# Adaptability of Fuzzy Sliding-Mode Control Design

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*Abstract:* - In this paper, a design approach adaptability of fuzzy sliding-mode controller (AFSMC) with a decoupled method is proposed. The decoupled method provides a simple way to achieve asymptotic stability for a class of fourth-order nonlinear system. Moreover, heuristic sliding factors are implemented as functions. Therefore, the sliding factor in hybrid fuzzy sliding mode controller is given without trial-and-error. Using this approach, the response of system will converge faster than that of previous reports. The simulation of a cart-pole system is presented to demonstrate the effectiveness and robustness of the method.

*Key-Words:* - fuzzy control, sliding mode

## 1 Introduction

Fuzzy logic controllers (FLC's) are one of useful control schemes for plants having difficulties in deriving mathematical models or having performance limitations with conventional linear control schemes [1]. A fuzzy logic controller is designed on the basis of human experience, which means a mathematical model is not required for controlling a system. Due to this advantages, a fuzzy logic-based control has been implement for many industrial applications [2-3].

In recent years, there have been attempts to design the FLC based on the sliding mode control (SMC) law [4-6]. They have shown that the boundary layer can reach in finite time and the ultimate boundedness of states is obtained asymptotically even though there exist some disturbance of dynamic uncertainties of the system. Palm showed that the analogy between a simple FLC and sliding mode controller with a boundary layer [7]. Hwang et al. proposed a fuzzy sliding mode controller and opened a way of designing an FLC for higher order nonlinear system [8]. The sliding mode control provides a good performance in tracking of some nonlinear systems. Nevertheless, a notorious characteristic of sliding mode control approach is the discontinuity around the switching hyperplane, that means some of the state variable are vibrant. One of the methods to cope with the problem is to utilize a feedforward compensator to offset unpredictable affect of system uncertainties.

Researches also show that the FSMC has the following advantages: (1) It can well control most of complex systems without knowing their exact mathematical models. (2) The dynamic behavior of the controlled system can be approximately dominated by a fuzzified sliding surface. (3) FMSC can not only increase the robustness to system uncertainties but also decrease the chattering phenomenon in the conventional sliding mode controller. Moreover, another problem of designing fuzzy controllers is applied to higher order systems. The large majority of fuzzy controllers are limited to systems with dominantly second-order dynamics.

The action of such fuzzy controllers are equivalent to that of full-state feedback controllers for second-order systems and, hence, these systems can always be stabilized. However, for a fourth-order system, such as the cart-pole system, the system may not be stabilized by using a PID controller and, therefore, using a conventional fuzzy controller will result in a large number of rules. For these systems, the instincts sense is that some rules may be not flexibility if a stabilizing rule base is determined.

In most studies, the fuzzy controller of second-order systems is designed on a phase plane built by error $e$ and change of error $\dot{e}$ that are produced from the states $x$ and $\dot{x}$. For example, in a cart-pole system only the pole subsystem is considered ignoring the cart subsystem and it is thus impossible to achieve a good control around the set point (distance=0).

In this study, a decoupled fuzzy controller design is proposed. This controller guarantees some properties, such as the robust performance and stability properties. Further, a class of fourth-order nonlinear systems is investigated. Lo and Kuo [9] proposed a method called "decoupled fuzzy sliding-mode control" to cope with the above issue. However, The sliding factor of hybrid sliding surface is always given by try-and-error. Therefore, using heuristic sliding factors are implemented as functions. It is easy to find sliding factors and the system can achieve asymptotic stability and it will converge faster.

The rest of the paper is divided into five sections. In Section 2, the systems are described. In Section 3, the adaptability complexity hybrid fuzzy sliding-mode control is presented. In Section 4, the proposed controller is used to control a cart-pole system. Finally, we conclude with Section 5.

## 2 System description

Consider a second-order nonlinear system, which can be represented by the following state-space model in a canonical form:

$$\begin{aligned}\dot{x}_1(t) &= x_2(t)\\ \dot{x}_2(t) &= f(\mathbf{x}) + b(\mathbf{x})u + d(t)\\ y(t) &= x_1(t)\end{aligned} \tag{1}$$

where $\mathbf{x} = [x_1\ x_2]^T$ is the state vector, $f(\mathbf{x})$ and $b(\mathbf{x})$ are nonlinear functions, $u$ is the control input, and $d(t)$ is external disturbance. The disturbance is assumed to be bounded as $|d(t)| \le D(t)$ .

For this kind of the second order system, we can use many kinds of control methods, such as, fuzzy control, PID control, sliding mode control…etc. A control law $u$ can be easily designed to make the second order system (1) arrive at our control goal. However, for such nonlinear models as a cart-pole system, the system dynamic representation is generally not in a canonical form exactly. Rather, it has a form shown below:

$$\begin{aligned}\dot{x}_1(t) &= x_2(t)\\ \dot{x}_2(t) &= f_1(\mathbf{x}) + b_1(\mathbf{x})u_1 + d_1(t)\\ \dot{x}_3(t) &= x_4(t)\\ \dot{x}_4(t) &= f_2(\mathbf{x}) + b_2(\mathbf{x})u_2 + d_2(t)\end{aligned} \tag{2}$$

where $\mathbf{x} = [x_1\ x_2\ x_3\ x_4]^T$ is the state vector, $f_1(\mathbf{x})$, $f_2(\mathbf{x})$ and $b_1(\mathbf{x})$, $b_2(\mathbf{x})$ are nonlinear functions, $u_1$, $u_2$ are the control inputs, and $d_1(t)$, $d_2(t)$ are external disturbances. The disturbances are assumed to be bounded as $|d_1(t)| \le D_1(t)$ , $|d_2(t)| \le D_2(t)$ . From (2), one can design $u_1$ and $u_2$ respectively, however, this approach is only utilized to control a subsystem in (2). For example, if the model is a cart-pole system, we only control either the pole or the cart of a system such as (2). Hence, the idea of decoupling is employed to design a control $u$ to govern the whole system.

The switching line is defined by:

$$s : \dot{x} + c_1 x = 0 \tag{3}$$

For the second order system (1), a switching line is chosen as

$$s = c_1 x_1 + x_2 \tag{4}$$

By taking the time derivative of both sides of (4), we can obtain

$$\dot{s} = c_1\dot{x}_1 + \dot{x}_2 = c_1 x_2 + f(\mathbf{x}) + b(\mathbf{x})u + d \tag{5}$$

Then, multiplying both sides of the above equation by gives

$$s\dot{s} = sc_1 x_2 + sf(\mathbf{x}) + sb(\mathbf{x})u + sd \tag{6}$$

Here, we assume that $b(\mathbf{x})>0$. In (5), it is seen that $\dot{s}$ increases as $u$ increases and vice versa. Equation (6) provides the information that if $s>0$, the decreasing $u$ will make $s\dot{s}$ decrease and that if $s<0$, the increasing $u$ will make $s\dot{s}$ decrease.

## 3 Design decoupled fuzzy logic controller

In this section, the idea of the signed distance of fuzzy logic control is used in section 3. For implementation, a triangular type membership function is chosen for the aforementioned fuzzy variables, as shown in Fig. 1. In Eqn. (2), we first define one switching line as

$$s_1 = c_1(x_1 - z) + x_2 \tag{7}$$

and another switching line as

$$s_2 = c_2 x_3 + x_4 \tag{8}$$

The control objective is to drive the system state to the original equilibrium point. The switching line variables $s_1$ and $s_2$ are reduced to zeros gradually at the same time by an intermediate variable $z$, as illustrated in Fig. 2.

In equation (7), $z$ is a value transferred from $s_2$, it has a value proportional to $s_2$ and has the range proper to $x_1$. Equation (7) denotes that the control objective of $u_1$ is changed from $x_1 = 0$, $x_2 = 0$ to $x_1 = z$, $x_2 = 0$. Because the controller $u = u_1$ is used to govern the whole system, the bound of $x_1$ can be guaranteed by letting

$$|z| \le Z_u, \qquad 0 < Z_u < 1 \tag{9}$$

where $Z_u$ is the upper bound of $abs(z)$. Equation (9) implies that the maximum absolute value of $x_1$ will be limited.

Summarizing what we have mentioned above, $z$ can be defined as

$$z = sat(s_2/\Phi_z)\cdot Z_u, \quad 0 < Z_u < 1 \tag{10}$$

where $\Phi_z$ is the boundary layer of $s_2$ to smooth $z$, $\Phi_z$ transfers $s_2$ to the proper range of $x_1$, and the definition of $sat(.\ \ )$ function is

$$sat(\varphi)=\begin{cases}\operatorname{sgn}(\varphi), & if\ |\varphi|\ge 1\\ \varphi, & if\ |\varphi|<1\end{cases} \qquad (11)$$

Notice that $z$ is a decaying oscillation signal because $Z_u$ is a factor less than one.

The sliding factors $c_1$ and $c_2$ are defined as functions of the output $u$ of a fuzzy controller. It is straightforward to find that $c_1$ and $c_2$ can be characterized by the simple functions [10]

$$c_1 = f_1(u)=\begin{cases}\dfrac{\alpha_1}{|u|}, & \text{if } |u|\ge \beta_1\\ \alpha_1/\beta_1, & \text{otherwise}\end{cases} \qquad (12)$$

$$c_2 = f_2(u)=\begin{cases}\dfrac{\alpha_2}{|u|}, & \text{if } |u|\ge \beta_2\\ \alpha_2/\beta_2, & \text{otherwise}\end{cases} \qquad (13)$$

*Remark1*. Consider equation (7). If $s_1 = 0$, then $x_1 = z$, $x_2 = 0$. Since $z$ is a value transferred from $s_2$, when $s_2 \to 0$, then $z \to 0$ and $x_1 \to 0$. From equation (8), if the condition $s_1 \to 0$, the control objective can be achieved.

*Remark2*. The equation (12) and (13). $\alpha_1$ and $\alpha_2$ are constant factors designed to satisfy the stability requirement, and $\beta_1$ and $\beta_2$ are arbitrary positive constant ( $1 \ge \beta_1, \beta_2 > 0$ ) selected to avoid the situation of a zero value of $|u|$ in the denominator.

*Remark3*. The functional scaling factor is generated with heuristic analyses. It is found that the scaling factors can be defined as simple function of the output of the AFMSC, according to the heuristic analyses of system dynamics. Thus, the scaling factor can be adjusted to structure the AFMS without a priori knowledge about the system plant. Also the functional scaling factor proposed here will no trial-and-error.

## 4. Simulation and Experiment of the Seesaw System

In this section, we shall demonstrate that the decoupled SFLC is applicable to the cart-pole system [9] to verify the theoretical development.

The structure of an inverted pendulum is illustrated in Fig.5 and its dynamic is described below:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \frac{m_t g \sin x_1 - m_p L \sin x_1 \cos x_1 x_2^2 + \cos x_1 \cdot u}{L\cdot(\frac{4}{3}m_t - m_p \cos^2 x_1)} + d$$

$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = \frac{\frac{4}{3}m_p L x_2^2 \sin x_1 + m_p g \sin x_1 \cos x_1}{\frac{4}{3}m_t - m_p \cos^2 x_1} + \frac{4}{3\cdot(\frac{4}{3}m_t - m_p\cos^2 x_1)}u + d \qquad (14)$$

where $x_1 = \theta$ the angle of the pole with respect to the vertical axis; $x_2 = \dot{\theta}$ the angle velocity of the pole with respect to the vertical axis; $x_3 = x$ the position of the cart; $x_4 = \dot{x}$ the velocity of the cart; $m_t = m_c + m_p$ .

In what follows, we define the following variables:

$$s_1 = c_1(\theta - z) + \dot{\theta} = c_1(x_1 - z) + x_2 \qquad (15)$$

$$s_2 = c_2 x + \dot{x} = c_2 x_3 + x_4 \qquad (16)$$

and

$$z = sat(s_2/\Phi_z)\cdot Z_u, \quad 0 < Z_u < 1 \qquad (17)$$

In the simulation, the following specifications are used: $m_p = 0.05kg$, $m_c = 1kg$, $L = 0.5m$, $g = 9.8m/s^2$, $\Phi_z = 15$, $Z_u = 0.9425$, $|d| \le 0.0873, \mathrm{K}_1 = 1, \mathrm{K}_2 = 40$ initial values are

$$\theta = -60^\circ,\ \dot{\theta} = 0,\ x = 0,\ \dot{x} = 0$$

and the parameters and are designed as

$$\alpha_1 = \alpha_2 = 1/200,\ \ \beta_1 = \beta_2 = 1$$

Fig. 4 through Fig. 6 shows the simulation result. It is found that the pole and the cart can be stabilized to the equilibrium point.

## 5 Conclusion

Adaptability of fuzzy SMC has been presented. The sliding factor is given without try-and-error in hybrid fuzzy sliding-mode controller. Using the method, heuristic sliding factors are implemented as functions. The response of system will converge faster than that of previous reports. Next, simulation results show that the pole and the cart can be stabilized to the equilibrium. The heuristic sliding factors were implemented as simple functions of the output of the fuzzy controller to avoid increasing the complexity of the systems.

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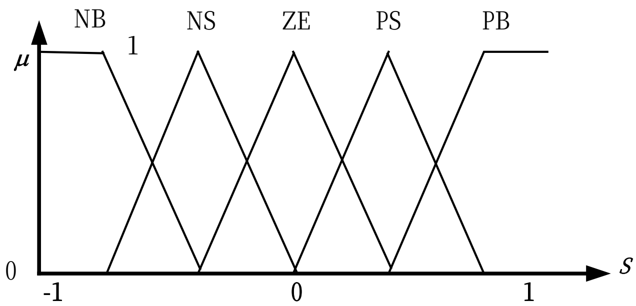

Fig 1. Fuzzy variable of triangular type.

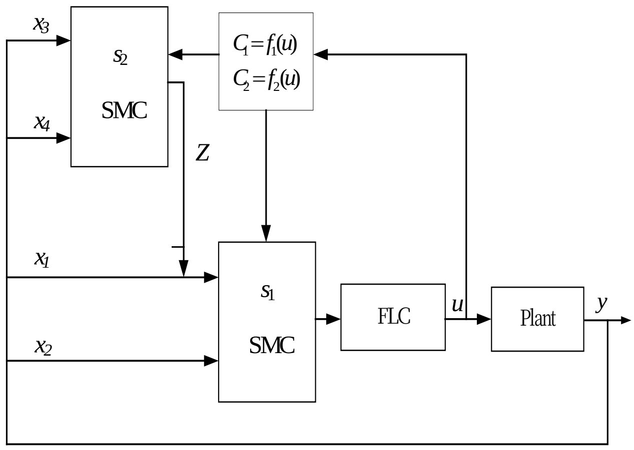

Fig. 2. The block of the hybrid adaptability SFLC.

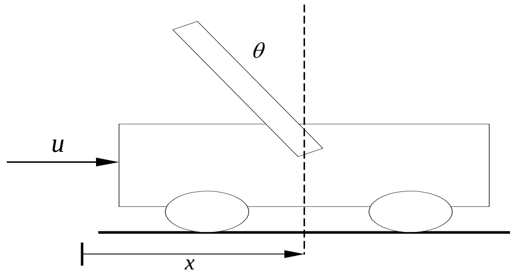

Fig. 3. Structure of an inverted pendulum system.

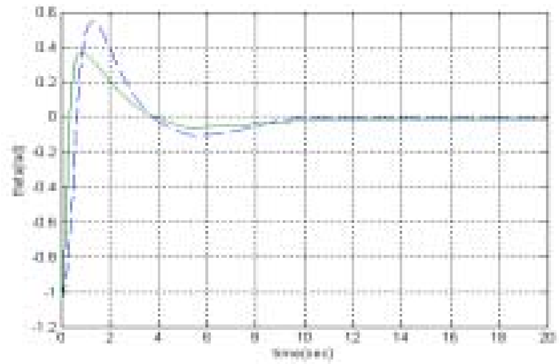
Fig. 4. Angle evolution of the pole.
(the FSMC method [9] ------- ,
the AFSMC method ______ )

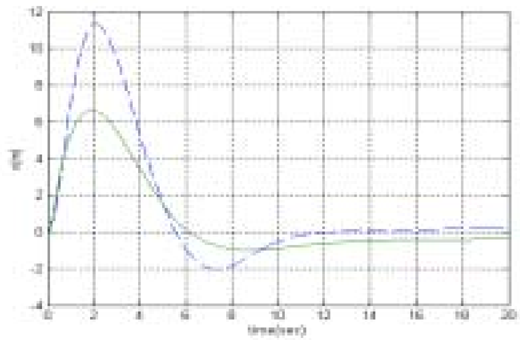
Fig. 5. Position evolution of the cart.
(the FSMC method [9] ------- ,
the AFSMC method ______ )

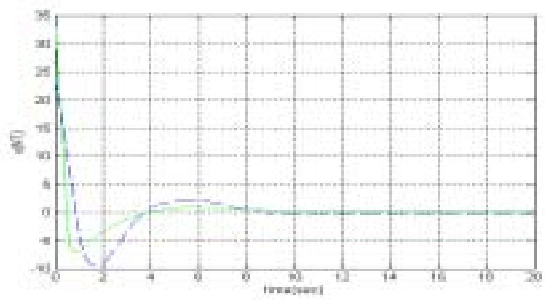
Fig. 6. Control output of an inverted pendulum system.
(the FSMC method [9] ------- ,
the AFSMC method ______ )