}}{V_{cog}}) \tag{10}$$

$$F_{srl} = C_{rl}.\alpha_{rl} = C_{rl}(\delta_{wr} - \beta + \frac{l_r\dot{\psi}}{V_{cog}}) \tag{١١}$$

$$F_{srr} = C_{rr}.\alpha_{rr} = C_{rr}(\delta_{wr} - \beta + \frac{l_r\dot{\psi}}{V_{cog}}) \tag{12}$$

Hence the wheel turn angle and the longitudinal wheel forces F_{lij} are utilized as control inputs for vehicle dynamic control by steering. So the state space variables of the reduced nonlinear four wheels steered two track model become:

$$f_1 = \dot{V}_{cog} = \frac{1}{M_{cog}}\{F_{lfl}\cos(\beta-\delta_{wf}) + F_{lfr}\cos(\beta-\delta_{wf}) + F_{lfr}\cos(\beta-\delta_{wr}) + F_{lrr}\cos(\beta-\delta_{wr}) - C_{aer}.A_l.\frac{\rho}{2}.V^2{}_{cog}\cos\beta + [C_{fl}\delta_{wf} - C_{fl}\beta - C_{fl}\frac{l_f\dot{\psi}}{V_{cog}}]\sin(\beta-\delta_{wf}) + [C_{fr}\delta_{wf} - C_{fr}\beta - C_{fr}\frac{l_f\dot{\psi}}{V_{cog}}]\sin(\beta-\delta_{wf}) + [C_{rl}\delta_{wr} - C_{rl}\beta - C_{rl}\frac{l_r\dot{\psi}}{V_{cog}}]\sin(\beta-\delta_{wr}) + [C_{rr}\delta_{wr} - C_{rr}\beta - C_{rr}\frac{l_r\dot{\psi}}{V_{cog}}]\sin(\beta-\delta_{wr})\} \quad (13)$$

$$f_2 = \dot{\beta} = \frac{1}{M_{cog}.V_{cog}}.\{Sin(\beta-\delta_{wf})[-F_{lfl} - F_{lfr}] + Sin(\beta-\delta_{wr})[-F_{lrl} - F_{lrr}] + \cos(\beta-\delta_{wf})[C_{fr} + C_{fl}].[\delta_{wf} - \beta - \frac{l_f\dot{\psi}}{V_{cog}}] + \cos(\beta-\delta_{wr})[C_{rr} + C_{rl}].[\delta_{wr} - \beta - \frac{l_r\dot{\psi}}{V_{cog}}] \quad (14)$$

$$f_3 = \dot{\psi} = \frac{1}{2J_z.V_{cog}}.\{\begin{pmatrix} C_{fl}\delta_{wf}V_{cog}b_f \\ -C_{fl}\beta V_{cog}b_f \\ -C_{fl}l_f\dot{\psi}b_f \\ +2F_{lfl}V_{cog}l_f \end{pmatrix}\sin\delta_{wf} + \begin{pmatrix} 2C_{fl}\delta_{wf}l_fV_{cog} - 2C_{fl}\beta V_{cog}l_f \\ -2C_{fl}l_f{}^2\dot{\psi} - F_{lfl}b_fV_{cog} \end{pmatrix}\cos\delta_{wf} + \begin{pmatrix} -C_{fr}\delta_{wf}V_{cog}b_f + C_{fr}\beta V_{cog}b_f \\ +C_{fr}l_f\dot{\psi}b_f + 2F_{lfr}V_{cog}l_f \end{pmatrix}\sin\delta_{wf} + \begin{pmatrix} 2C_{fr}\delta_{wf}l_fV_{cog} - 2C_{fr}\beta V_{cog}l_f \\ -2C_{fr}l_f{}^2\dot{\psi} + F_{lfr}b_fV_{cog} \end{pmatrix}\cos\delta_{wf} + \begin{pmatrix} -C_{rr}\delta_{wr}V_{cog}b_r + C_{rr}\beta V_{cog}b_r \\ -C_{rr}l_r\dot{\psi}b_r - 2F_{lrr}V_{cog}l_r \end{pmatrix}\sin\delta_{wr} + \begin{pmatrix} -2C_{rr}\delta_{wr}l_rV_{cog} + 2C_{rr}\beta V_{cog}l_r \\ -2C_{rr}l_r{}^2\dot{\psi} + F_{lrr}b_rV_{cog} \end{pmatrix}\cos\delta_{wr} + \begin{pmatrix} C_{rl}\delta_{wr}V_{cog}b_r - C_{rl}\beta V_{cog}b_r \\ +C_{rl}l_r\dot{\psi}b_r - 2F_{lrl}V_{cog}l_r \end{pmatrix}\sin\delta_{wr} + \begin{pmatrix} -2C_{rl}\delta_{wr}l_rV_{cog} + 2C_{rl}\beta V_{cog}l_r \\ -2C_{rl}l_r{}^2\dot{\psi} - F_{lrr}b_rV_{cog} \end{pmatrix}\cos\delta_{wr} + \beta\begin{pmatrix} 2n_{lf}V_{cog}(C_{fl}+C_{fr}) \\ +2n_{lr}V_{cog}(C_{rl}+C_{rr}) \end{pmatrix} - 2\delta_{wf}n_{lf}V_{cog}(C_{fl}+C_{fr}) - 2\delta_{wr}n_{lr}V_{cog}(C_{rl}+C_{rr})\}. \quad (15)$$

3 Controller Design

In the state space form the reduced nonlinear two track model can be written as:

$$\begin{aligned} \underline{\dot{x}} &= \underline{A}(\underline{x},\underline{u})\underline{x} + \underline{B}(\underline{x},\underline{u})\underline{u} \\ \underline{y} &= \underline{C}(\underline{x},\underline{u})\underline{x} \end{aligned} \quad (16)$$

The state vector is:

$$\underline{x} = [V_{cog} \;\; \beta \;\; \dot{\psi}]^T \quad (17)$$

While the control output is:

$$\underline{y} = [V_{cog} \;\; \dot{\psi}]^T \quad (18)$$

This nonlinear state space equation has to be optimally controlled. Regarding this will we would need to express our state variables in terms of Taylor series around an actual operating point. Since the non linear state space equations are rather complicated a first order Taylor series would be appropriate.
The state space equations are rewritten as:

$$\underline{\dot{x}} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \dot{V}_{cog} \\ \dot{\beta} \\ \ddot{\psi} \end{bmatrix} = \underline{f}(\underline{x},\underline{u}) = \begin{bmatrix} \underline{f}_1(\underline{x},\underline{u}) \\ \underline{f}_2(\underline{x},\underline{u}) \\ \underline{f}_3(\underline{x},\underline{u}) \end{bmatrix} \quad (19)$$

$$\underline{y} = \underline{c}(\underline{x},\underline{u})\;\underline{x} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\underline{x}$$

where:

$$\underline{f}(\underline{x},\underline{u}) = \underline{f}(\underline{x}_0,\underline{u}_0) + \left.\frac{\partial \underline{f}(\underline{x},\underline{u})}{\partial \underline{x}}\right|_{\substack{\underline{x}=\underline{x}_0 \\ \underline{u}=\underline{u}_0}}.(\underline{x}-\underline{x}_0) + \left.\frac{\partial \underline{f}(\underline{x},\underline{u})}{\partial \underline{u}}\right|_{\substack{\underline{x}=\underline{x}_0 \\ \underline{u}=\underline{u}_0}}.(\underline{u}-\underline{u}_0) \quad (20)$$

In this equation $\frac{\partial \underline{f}(\underline{x},\underline{u})}{\partial \underline{x}}$ and $\frac{\partial \underline{f}(\underline{x},\underline{u})}{\partial \underline{u}}$ are Jaccobian matrices which are defined as:

$$\frac{\partial \underline{f}(\underline{x},\underline{u})}{\partial \underline{x}} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} \\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} & \frac{\partial f_3}{\partial x_3} \end{bmatrix} \quad (21)$$

$$\frac{\partial \underline{f}(\underline{x},\underline{u})}{\partial \underline{u}} = \begin{bmatrix} \frac{\partial f_1}{\partial u_1} & \frac{\partial f_1}{\partial u_2} & \frac{\partial f_1}{\partial u_3} & \frac{\partial f_1}{\partial u_4} & \frac{\partial f_1}{\partial u_5} & \frac{\partial f_1}{\partial u_6} \\ \frac{\partial f_2}{\partial u_1} & \frac{\partial f_2}{\partial u_2} & \frac{\partial f_2}{\partial u_3} & \frac{\partial f_2}{\partial u_4} & \frac{\partial f_2}{\partial u_5} & \frac{\partial f_2}{\partial u_6} \\ \frac{\partial f_3}{\partial u_1} & \frac{\partial f_3}{\partial u_2} & \frac{\partial f_3}{\partial u_3} & \frac{\partial f_3}{\partial u_4} & \frac{\partial f_3}{\partial u_5} & \frac{\partial f_3}{\partial u_6} \end{bmatrix} \quad (22)$$

While the input vector $\underline{u}_F$ is:

$$\begin{aligned} \underline{u} &= \begin{bmatrix} u_1 & u_2 & u_3 & u_4 & u_5 & u_6 \end{bmatrix}^T \\ &= \begin{bmatrix} F_{lfl} & F_{lfr} & F_{lrl} & F_{lrr} & \delta_{wf} & \delta_{wr} \end{bmatrix}^T \end{aligned} \quad (23)$$

In order to enhance the control of the system we would use a new method separating these matrices into two parts: the first part is the forces applied to tyres and the second part is the input steering angles:

$$\begin{aligned} \underline{u}_F &= \begin{bmatrix} u_1 & u_2 & u_3 & u_4 \end{bmatrix}^T \\ &= \begin{bmatrix} F_{lfl} & F_{lfr} & F_{lrl} & F_{lrr} \end{bmatrix}^T \\ \underline{u}_{\delta w} &= \begin{bmatrix} u_5 & u_6 \end{bmatrix}^T = \begin{bmatrix} \delta_{wf} & \delta_{wr} \end{bmatrix}^T \end{aligned} \quad (24)$$

meaning that the driver influences the steering (i.e. δ_w) and the controller affects the brake forces $F_{Li,j}$, thus the effect of two can be separated. The Jaccobian is:

$$\left.\frac{\partial \underline{f}(\underline{x},\underline{u})}{\partial \underline{u}}\right|_{\substack{\underline{x}=\underline{x}_0 \\ \underline{u}=\underline{u}_0}} .\Delta \underline{u} =$$

$$\left.\left[\begin{array}{cccc|cc} \frac{\partial f_1}{\partial u_1} & \frac{\partial f_1}{\partial u_2} & \frac{\partial f_1}{\partial u_3} & \frac{\partial f_1}{\partial u_4} & \frac{\partial f_1}{\partial u_5} & \frac{\partial f_1}{\partial u_6} \\ \frac{\partial f_2}{\partial u_1} & \frac{\partial f_2}{\partial u_2} & \frac{\partial f_2}{\partial u_3} & \frac{\partial f_2}{\partial u_4} & \frac{\partial f_2}{\partial u_5} & \frac{\partial f_2}{\partial u_6} \\ \frac{\partial f_3}{\partial u_1} & \frac{\partial f_3}{\partial u_2} & \frac{\partial f_3}{\partial u_3} & \frac{\partial f_3}{\partial u_4} & \frac{\partial f_3}{\partial u_5} & \frac{\partial f_3}{\partial u_6} \end{array}\right]\right|_{\substack{\underline{x}=\underline{x}_0 \\ \underline{u}=\underline{u}_0}} . \begin{bmatrix} \Delta \underline{u}_F \\ \Delta \underline{u}_\delta \end{bmatrix} \quad (25)$$

$$= \underline{M}_F \Big|_{\substack{\underline{x}=\underline{x}_0 \\ \underline{u}=\underline{u}_0}} .(\underline{u}_F - \underline{u}_{F0}) + \underline{m}_\delta \Big|_{\substack{\underline{x}=\underline{x}_0 \\ \underline{u}=\underline{u}_0}} .(\delta_w - \delta_{w0})$$

A control law is then defined for small deviations from the operating point $\underline{x}$-$\underline{x}_0$.

The following control law is used:

$$\Delta u_F = -K_c . \Delta x \quad (26)$$

where K_c is the feedback matrix.

Substituting into the linearized state space equation we will have:

$$\begin{aligned} \Delta \underline{\dot{x}} &= \left.\frac{\partial \underline{f}(\underline{x},\underline{u})}{\partial \underline{x}}\right|_{\substack{\underline{x}=\underline{x}_0 \\ \underline{u}=\underline{u}_0}} .\Delta \underline{x} - \underline{M}_F \Big|_{\substack{\underline{x}=\underline{x}_0 \\ \underline{u}=\underline{u}_0}} .K_c .\Delta \underline{x} \\ &+ m_\delta \Big|_{\substack{\underline{x}=\underline{x}_0 \\ \underline{u}=\underline{u}_0}} .\Delta \delta_w \end{aligned} \quad (27)$$

hence:

$$\Delta \underline{\dot{x}} = \left(\left.\frac{\partial \underline{f}(\underline{x},\underline{u})}{\partial \underline{x}}\right|_{\substack{\underline{x}=\underline{x}_0 \\ \underline{u}=\underline{u}_0}} - \underline{M}_F \Big|_{\substack{\underline{x}=\underline{x}_0 \\ \underline{u}=\underline{u}_0}} .K_c \right) .\Delta \underline{x} + m_\delta \Big|_{\substack{\underline{x}=\underline{x}_0 \\ \underline{u}=\underline{u}_0}} .\Delta \delta_w \quad (28)$$

where the term inside brackets represents the system matrix for the closed loop. The driver input shall be regarded as the superimposed noise. The dynamic characteristics of this system can be set using pole placement. A desired matrix $\underline{G}$ is defined, which has the desired system characteristics, i.e. the desired pole positions:

$$\left.\frac{\partial \underline{f}(\underline{x},\underline{u})}{\partial \underline{x}}\right|_{\substack{\underline{x}=\underline{x}_0 \\ \underline{u}=\underline{u}_0}} - \underline{M}_F \Big|_{\substack{\underline{x}=\underline{x}_0 \\ \underline{u}=\underline{u}_0}} .K_c = \underline{G} \quad (29)$$

Now by proper determination of operating point and also the destination state all needed state space parameters are determined.

The state-space representation of the system in Equation 1 can be written as:

$$\underline{\dot{x}} = A\,\underline{x} + B\,\underline{u} \quad (30)$$

The LQR problem is to find the optimal gain matrix such that the state-feedback law minimizes the quadratic cost function.

$$J = \int_0^\infty (x^T Q\ x + u^T R\ u) \quad (31)$$

The matrices Q and R are referred to as the weighting matrices on the state and the input respectively. A smaller Q increased the relative weighting on the input matrix. This should decrease the magnitude of the input necessary to maintain control. In order to insure that all the states go to zero as time goes to infinity, Q must be chosen to be a positive-definite matrix. R is also chosen as a positive-definite matrix to insure the control is finite. The weighting matrices are chosen based on matlab simulations and driving simulator tests such that

$$Q = \begin{bmatrix} 10^5 & 0 & 0 \\ 0 & 10^3 & 0 \\ 0 & 0 & 3\times 10^3 \end{bmatrix} \quad (32)$$

$$R = \begin{bmatrix} 0.05 & 0 \\ 0 & 0.05 \end{bmatrix}$$

constant gain optimal control is

$$u^*(t) = \delta_d(t) = -R^{-1} B^T P x(t) \tag{33}$$

where, P is the steady-state solution to the matrix differential Riccatti equation of the form :

$$\dot{P} = -A^T P - PA + PBR^{-1} B^T P - Q \tag{34}$$

The boundary condition at terminal time is zero, such that:

$$P(t_f) = 0 \tag{35}$$

4 Simulation Results

In this section, the closed-loop responses using the system parameters shown in Table will be presented.

Table Vehicle Parameters

Vehicle mass M_{cog}	*1600 kg*
Mass moment of Inertia J_z	*2300 kg.m*^{2}
Distance from CoG to front axle l_f	*1.2m*
Distance from CoG to rear axle l_r	*1m*
Distance between wheels on front and rear axles b_f, b_r	*1.25m*
Front and rear tire longitudinal casters: n_{lf}, n_{lr}	*0.05m*
Effective vehicle surface A_l	*1.5m*^{2}

In fig. 2-4 we can see the results of the model operating in a lane change manoeuvre in a highway. The vehicle is changing the position from the primary velocity of *25 m/s* and body side slip of 6 degrees while the yaw rate is *0.15 rad/sec* to the final state that the velocity has increased to 35 m/s, the body side slip angle of *4.5* degrees and the yaw rate of *0.136 rad/sec*. The inputs of the system are the steering angles (shown in figs.5,6) applied to front and rear wheels besides that the 4 longitudinal forces applied to the 4 wheels of the vehicle of which the first one F_{lfl} is depicted in fig.7 are determined from the feedback law as expressed before. By proper definition of the feedback matrix, the eigenvalues of the G matrix (which determine the stability of the system) are: -0.7477, -0.2783 + 0.5044i and -0.2783 - 0.5044i.

5 Conclusion

In this paper, a three degree of freedom model for a four wheel steering system was considered and an optimal controller was designed to control the vehicle during its lane change manoeuvres in highways. Besides the control inputs were separated into two parts; the longitudinal tyre forces and wheel steering angles. The first part is determined by a feedback matrix from the state variables and hence the stability of the system can be determined by proper definition of this feedback matrix. Simulation results reveal that the model has a very good and effective performance.

References

[1] Y. Wang and M. Nagai, "Integrated Control of Four-Wheel-Steer and Yaw Moment to Improve Dynamic Stability Margin", Proc. 35th IEEE-CDC, pp.1783-1784, 1996.

[2] Mostavi M., Kazemi R. and Shariatpanahi M. “An optimal control of four wheel steering vehicles" to appear in WSEAS Conf. Greece, Athens, July 2004.

[3] Peng, H., and M. Tomizuka, “Vehicle Lateral Control for Highway Automation,” Proc. of the Am. Ctrl Conf., San Diego, 1990, pp. 788-794.

[4] R. Uinter “Optimal Control” Birkhaus, Boston, 2000.

[5] J. Yang,X.Y. Zhou “Stocastic controls-Hamiltonian systems and HJB equations” Springer,New York,1999

[6] A. Daiss and U. Kiencke, "Estimation of Tire Slip during combined Cornering and Braking Observer Supported Fuzzy Estimation", Proc. of 13the IFAC World Congress, 8b-02-2, pp.41- 46, 1996.

[7] L. R. Ray, "Nonlinear Tire Force Estimation and Road Friction Identification: Simulation and Experiments", Automatica, Vol.33, No.10, pp.1819-1833, 1997.

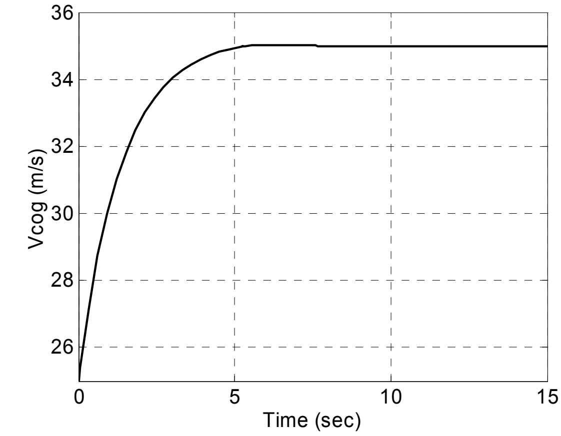

Fig. 2. Vehicle velocity during lane change maneuver

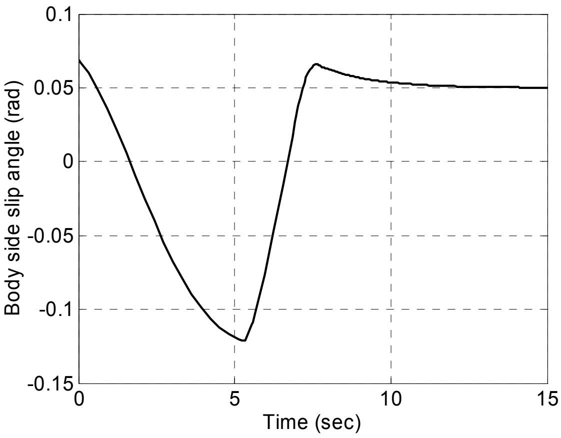

Fig. 3. Vehicle body side slip angle during lane change maneuver

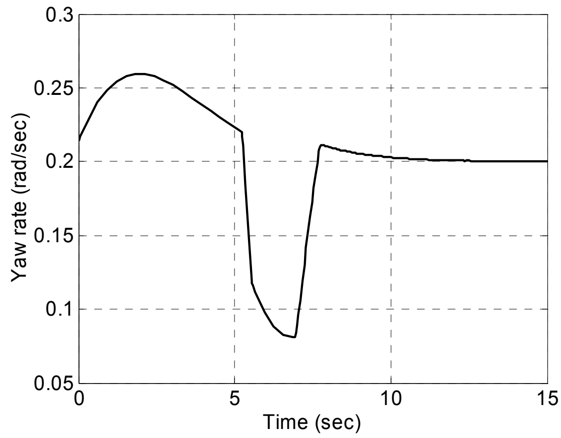

Fig. 4. Yaw rate during lane change maneuver

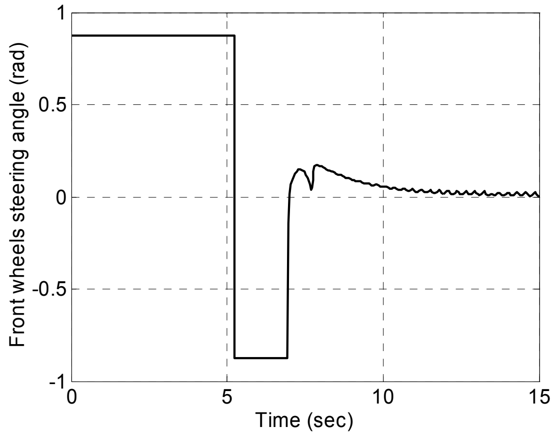

Fig. 5. Front wheels steering angle during lane change maneuver

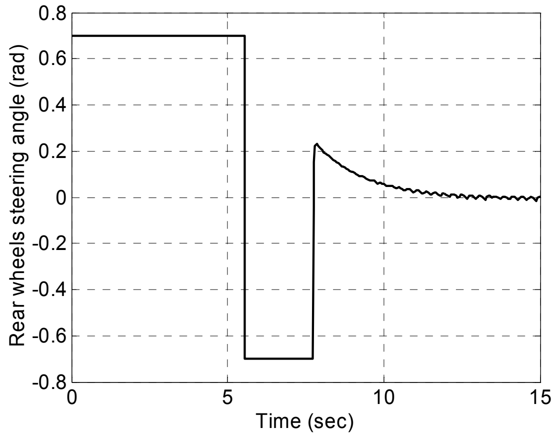

Fig. 6. Rear wheels steering angle during lane change maneuver

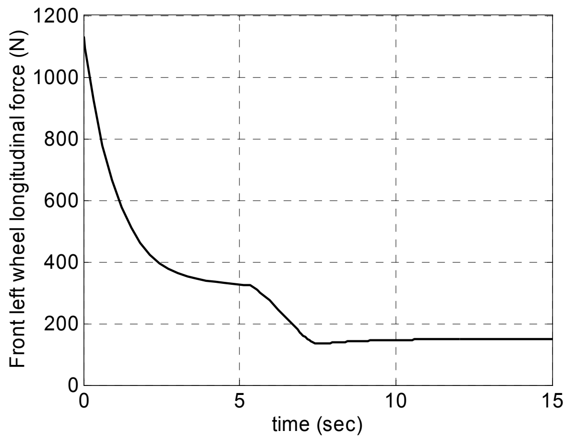

Fig. 7. Front left wheel longitudinal force during lane change maneuver