# Four Layers Opened Slot Resonator

HUMBERTO C. CHAVES FERNANDES, ÉRICO C. BRAZ AND MANOEL B. L. AQUINO
Department of Electrical Engineering - Technological Center - Federal University of Rio Grande do Norte - UFRN - P.O.Box: 1583 - 59.078-970 Natal, RN, Brazil. Phone: +55 (021) 84 215.3731.

*Abstract* — The analysis and results of rectangular slot resonator with four layers are presented. The full wave Transversal Transmission Line (TTL) method, is used in the analysis to obtain the complex resonant frequency. This complex resonant frequency is calculated through double spectral variables using jointly the moment method. Computational results are obtained for the with and length as functions of the complex frequency for different layer's thickness of the slot resonator with four layers.

*Index Terms* — Transverse Transmission Line (TTL) method, Slot Resonator, complex resonant frequency.

## I. INTRODUCTION

The rectangular slot resonator with four layers substrate without and with loss, with width w and length *l,* is shown in the Fig. 1. For the analysis the full wave TTL method, jointly with the Galerkin's procedure and adequate basis function are used to obtain the concise and general equations of the electromagnetic fields, allowing the calculation of the complex resonant frequency . This complex resonant frequency is calculated through double spectral variables. Computational results are obtained for the complex frequency as functions of the resonator with and length for different layer's thickness of the slot resonator with four layers.

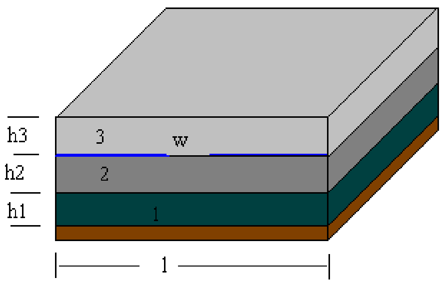


Fig. 1 – Spatial view of the four layes slot resonator where the fourth layer is the air.

## II. THEORY

At the slot resonator using the TTL method the field equations are applied for double Fourier transformed defined as:

$$\tilde{f}(\alpha_n, y, \beta_k) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y,z)\cdot e^{j\alpha_n x}\cdot e^{j\beta_k z}\, dxdz \quad (1)$$

Where $\alpha_n$ is the spectral variable in the "x" direction and β spectral variable in the "z" direction.

After using the Maxwell's equations in the spectral domain, the general equations of the electric and magnetic fields in the method TTL, are obtained as:

$$\tilde{E}_{xi} = \frac{1}{\gamma_i^2 + k_i^2}\left[-j\alpha_n \frac{\partial}{\partial y}\tilde{E}_{yi} + \omega\mu\beta_k \tilde{H}_{yi}\right] \quad (2.1)$$

$$\tilde{E}_{zi} = \frac{1}{\gamma_i^2 + k_i^2}\left[-j\beta_k \frac{\partial}{\partial y}\tilde{E}_{yi} - \omega\mu\alpha_n \tilde{H}_{yi}\right] \quad (2.2)$$

$$\tilde{H}_{xi} = \frac{1}{\gamma_i^2 + k_i^2}\left[-j\alpha_n \frac{\partial}{\partial y}\tilde{H}_{yi} - \omega\varepsilon\beta_k \tilde{E}_{yi}\right] \quad (2.3)$$

$$\tilde{H}_{zi} = \frac{1}{\gamma_i^2 + k_i^2}\left[-j\beta_k \frac{\partial}{\partial y}\tilde{H}_{yi} + \omega\varepsilon\alpha_n \tilde{E}_{yi}\right] \quad (2.4)$$

where:

i = 1, 2, 3, 4 represent four dielectric regions of the structure;

$$\gamma_i^2 = \alpha_n^2 + \beta_k^2 - k_i^2 \quad (2.5)$$

is the propagation constant in "y" direction; $\alpha_n$ is the spectral variable in "x" direction and $\beta_k$ is the spectral variable in "z" direction.

$k_i^2 = \omega^2\mu\varepsilon = k_0^2\varepsilon_{ri}^*$ is the wave number of i[th] dielectric region;

$\varepsilon_{ri}^* = \varepsilon_{ri} - j\dfrac{\sigma_i}{\omega\varepsilon_0}$ is the relative dielectric constant of the material with losses;

$\omega = \omega_r + j\omega_i$ is the complex angular frequency;

$\varepsilon_i = \varepsilon_{ri}^* \cdot \varepsilon_0$ is the dielectric constant of the material;

The equations above are applied to the resonator being the fields $E_y$ and $H_y$ calculated, through the solution of the Helmoltz equations in the spectral domain [2]-[4]:

$$\left(\frac{\partial^2}{\partial y^2} - \gamma^2\right)\tilde{E}_y = 0 \quad (3.1)$$

$$\left(\frac{\partial^2}{\partial y^2}-\gamma^2\right)\tilde{H}_y=0 \qquad (3.2)$$

The solutions of those equations for the four regions of the structure are given as:

For the region 1:

$$\tilde{E}_{y1}=A_{1e}\cdot\cosh\gamma_1 y \qquad (4.1)$$

$$\tilde{H}_{y1}=A_{1h}\cdot sinh\gamma_1 y \qquad (4.2)$$

For the region 2:

$$\tilde{E}_{y2}=A_{2e}\cdot\mathrm{senh}\gamma_2 y+B_{2e}\cdot\cosh\gamma_2 y \quad (4.3)$$

$$\tilde{H}_{y2}=A_{2h}\cdot\mathrm{senh}\gamma_2 y+B_{2h}\cdot\cosh\gamma_2 y \quad (4.4)$$

For the region 3:

$$\tilde{E}_{y3}=A_{3e}\cdot\mathrm{senh}\,\gamma_3 y+B_{3e}\cdot\cosh\gamma_3 y \quad (4.5)$$

$$\tilde{H}_{y3}=A_{3h}\cdot\mathrm{senh}\,\gamma 3y+B_{3h}\cdot\cosh\gamma 3y \quad (4.6)$$

For the region 4:

$$\tilde{E}_{y3}=A_{3e}\cdot e^{-\gamma_3 y} \qquad (4.7)$$

$$\tilde{H}_{y3}=A_{3h}\cdot e^{-\gamma_3 y} \qquad (4.8)$$

Substituting these solutions in the field equations (2.1) the (2.4), as function of the unknown constants $A_{21}$, $A_{22}$, $B_{21}$ and $B_{22}$ are obtained, for example, for the region 2:

$$\tilde{E}_{x2}=\frac{-j}{K_2^2+\gamma_2^2}\begin{bmatrix}(jw\mu_0\beta_k B_{21}+\alpha_n\gamma_2 A_{22})\cosh(\gamma_2 y))+\\(jw\mu_0\beta_k B_{22}+\alpha_n\gamma_2 A_{21})\mathrm{senh}(\gamma_2 y))\end{bmatrix} \qquad (4.9)$$

$$\tilde{H}_{x2}=\frac{-j}{K_2^2+\gamma_2^2}\begin{bmatrix}(jw\varepsilon_2\beta_k A_{21}+\alpha_n\gamma_2 B_{22})\cosh(\gamma_2 y))+\\(jw\varepsilon_2\beta_k A_{22}+\alpha_n\gamma_2 B_{21})\mathrm{senh}(\gamma_2 y))\end{bmatrix} \qquad (4.10)$$

For the determination of the unknown constants, the boundary conditions are applied for the regions 1, 2 and 3, and 4 , as exemple:

For the regions 1 and 2: y = h1

$$\tilde{E}_{x1}=\tilde{E}_{x2} \qquad (5.1)$$

$$\tilde{E}_{z1}=\tilde{E}_{z2} \qquad (5.2)$$

$$\tilde{H}_{x1}=\tilde{H}_{x2} \qquad (5.3)$$

$$\tilde{H}_{z1}=\tilde{H}_{z2} \qquad (5.4)$$

For the regions 2 and 3: y =d ; (g=h1+h2)

$$\tilde{E}_{x2}=\tilde{E}_{x3}=\tilde{E}_{xg} \qquad (5.5)$$

$$\tilde{E}_{z2}=\tilde{E}_{z3}=\tilde{E}_{zg} \qquad (5.6)$$

After several calculations are obtained, for two region :

$$A_{21}=\frac{\varepsilon_1\cosh(\gamma_2 y)}{\varepsilon_2\gamma_1\,\mathrm{senh}(\gamma_1 g_1)\cosh(\gamma_2 g_2)+\gamma_2\frac{\varepsilon_1}{\varepsilon_2}\cosh(\gamma_1 g_1)\mathrm{senh}(\gamma_2 g_2)}*\left[j(\alpha_n\tilde{E}_{xg}+\beta_k\tilde{E}_{zg}\right] \qquad (6.1)$$

$$A_{22}=\frac{\gamma_1\,\mathrm{senh}(\gamma_2 y)}{\gamma_2\gamma_1\,\mathrm{senh}(\gamma_1 g_1)\cosh(\gamma_2 g_2)+\gamma_2\frac{\varepsilon_1}{\varepsilon_2}\cosh(\gamma_1 g_1)\mathrm{senh}(\gamma_2 g_2)}*\left[j(\alpha_n\tilde{E}_{xg}+\beta_k\tilde{E}_{zg}\right] \qquad (6.2)$$

$$B_{21}=-\frac{\mathrm{senh}(\gamma_1 g_1)}{\omega\mu_0\,\mathrm{senh}(\gamma_1 g_1)\cosh(\gamma_2 g_2)+\frac{\gamma_1}{\gamma_2}\cosh(\gamma_1 g_1)\mathrm{senh}(\gamma_2 g_2)}*\left[-\beta_k\tilde{E}_{xg}+\alpha_n\tilde{E}_{zg}\right] \qquad (6.3)$$

$$B_{22}=-\frac{\gamma_1}{\gamma_2}\frac{\cosh(\gamma_1 g_1)}{\omega\mu_0\,\mathrm{senh}(\gamma_1 g_1)\cosh(\gamma_2 g_2)+\frac{\gamma_1}{\gamma_2}\cosh(\gamma_1 g_1)\mathrm{senh}(\gamma_2 g_2)}*\left[-\beta_k\tilde{E}_{xg}+\alpha_n\tilde{E}_{zg}\right] \qquad (6.4)$$

The general equations of the electromagnetic fields are then obtained.

The following equations (7.1) and (7.2) relate the current densities in the sheets ( $\tilde{J}xt$ and $\tilde{J}zt$ ) and the magnetic fields in the interface y = h1+h2:

$$\tilde{H}_{x2}-\tilde{H}_{x3}=\tilde{J}_{zt} \qquad (7.1)$$

$$\tilde{H}_{z2}-\tilde{H}_{z3}=-\tilde{J}_{xt} \qquad (7.2)$$

With the substitutions of the magnetic field equations, and after some calculations are obtained,

$$Y_{xx}\tilde{E}_{xg}+Y_{xz}\tilde{E}_{zg}=\tilde{J}_{zg} \qquad (8.1)$$

$$Y_{zx}\tilde{E}_{xg}+Y_{zz}\tilde{E}_{zg}=\tilde{J}_{xg} \qquad (8.2)$$

that in matricial form:

$$\begin{bmatrix}Y_{xx} & Y_{xz}\\ Y_{zx} & Y_{zz}\end{bmatrix}\begin{bmatrix}\tilde{E}_{xg}\\ \tilde{E}_{zg}\end{bmatrix}=\begin{bmatrix}\tilde{J}_{zg}\\ \tilde{J}_{xg}\end{bmatrix} \qquad (9)$$

The " Y " admittance functions are the dyadic Green functions of the antenna slot resonator and they are given as:

$$Y_{xx}=-\frac{j}{\varpi\mu_0(\gamma_2^2+k_2^2)}\left[-\beta_k^2\gamma_2E+k_2^2\alpha_n^2.F\right]+\frac{j}{\varpi\mu_0\gamma_3}\left[\alpha_n^2k_3^2E-\beta_k^2\gamma_3^2.D\right] \quad (9.1)$$

$$Y_{xz}=\frac{-j\alpha n\beta k}{\varpi\mu_0\left(\gamma_2^2+k_2^2\right)}\left[A+k_2^2.(B)\right]-\frac{\alpha n\beta k}{\varpi\mu_0\gamma_3\left(k_3^2+\gamma_3^2\right)}\left[k_3^2.C+\gamma_3^2.D\right] \quad (9.2)$$

$$Y_{zx}=\frac{-j\alpha n\beta k}{\varpi\mu_0\left(\gamma_2^2+k_2^2\right)}\left[A+k_2^2.(B)\right]-\frac{\alpha n\beta k}{\varpi\mu_0\gamma_3\left(k_3^2+\gamma_3^2\right)}\left[k_3^2.C+\gamma_3^2.D\right] \quad (9.3)$$

$$Y_{zz}=\frac{j}{\varpi\mu_0\left(\gamma_2^2+k_2^2\right)}\left[\alpha n^2.A-\beta k^2.k_2^2.\mathbf{B}\right]-\frac{j}{\varpi\mu_0\gamma_3\left(k_3^2+\gamma_3^2\right)}\left[\alpha n^2\gamma_3^2.C-\beta k^2\gamma_3^2.D\right] \quad (9.4)$$

$$A=\frac{\gamma_1\cdot\gamma_2}{\gamma_2 tgh(\gamma_1\cdot h_1)+\gamma_1 tgh(\gamma_2 h_2)}+\frac{\gamma_2^2}{\dfrac{\gamma_2}{tgh(\gamma_2\cdot h_2)}+\dfrac{\gamma_1}{tgh(\gamma_1\cdot h_1)}} \quad (9.4.1)$$

$$B=\left(\frac{\varepsilon_1}{\gamma_1\cdot\varepsilon_2 tgh(\gamma_1\cdot h_1)+\gamma_2\cdot\varepsilon_1 tgh(\gamma_2\cdot h_2)}+\frac{\gamma_1\cdot\varepsilon_2}{\dfrac{\gamma_1.\gamma_2.\varepsilon_2}{tgh(\gamma_2.h_2)}+\dfrac{\gamma_2^2.\varepsilon_1}{tgh(\gamma_1.h_1)}}\right) \quad (9.4.2)$$

$$C=\left(\frac{\dfrac{\gamma_3.\varepsilon_4}{\gamma_4\varepsilon_3}+tgh(\gamma_3h_3)}{1+\dfrac{\gamma_3\varepsilon_4.tgh(\gamma_3h_3)}{\gamma_4\varepsilon_3}}\right) \quad (9.4.3)$$

$$D=\left(\frac{\dfrac{\gamma_4}{\gamma_3}+tgh(\gamma_3h_3)}{1+\dfrac{\gamma_4.tgh(\gamma_3h_3)}{\gamma_3}}\right) \quad (9.4.4)$$

$$E=\left(\frac{\gamma_1+\gamma_2.tgh(\gamma_1h_1).tgh(\gamma_2h_2)}{\gamma_2 tgh(\gamma_1h_1)+\gamma_2.tgh(\gamma_2h_2)}\right) \quad (9.4.5)$$

The electric fields in the interface, are expanded in terms of known base functions through as [3],[5]:

$$\tilde{E}_{xg}=\sum_{i=1}^{n}a_{xi}\cdot\tilde{f}_{xi}(\alpha_n,\beta_k) \quad (10.1)$$

$$\tilde{E}_{zg}=\sum_{j=1}^{m}a_{zj}\cdot\tilde{f}_{zj}(\alpha_n,\beta_k) \quad (10.2)$$

where $a_{xi}$ and $a_{zj}$ are unknown constant and the n and m terms are integer and positive numbers that can be done equal to 1, as in the equations (11) and (12) following:

$$\tilde{E}_{xg}=a_x\cdot\tilde{f}_x(\alpha_n,\beta_k) \quad (11.1)$$

$$\tilde{E}_{zg}=a_z\cdot\tilde{f}_z(\alpha_n,\beta_k) \quad (11.2)$$

Were chosen base functions in the space domain as:

$$f_x(x,z)=f_x(x).f_x(z) \quad (12.1)$$

$$f_x(x)=\frac{1}{\sqrt{\left(\frac{w}{2}\right)^2-x^2}} \quad (12.2)$$

$$f_x(z)=\cos\left(\frac{\pi z}{l}\right) \quad (12.3)$$

whose transformed of Fourier are:

$$\tilde{f}_x(\alpha_n)=\pi\cdot J_o\left(\alpha_n\frac{w}{2}\right) \quad (13.1)$$

$$\tilde{f}_x(\beta_k)=\frac{2\pi l\cdot\cos\left(\frac{\beta_k l}{2}\right)}{\pi^2-(\beta_k l)^2} \quad (13.2)$$

$$\tilde{f}_x(\alpha_n,\beta_k)=\frac{2\pi^2 l\cdot\cos\left(\frac{\beta_k l}{2}\right)}{\pi^2-(\beta_k l)^2}\cdot J_o\left(\alpha_n\frac{w}{2}\right) \quad (13.3)$$

where $J_0$ is the function of Bessel of first species and order zero.

The Gallerkin method is applied to eq. (9), the eliminated current densities and the new equation in matrix's form are obtained [5], [7].

$$\begin{bmatrix}K_{xx} & K_{xz}\\ K_{zx} & K_{zz}\end{bmatrix}.\begin{bmatrix}a_x\\ a_z\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix} \quad (14.1)$$

where,

$$K_{xx}=\sum_{-\infty}^{\infty}\tilde{f}_x\cdot Y_{xx}\cdot\tilde{f}_x^{*} \quad (14.2)$$

$$K_{xz}=\sum_{-\infty}^{\infty}\tilde{f}_z\cdot Y_{xz}\cdot\tilde{f}_x^{*} \quad (14.3)$$

$$K_{zx}=\sum_{-\infty}^{\infty}\tilde{f}_x\cdot Y_{zx}\cdot\tilde{f}_z^{*} \quad (14.4)$$

$$K_{zz}=\sum_{-\infty}^{\infty}\tilde{f}_z\cdot Y_{zz}\cdot\tilde{f}_z^{*} \quad (14.5)$$

The solution of the characteristic equation of the determinant of (14) supplies the complex resonant frequency [8]-[10].

## III. RESULTS

The Fig.2 shows the resonant frequency as function of length slot for different substrate thickness at layer 1.

The Fig. 3 shows the resonant frequency as function of width slot for different substrate thickness at layer 3.

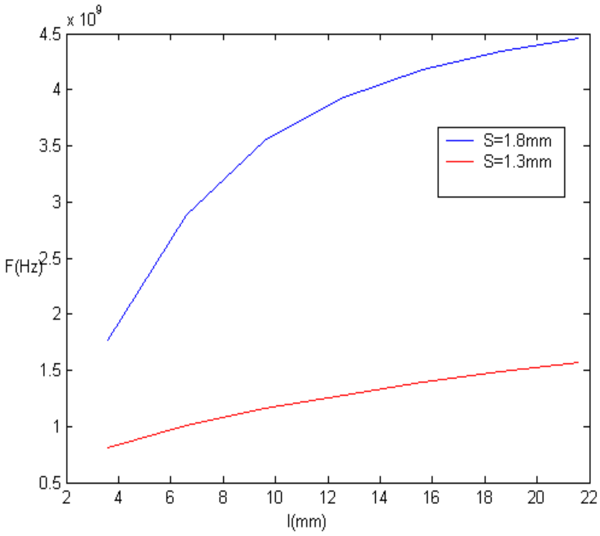

Fig. 2 Resonant frequency as function of length slot for a slot resonator with four layers.

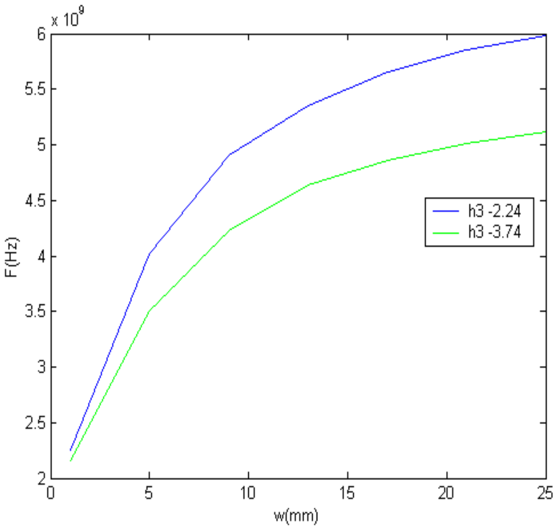

Fig. 3 Resonant frequency as function of width slot for a slot resonator with four layers.

## IV. CONCLUSION

The full wave method of the Transverse Transmission Line - TTL, was used for obtaining the numeric results of the planar slot resonator with four layer's substrate. According this concise and effective procedure the complex resonant frequency was obtained. The possibilities of the change different materials are the greater advantage of multiple layers slot resonator.

This work receive financial support from CNPQ.

## VI. REFERENCES

[1] A. K. Agrawal and B. Bhat, "Resonant Characteristics and End Effects of a Slot Resonator in Unilateral Fin Line", Proc. IEEE, Vol. 72, pp. 1416-1418, Oct. 1984.

[2] Humberto C. C. Fernandes, Sidney A. P Silva and José P. Silva, "Coupling Analisys at the Coupler and Unilateral Edge-Coupled Fin Line", International Conference on Millimeter And Submillimeter Waves and Applications II, SPIE's 1998 International Symposium on Optical Science, Engineering and Instrumentation, San Diego, CA, USA. Conf. Proc. pp. 53-54, July 1998.

[3] H. C. C. Fernandes and C. B. Fonseca "Phased Array with photonic material", International Microwave and Optoelectronics Conference- IMOC 2003, vol II, pp 737-740, foz do iguaçu-Brazil, Sep. 2003.

[4] H. C. C. Fernandes, S.A.P. Silva and O. S. D. Costa, "3D Complex Propagation of Coupled Unilateral and Antipodal Arbitrary Finlines", CBMAG'96- Brazilian Electromagnetism Congress, Ouro Preto-MG, pp. 159-162 , Nov. 1996.

[5] B. Baht and S. K. Koul, "Analysis, design and applications of finlines", Artech House, 1987.

[6] Sidney A. P. Silva and H. C.C. Fernandes, " Functions of basis in analisys of the acoupled unilateral fin line coupler ", , IV SPET-symposium of Research and Extension in Technology, Natal-RN, Annals pp. 79-81, Nov. 1998.

[7] H. C. C. Fernandes, "Planar structures general in guides of waves Millimeters: finlines ", Thesis of Doctorate, FEC, UNICAMP, 189p., Campinas-SP, july de 1984.

[8] H. C. C. Fernandes, S.A.P. Silva and L. C. Freitas Júnior, "Complex Propagation Results in 3-D of the Generic Arbitrary Bilateral Finlines", 22th International Conference on Infrared and Millimeter Waves, Wintergreen, VA, USA, Conf. Proc. pp.340-341, 20-25 de Jul. 1997.

[9] H. C. C. Fernandes and C. B. Fonseca "Phased Antenna Array by Dynamic Analysis", International Microwave and Optoelectronics Conference, Belém, Brazil, Aug. 2001.

[10] H. C. C. Fernandes, G. O. Batista, J. E. C. Neto, "Accurate Analysis of planar Microstrip Antenna Array", International Journal of Infrared and Milimeters Waves, vol 24, pp 1159-1169, july 2003.