RESIDUAL GENERATION SYNTHESIS FOR FAULT DETECTION AND ISOLATION

KECHIDA SIHEM [1], DEBBACHE NASR EDDINE[3], GHOUL HADIBY RACHIDA[2,3], TEBBIKH HICHAM [1]

[1]Laboratory of Automatics and Informatics of Guelma "LAIG",
Department of electronics, Faculty of engineering and science , University of Guelma, Algeria .

[2]Laboratory of Automatics and Informatics of Guelma "LAIG",
[3]Department of electronics, faculty of engineering, University of Annaba, Algeria.

Phone (213) 37 20 02 65 Fax : (213) 37 20 72 68

Abstract - The Taking care of fault that might induce a downgrading in complex automated systems performances, by detection and diagnosis procedures has become a vital necessity. The aim is to maintain an acceptable behavior of the system in abnormal operating conditions. Fault Detection and Isolation procedures **(FDI)** coupled with regulation / control procedures insure an adequate tolerance level to automated system fault.

In this paper, we focus on the FDI procedures based on the analytical redundancy, i.e. on the use of a mathematical model of the system, and more particularly to fault indicating signal synthesis (called residual) which is the essential step in any analytical FDI procedure. Residuals must satisfy a double requirement : insensitive (robust) to modelling uncertainties (in ordre to reduce false alarm rate), and highly sensitive to faults (in ordre to reduce the non–detection rate), while having appropriated structures to facilitate fault isolation.

KeyWords : Fault detection and isolation, diagnosis, residual generator, coprime factorization, robustness.

1. INTRODUCTION

The modern technology advances to a point where it is possible and extensively desirable to improve reliability and the technical process safety. This is achieved by computer implanted FDI procedures. However, the malfunction of actuators, sensors and of the process components, as well as erroneous actions of human operators can have some disastrous consequences in high risk systems such as: spatial engines (Astronomy), aircrafts (Aviation), nuclear reactors, chemical plants.

Thus, each failure or fault can lead to shutdowns or a rupture of service and consequently a plant output reduction. There is an improvement of consciousness and attitude to possible disaster provoked by failures that could enable a failure tolerating system development.

Such system must maintain a optimal performance during normal operating conditions and must handl encountered critical situations during which the system's conditions are abnormal that is by performing of detection and diagnosis procedures and reconfiguration according to accurate software programs.

2. FDI PROBLEM FORMULATION

Since the FDI system operating quality depends mainly on the model's quality, it is more important to start with a realistic and minitious specification of the given process. It will be the basis for the FDI problem fundamental solution.

In general, we adopt the following linear continuous state representation to describe the nominal behavior of the system.

$$\begin{cases} \dot{x}(t)=A.x(t)+B.u(t) \\ y(t)=C.x(t)+D.u(t) \end{cases} \quad (1)$$

where $x \in \Re^n$, $u \in \Re^r$,$y \in \Re^m$ being respectively state, input, and observation vectors. The A, B, C and D are known matrices.

The input / output corresponding model is:

$$y_0(s)=[\ C.(sI - A)^{-1}\ B+ D\ ].u(s)= G_u(s).u(s) \quad (2)$$

where $G_u(s) = [\ C.(sI - A)^{-1}\ B+ D\ ]= [\ A, B, C, D\ ]$
is the nominal transfer matrice which is rational proper and (mxr) dimension.

In a special case of the linear systems including faults effects and additives type perturbations, the state equations take the following form[1] :

$$\begin{cases} \dot{x}(t)=A.x(t)+B.u(t)+E_1.d(t)+K_1.f(t) \\ y(t)=C.x(t)+D.u(t)+E_2.d(t)+K_2.f(t) \end{cases} \quad (3)$$

With A, B, C, D, E_1 ,E_2 K_1 and, K_2 are known matrices with appropriate dimensions.
$d \in \Re^p$, $f \in \Re^q$ are respectively the unkown perturbations and faults vectors.

In an equivalent way, we have:

$$y(s) = G_u(s).u\ (s) + G_f\ (s).f\ (s) + G_d\ (s).d\ (s) \quad (4)$$

with

$$G_u\ (s) = C.(sI - A)^{-1}\ B + D \quad (5)$$

$$G_d\ (s) = C.(sI - A)^{-1}\ E_1 + E_2 \quad (6)$$

$$G_f\ (s) = C.(sI - A)^{-1}\ K_1 + K_2 \quad (7)$$

Where y(s) , f(s) et d(s) are respectively the Laplace transformation of y(t), f(t) and d(t).

2.1 Residual Generation

The aim of residual generation is to produce a vector r which is structured such that fault effects are independent from each other and also independent from those of the unknown inputs. The most favorable case corresponds to the perfect decoupling.

The most general form of a linear residual generator in the frequency domain is [2]:

$$r(s) = H(s).\begin{bmatrix} u(s) \\ y(s) \end{bmatrix} = [\, H_u(s) \;\; H_y(s)\,].\begin{bmatrix} u(s) \\ y(s) \end{bmatrix} \quad (8)$$

where H_u and H_y are transfer matrices which must stable.

In ordre to make the r(s) becomes zero for the fault free case (i.e. to achieve requirements in equation (11)), H_u, H_y must satisfy:

$$H_y(s).G_u(s) + H_u(s) = 0 \quad (9)$$

The residual generator priorly uses knowledge contained in the model of the system and processes on line y(t) and u(t) measurements to give in output a residual signal helping to test the coherence of measurements issued from the system and its model.

Thus, the residual has the following mathematical definition:

$$\lim_{t \to \infty} r(t) = 0 \quad \text{for} \quad f(t) = 0, \quad d \neq 0 \quad (10)$$

$$r(t) \neq 0, \quad \text{for} \quad f(t) \neq 0 \quad (11)$$

2.2 Residual evaluation

The generated residual is then used to form appropriate decision functions, noted by J(r). They are valued by an decision unit. The FDI procedure is realised if the following specifications are fulfilled:

$$J(r) < J_{th} \quad \text{for} \quad f(t) = 0, \quad (12)$$

$$J(r) > J_{th} \quad \text{for} \quad f(t) \neq 0 \quad \text{(fault Detection)} \quad (13)$$

$$J(r_i) < J_{thi} \quad \text{for} \quad f_i(t) = 0, \quad (i = 1, \ldots, q) \quad (14)$$

$$J(r_i) > J_{thi} \quad \text{for} \quad f_i(t) \neq 0 \quad \text{(fault Isolation)} \quad (15)$$

Where J_{th} as well as J_{thi} define thresholds.

In this case, the problem of perfect fault detection and isolation (PFDI), (robust residual generation) may be formulated as :

1. For a fault detection, the fault effect on the residual must be decoupled from the unknown input effect.
2. For a fault isolation, the fault effect on the residual must be decoupled from the modelling uncertainties effects and from other faults.

The residual generator synthesis then consists on determining the transfer matrices $H_u(s)$ and $H_y(s)$ which must satisfy (9) as well as the required specifications (robustness, isolability...).

3. CONSTRUCTION OF RESIDUAL GENERATORS

From the Bezout identity [3] given by the following matrice representation :

$$\begin{bmatrix} Y(s) & X(s) \\ -\tilde{N}(s) & \tilde{M}(s) \end{bmatrix} . \begin{bmatrix} M(s) & -\tilde{X}(s) \\ N(s) & \tilde{Y}(s) \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \quad (16)$$

$$\begin{bmatrix} M(s) & \tilde{X}(s) \\ -N(s) & \tilde{Y}(s) \end{bmatrix} . \begin{bmatrix} Y(s) & -X(s) \\ \tilde{N}(s) & \tilde{M}(s) \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}$$

A double coprime factorization of transfer matrice G(s) is written as

$$G_u(s) = N_u(s)\, M_u^{-1}(s) = \tilde{M}_u^{-1}(s).\, \tilde{N}_u(s) \quad (17)$$

where $N_u(s)$, $M_u(s)$, $\tilde{N}_u(s)$ et $\tilde{M}_u(s)$ are the right and left coprime factors and belong to the set of proper stable rationnel matrices noted by $\Re H\infty$[4].

From the left factorisation, we can written :

$$\tilde{M}_u.y - \tilde{N}_u.u = 0 \quad (18)$$

Let's note that the equation (18) can be drawn straight from equation (2) and equation (17).

In the occurence of faults in actuators and /or sensors, the relation (18) does not hold anymore and we get

$$r(s) = \tilde{M}_u.y - \tilde{N}_u.u \quad (19)$$

r(s) is called the residual vector which is given by a set of dynamical relations (residual generator) operating on inputs - outputs data with the help of proper stable rational functions.

In normal functionning (non failing sensors and actuators), r(s) is zero, in the ideal case (without modelisation errors and without noise). However, all or one vector component of r(s) will deviate in presence of failures on either one of the sensors or actuators.

Therefore, the vector r (s) can be used as a residual to detect the presence of faults.

4. FAILURE ISOLATION

we consider the vote procedures use which are more general; they don't require any hypothesis on the failures mode.

To implement a vote procedure with complete isolation, redondancy relation structures have to satisfy these two criterias:

- Each component (sensor or actuator) appear in at least one equation.
- Each component is excluded from least a equation.

If these two conditions are satisfied, then the isolation task becomes simple: when a component fails, all relations including this component will be uncoherent while those where it is excluded remain coherent. Thus, all components featured in the coherent relations will be declared non failing and the component common to all relations is readily identified as failed.

4.1 Vote procedure using Hermite form

As illustrated, we assume that the actuators are good and we demonstrate sensor failure isolation.

Let the left factorization ($\widetilde{M}_u$, $\widetilde{N}_u$), we can always find unimodular matrice U_1 such as $U_1.\widetilde{M}_u$ has a upper triangular Hermite form.

Premultiplying (19) by U_1 (i.e. pour m = 3) yields :

$$r_c = U_1.\widetilde{M}_u.y - U_1\widetilde{N}_u.u \quad (20)$$

$$\begin{bmatrix} r_{c1} \\ r_{c2} \\ r_{c3} \end{bmatrix} = \begin{bmatrix} \widetilde{M}_{11} & 0 & 0 \\ \widetilde{M}_{21} & \widetilde{M}_{22} & 0 \\ \widetilde{M}_{31} & \widetilde{M}_{32} & \widetilde{M}_{33} \end{bmatrix} \begin{bmatrix} y1 \\ y2 \\ y3 \end{bmatrix} - U_1 . \widetilde{N}_u.u \quad (21)$$

we can easily arrive to the following concluions:

If $\mathbf{r_{c1} = r_{c2} = 0}$ and $\mathbf{r_{c3} \neq 0}$ then only sensor 3 is faulty.

If $\mathbf{r_{c1} = 0}$ but $\mathbf{r_{c2} \neq 0}$ and $\mathbf{r_{c3} \neq 0}$ then sensor 2 is certainly faultly and sensor 3 is eventually fautly

If $\mathbf{r_{c1} \neq 0}$ $\mathbf{r_{c2} \neq 0}$ and $\mathbf{r_{c3} \neq 0}$ then sensor 1 is certainly failed and either sensor 2 or sensor 3 or both are failed.

5. APPLICATION

In the last section, a FDI approach is proposed. To illustrate its implementation we give an exemple. The physical system considered is an airplane propulsion system GE 21 [5].

The values of A, B, C, D are the following:

$$A = \begin{bmatrix} -3.370 & 1.636 \\ -0.325 & -1.896 \end{bmatrix}; B = \begin{bmatrix} 0.586 & 1.419 & 1.252 \\ 0.410 & 1.118 & 0.139 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.267 & -0.025 & -0.146 \end{bmatrix}; C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0.731 & 0.786 \end{bmatrix}$$

and the transfer function is

$G_u(s) = [G_{11}\ G_{12}\ G_{13}] / (s^2+5.2660s+6.9212)$

where

$$G_{11} = \begin{bmatrix} 0.586s+1.7818 \\ 0.4100s+1.1913 \\ 0.2670s^2+2.1566s+4.0868 \end{bmatrix}$$

$$G_{12} = \begin{bmatrix} -1.4180s-0.8614 \\ 1.1180s+4.2288 \\ -0.0250s^2-0.2902s+2.212 \end{bmatrix}$$

$$G_{13} = \begin{bmatrix} 1.2520s+2.6012 \\ 0.1390s+0.0615 \\ -0.1460s^2+0.2556s+0.9393 \end{bmatrix}$$

we then get the following left factors :

$\widetilde{N}_u(s) = [N_{11}\ N_{12}\ N_{13}] / (s^2+10s+25)$

$$N_{11} = \begin{bmatrix} 0.0303s+0.1513 \\ -0.6444s-3.2221 \\ 0.267s^2+2.1856s+4.2531 \end{bmatrix}$$

$$N_{12} = \begin{bmatrix} -1.367s-6.8348 \\ 1.2167s+6.0836 \\ -0.025s^2-0.2929s-0.8395 \end{bmatrix}$$

$$N_{13} = \begin{bmatrix} 1.5559s+7.7794 \\ 0.7156s+3.5778 \\ -0.146s^2+0.2398s+4.849 \end{bmatrix}$$

And

$\widetilde{M}_u(s) =$

$$\frac{\begin{bmatrix} (s^2+9.891s+24.4594) & 0 & (-2.0814s-10.4071) \\ (3.2118s+16.059) & (s^2+10s+25) & (-3.9491s-19.7455) \\ (2.4452s+12.2259) & 0 & (s^2+5.3745s+1.8724) \end{bmatrix}}{(s^2+10s+25)}$$

The residual generator is then defined by the equations:

$\mathbf{r_1}(s) =$
$[(s^2+9.8915s+24.4576).y_1(s) + (-2.0814s-10.4071).y_3(s) + (0.0303s+0.1513).u_1(s)+(-1.3670s-6.8348).u_2(s)+ (1.5559s+7.7794).u_3(s)\ ] / d(s).$

$\mathbf{r_2}(s) =$
$[(3.2118s+16.0590).y_1(s) + (s^2+10s+25).y_2(s) + (-3.9491s-19.7455).y_3(s) + (-0.6444s - 3.2211).u_1(s) + (1.2167s+6.0836).u_2(s) + (0.7156s+3.5778).u_3(s)]/d(s).$

$\mathbf{r_3}(s) =$
$[(2.4452s+12.2259).y_1(s) + (s^2+5.3745s+1.8724).y_3(s) + (0.2670s^2+2.1856s+4.2531).u_1(s)+(-0.025s^2-0.2929s-0.8395).u_2(s) +(-0.146s^2+0.2398s+4.849).u_3(s)]/d(s).$

5.1 Sensors faiIures introduction

When the system is not affected of faults nor unknown inputs, the residual is null.

Let's consider the introduction of a biased type fault on :

a) Case 1: one sensor

	Fault	Time injection
Sensor 1	f1=0.07	3 seconds

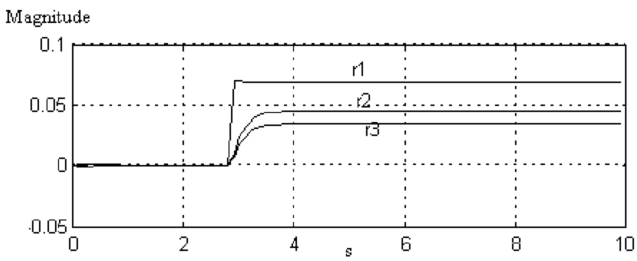

Fig.1. Response of residual r (t) corresponding to fault sensor 1.

b) Case 2: three sensors

	Fault	Time injection
Sensor 1	f1=0.07	2.5 seconds
Sensor 2	f2 =0.5	5 seconds
Sensor3	f3=0.5	7.5 seconds

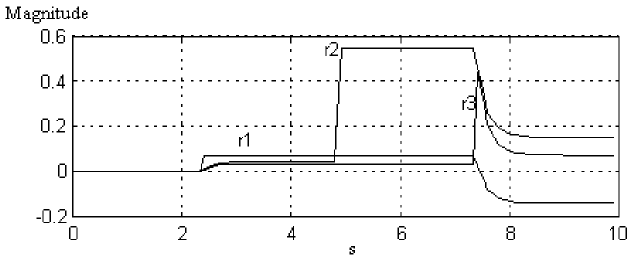

Fig.2. Response of residual r (t) corresponding to the faults sensors.

Residual figures clearly indicate the fault presence. The delay of detection will be related to chosen thresholds. we can however underline that the isolation cannot be achieved because the three residuals are sensitive to a one sensor fault.

The obtained results show that the residual generator is sensitive to the different faults, (even to noises), introduced to the different output components, but it is difficult to isolate such a fault or a noise. Because of that, we introduce an isolation procedure based on the Hermite form. The established algorithm gives left factors of $G_u(s)$ with $\widetilde{M}_H(s)$ of $\widetilde{M}_u(s)$ upper triangular form.

$$\widetilde{M}_H = [\ \widetilde{M}_{11H} \quad \widetilde{M}_{12H} \quad \widetilde{M}_{13H}\ ]$$

$$\widetilde{M}_{11H} = \begin{bmatrix} 3.2118s+16.059 \\ 0 \\ 0 \end{bmatrix}$$

$$\widetilde{M}_{12H} = \begin{bmatrix} 0 \\ -0.7613s^2-7.6131s-19.0328 \\ 0 \end{bmatrix}$$

$$\widetilde{M}_{13H} = \begin{bmatrix} 1.3135s^2+7.0595s+2.4594 \\ s^2+8.381s+16.9094 \\ -0.409s^3-4.1985s^2-13.5987s-14.1528 \end{bmatrix}$$

and

$$U_1 = \begin{bmatrix} 0 & 0 & 1.3135 \\ 0 & -0.7613 & 1 \\ 1 & 0 & -0.408s+2.0005 \end{bmatrix}$$

The residual is:

$\mathbf{r}_1(s) =$

$(3.21118s+16.059).y_1(s)+(1.3135s^2+7.0595s+2.4594).y_3(s)$

$\mathbf{r}_2(s) =$

$(-0.7613s^2-7.6131s-19.0328).y_2(s)+(s^2+8.38s+16.905).y_3(s)$

$\mathbf{r}_3(s) =$

$(-0.4090s^3 - 4.1985s^2-13.5987s-14.1528).y_3(s).$

From these equations a simple logic decision may be considered to isolate the failing sensors.

The corresponding residuals are illustrated by figures (3) and (4) which allow to determinate the instants of fault apparition as well as their isolation.

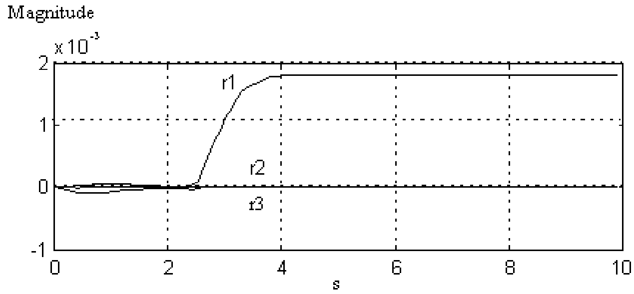

Fig .3. Fault isolation corresponding to first case.

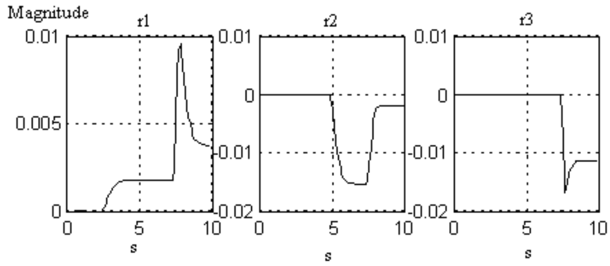

Fig.4. Faults isolation corresponding to second case.

6. CONCLUSION

In this paper, the problem related to desiging fault detection and isolation has been formulated and solved. It has been shown that the frequency domain approaches can be used effectively in treating this class of problem.
Using factorization techniques, residual generators have been developed. They are the basis of our studies.
The results obtained allow to conclude that the residual generator show good performance for fault detection and isolation for sensor type.
As far as, the fault actuator, farther work can confirm the validity for the diagnosis.
In the presence of unkown inputs or uncertainty the robustness problem remain, hence an application of H∞ optimization is still called for.

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