Echoplex Error Control System using Avalanche Transformations

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Abstract: - In this paper a new way of increasing the reliability of echoplex error detection by using avalanche Boolean transformations has been proposed. In echoplex scheme such transformations are used as error amplifier and decrease the probability of undetected errors. The analysis of efficiency of avalanche Boolean transformations utilization in echoplex error detection scheme is presented in this paper. It is shown that avalanche Boolean transforms can be realized effectively by Field Programmable Gate Array (FPGA) devices**.** In such a case it is proved that the utilization of the avalanche Boolean transforms in echoplex scheme does not worsen the time parameters of a duplex channel.

Key-Words: - Echoplex, error detection, SAC function, Control System error detection

1 Introduction

Echoplex is a widely used method of error detection in duplex channels. Utilization of this error detection method is foreseen by a number of data transmission protocols, in particular by the most widely used standart ITU-T ISDN [5].

Echoplex error control in essence consists of the following sequence of actions. The transmitter sends to the communication channel the n-bits code D_s and stores it in the memory. Receiver gets from the channel output and sends it back to the transmitter before it receives the next n bit code D_{s+1}. The transmiter gets echo code D_e (D_e is the echo code of D_r code) from channel output and compares it with the originally transmited D_s. If the stored code D_s and echo code D_e are equal $D_s = D_e$ then it is considered that the transfer is executed without errors. The situation when $D_s \neq D_e$, is classified as error code transmission D_s.

Echoplex utilization is most effective in the duplex channels. In comparison with other methods, which utilize the error detection in the block data transmission (checksum and CRC - Cyclic Redundancy Check), echoplex has some important advantages. In particular,

- ❖ Echoplex ensures a quick reaction of the data-transmission system in case an error appears. In addition the echoplex allows error control in real time systems, so it is effective for error detection during data transmission in automation control systems;
- ❖ Echoplex is using more effectively the data transmission channel when the appearance intensity of the errors is high since the volume of the repeatedly transferred information is substantially less in comparison with other methods, which utilize the error detection in the block data transmission;
- ❖ Echoplex ensures significantly high error detection probability, especially when the volume of the data is high, because the check is done in one cycle of control (n- bit code) and it is substantially less in comparison with other methods, which detect error in large length data blocks;

At the same time, echoplex has some disadvantages, most essential of which are:

- the possibility of the false error detection if the error appears at the reverse transfer of the echo-code;
- error in forward transmission is not detected, if at the reverse transmission the error occurs at the same bit position (error masking interaction).

In current practice situations where the data transmission rate is increased an important problem is the development of a method to increase the error detection reliability, including the echoplex method.

2 Analysis of echoplex error detection reliability model

For the analysis of echoplex error detection reliablility a binomial model of the errors appearance during data transmission in duplex channel is used [4]. According to binomial model the probability of error appearance for any bit of the transmitted code is equal to p. Respectively, the probability of appearance of k- multiple errors during n-bit code transmission is determined by the formula:

$$P_k = \binom{n}{k} \cdot p^k \cdot (1-p)^{n-k} \quad (1)$$

In practice the probability, p, of error appearance during the transfer of one bit is very small. The length of the transmited code is also small (in most cases n=8). Therefore, during reliability analysis we can suppose that only a single error appears during transfering of multiple n-bit codes. Apart from this, in case of multiple error appearance during forward transimission the probability that these errors will be masked by the same errors which appear during reverse transmission is limited to zero.

When the forward transmission is executed correctly, i.e., $D_r=D_s$, but an error appears during reverse transmission of the echo code D_r, D_e, then the D_s code stored in the trasmiter is different than the echo code $D_e \neq D_r$. In this case there is false error detection.

The probability P_f of the false error detection is determined by the product of the correct forward transmission probability with the probability of single error occurence during reverse n-bit code transmission.

$$P_f = (1-p)^n \cdot (1-(1-p)^n) \quad (2)$$

In case of a false error detection the transmitter repeats the transmission of the erroneous code. Such echoplex false error detection does not influence the error detection reliability, but it reduces the rate of data transmission [3]. The problem of error masking interaction is significantly more important.

The single error which appears in forward transmission will not be detected by echoplex, if it is masked by a single error which appears during reverse data transmission in the same bit position. The probability P_e of such situation is determined by the product of the probability of the error appearance during forward transmission of any of the n bits position of the code with the probability of the error appearance in a fixed bit possition during reverse transmission.

$$P_e = \binom{n}{1} \cdot p \cdot (1-p)^{n-1} \cdot p \cdot (1-p)^{n-1} = \\ = n \cdot p^2 \cdot (1-p)^{2\cdot(n-1)} \quad (3)$$

In nowadays the sphere of computer network utilization is constantly enlarging. There are many computer network applications which demand very high level of data transmission rebiability. Apart from this, there is a tendency to increase the data transmission rate which decreases the data transmission reliability. Thus, increasing the echoplex error detection reliability is a very actual problem.

3 Utilization of Avalanche tranformation in echoplex control scheme

The effectiveness of echoplex error detection in duplex channels can be even higher due to the increase of the error detection probability. To decrease the error masking interaction probability we propose to use a functional single error amplifier in echoplex scheme. Such an error amplifier can be realized as a system of orthogonal Boolean functions which satisfy Strict Avalanche Criterion (SAC) [1,2,6].

A Boolean function $f(x_1,\ldots,x_n)$ defined on a set Z with binary elements consisting of all possible 2^n n-tuples of n variables, satisfies the Strict Avalanche Criterion (SAC), if there is a 50% probability that the complement of a single incoming n-tuple data bit affects the output of the Boolean function by 50%, as follows:

$$\forall j \in \{1,\ldots,n\}: \\ \sum_{x_1,\ldots,x_n \in Z} (f(x_1,\ldots,x_j,\ldots,x_n) \oplus f(x_1,\ldots,\overline{x_j},\ldots,x_n)) = 2^{n-1} \quad (4)$$

Usually these type of functions are being used in cryptographical algorithms and their design methods have been developed in [1,2,6].

The system of Boolean functions $f_1(x_1,\ldots,x_n)$, $f_2(x_1,\ldots,x_n)$, …, $f_n(x_1,\ldots,x_n)$ is orthogonal if the linear combination (XORed) of any subset of them is balanced:

$$\forall \lambda_j \in \{0,1\}, j \in \{1,\ldots,n\}, \sum_{j=1}^{n} \lambda_j \neq 0: \\ \sum_{x_1,\ldots x_n \in Z} \bigoplus_{j=1}^{n} \lambda_j \cdot f_j(x_1,\ldots,x_n) = 2^{n-1} \quad (5)$$

If one of the n inputs of avalanche tranformations is changed then half of its outputs will be changed. This means, that there is an "avalanche amplifier" which by changing one incoming of the n-tuple data bit tranforms half of the outputs. Because every function of this system satisfies the Avalanche Criterion, these tranformations are called "avalanche".

Let's denote with F(D), the Boolean orthogonal avalanche transformation on the n-bits code D. The length of the transformed code R=F(D) is n bits long, too. The orthogonality of the F(D) tranformation indicates the one-to-one correspondence of codes D and R. The avalanche properties of the F(D) tranformation indicates that if one bit of the input code D is changed then on the average, n/2 bits of the output code R=F(D) will be changed also.

The proposed approach to increase the echoplex error detection reliability foresees the use of the following avalanche transformation. The transmitter executes the avalanche transformation $F(D_s)$ on code D_s which is sent to the channel. The transformed code $F(D_s)$ of the tranformation is stored in the transmiter's memory. The receiver, gets the code D_r from the channel output, it executes the avalanche tranformation on the code D_r and the result $F(D_r)$ is sent back to the echo channel. The transmitter receives the echo code R_e from echo the channel and compares it with the code $F(D_s)$ stored in memory. If the above codes are equal, i.e. $R_e= F(D_s)$, then the transmission is considered to be executed without errors. Otherwise the transfer is classified as erroneous.

The proposed scheme of Avalanche transformation utilization in echoplex error detection is shown in Fig. 1.

The Boolean avalanche transformations in the proposed echoplex scheme are being used as "single error amplifier" and their utilization ensures increasing of the Hamming distance between the correct and erroneous echo-codes. Correspondently, the probabilities of masking error appearance during forward transmission by an error appearing during reverse transmission are decreased.

Thus, from the theoretical point of view, the source of increasing the echoplex error detection reliability optimises the coding control information under the condition that a single error is predominant. The optimisation of this coding control information has been achieved by the utilization of the Avalanche transformations which "amplify" single error [4].

When code D_s is correctly transmitted in the forward direction then code D_r received at the channel output is identical to D_s : $D_r = D_s$. Respectively, code $F(D_r)$, which is obtained as a result of avalanche transformation on D_r will be identical to code $F(D_s)$ which is stored in the transmitter's memory: $F(D_r)=F(D_s)$.

If during reverse transmission of $F(D_r)$ no error appears then the transmitter gets from the channel output the echo-code R_e which is idential to $F(D_r)$ i.e., $R_e=F(D_r)$. In this case the code R_e will be identical to the code $F(D_s)$ which is stored in the transmitter's memory: $R_e=F(D_s)$.

If a single error appears during transmission of the code D_s in the forward direction then the receiver gets from the channel output the code D_r , which differs from D_s in one bit. Since the functional transformation performed by the receiver on code D_r has the avalanche properties then the result $F(D_r)$ of this transformation will be different from the code $F(D_s)$ on the average in n/2 bits.

If no error appears during the reverse transmission of $F(D_r)$, then the transmitter gets from the channel output the echo-code R_e which is identical to $F(D_r)$ i.e., $R_e=F(D_r)$. In this case the code R_e will be different from the code $F(D_s)$ which is stored in the transmitter's memory in n/2 bits: $R_e \neq F(D_s)$. This case is classified as erroneous code transmission.

If a single error appears during the reverse transmission of $F(D_r)$, then the transmitter gets from the echo-channel the output echo-code R_e which is different from $F(D_r)$ in one bit. Obviously in this case the code R_e will be different from the code $F(D_s)$ which is stored in the transmitter's memory in (n/2)-1 or (n/2)+1 bits. It is clear that $R_e \neq F(D_s)$. This case is classified as erroneous code transmission.

Thus, the proposed approach ensures the increase of the echoplex error detection reliability by optimizing the codings of $F(D_s)$ and $F(D_r)$ which are used for error detection.

4 Effectiveness analysis of using Avalanche Transformations for increasing echoplex error detection reliability

Here the important aspect of the effectiveness of the proposed approach is obtain and the analysis of the error detection probability estimation in echoplex scheme which is using the avalanche transformation. Theoretical and experimental studies have been performed for obtaning such estimations.

According to the above mentioned properties of Strict Avalanche Transformations, if one from its n bit inputs changes then about n/2 of its outputs will change also. However, there is a nonzero probability that a little more or less than n/2 of the output bits will change.

If a single bit of the Avalanche Transformations on the input code is changed then the probability that every Boolean SAC-function of the transformation will change its value is 0.5. Since the system of Boolean functions is orthogonal then every Boolean function will change its own value independent to each other. Therefore, the change of the output value of the Boolean functions of the Avalanche Transformations can be described by the probabilistic binomial model. According to this model, the probabilities P_h that h outputs of the

Avalanche transformations will change by changing only one input can be determined by the following formula:

$$P_h = \frac{\binom{n}{h}}{2^n}, \forall h \in \{1,..., n\} \quad (6)$$

The single error which appears during the transmission of the code D_s in the forward direction will not be detected in the proposed echoplex scheme if the codes $F(D_s)$ and $F(D_r)$ are diffetent in h bits and exactly h errors appear during the reverse transmission of the code D_r and these errors are located in the same bit positions, where codes $F(D_s)$ and $F(D_r)$ are diffetent.

If we suppose that during the transmission of one code only one error appears, then h=1. Thus, the probability P_e^f that the single error which appears during the forward transmission will not be detected by the proposed echoplex scheme can be represented as the multiplication of the probability of a single error appearance in any of the n bits during the forward transmission ($n \cdot p \cdot (1-p)^{n-1}$) with the probability that $F(D_s)$ and $F(D_r)$ are different in one bit ($n/2^n$) with the probability of a single error appearance in a fixed bit position during the reverse transmission ($p \cdot (1-p)^{n-1}$):

$$P_e^f = \frac{n^2 \cdot p^2 \cdot (1-p)^{2\cdot(n-1)}}{2^n} \quad (7)$$

The comparison between the obtained expression (7) and the formula (3) for the probability of an undetected single error appearance by the traditional echoplex method does show that the utilization of the proposed approach makes it possible to increase the echoplex error detection reliability to q_0 times. The numerical value q_0 is determined by the following expression:

$$q_0 = \frac{P_e}{P_e^f} \approx \frac{2^n}{n} \quad (8)$$

For example, for the most widely used in practice code length n=8, the utilization of the proposed scheme of error detection using the Boolean Avalanche transformations ensures the increase of the echoplex error detection reliability to 32 times. When the code length equals to 16 the probability of the undetected error appearance is decreased by 4096 times.

The results of the statistical simulation of the proposed echoplex error detection scheme used in duplex channels are near to the previously presented theoretical results. Thus, the presented analysis shows that the proposed scheme of the used Boolean avalanche transformations in the echoplex method ensures significant increase error detection reliability for the echoplex.

5 Analysis of implementation variants of echoplex

The important aspect of the effectiveness of execution time of the proposed error detection scheme using the avalanche transformations on the data transmission rate is the analysed and presended below.

In order to implement in practice the proposed error detection scheme it is necessary to previously design Boolean avalanche transformations. This can be made by using known formalized methods of the Boolean SAC-functions synthesis [1,2,6].

The practical implementation of the proposed approach requires to design the Boolean avalanche transformations scheme in the transmitter and receiver component of the telecomunication system. The Boolean avalanche transformations can be implemented by software or hardware. From the influence on data transmission rate point of view the hardware avalanche transformation using FPGA-implementation is the most effective.

The analysis shows that using one of the existing Altera FPGA-devices ensures the implementation of orhogonal system of SAC-functions when their number is less or equal than 32. The calculation of the Avalanche transformations Boolean SAC-functions is executed in parallel.

In case of FPGA-implementation the execution time of avalanche transformation does not exceed tens nanoseconds and therefore does not affect the data transmission rate for the majority of real duplex channels.

In case of implementing the avalanche transformations by software when the length of transmitted codes is small (n=8) the calculation of the Boolean SAC-functions system can be realized by way of a table. The truth tables of the SAC-functions may be stored in the transmitter/receiver EPROM or loaded to the transmitter/receiver RAM. If n=8 then the truth tables of the SAC-functions occupy storage capacity which is equal to 256 bytes. In case of the avalanche transformation implemented by software the execution time is around to tens of nanosecons and less than the data transmission rate for the majority of duplex channels.

Thus, the avalanche transformation implementetion by hardware or software in the prorosed scheme error detection by the echoplex method does not affect the time characteristics of the data transmission channel. Therefore, the proposed method for error detection can be used within the framework of the existing standards of echopex control data transmission, such as ISDN, e.t.c.

6 Conclusion

The results of the presented studies which are directed toward an increase of the echoplex error detection reliability for duplex data transmission channels show that the utilization of the avalanche transformstions is a very effective way of solving this problem. In order to realize the avalanche tranformations it is proposed to use an orhogonal system of Boolean SAC-functions. An echoplex error detection scheme which utilizes avalanche transformations has been developed. In the proposed echoplex error detection scheme the avalanche transformations amplify the single error in such a way that they prevent the masking of the forward transmission error by an echo-transmission error.

Using theoretical and exsperemental studies it has been proved that the utilization of the proposed echoplex error detection scheme for n-bit length transmitted code ensures the increase of the error detection reliability by $\frac{2^n}{n}$ times.

Analysis of avalanche transformation implementation by hardware and software has also been shown that the utilization of the proposed approach to increase the echoplex error detection reliability practically does not affect to data trasmission rate.

References:

[1] Bardis N., Bardis E., Markovskyy A., Spyropoulos A., Design of Boolean Function from a Great Number of Variables Satisfying Strict Avalanche Criterion. *Recent Advances in Signal Processing and Communications - IMACS/IEEE,* ISBN: 960-8052-03-3, pp. 107-112, 1999.

[2] Bardis N.G, Bardis E.G., Markovskaja N.A., Polymenopoulos A., Design and Implementation of Boolean Balanced Functions Satisfying Strict Avalanche Criterion (SAC)", *WSES Press – Problem in Applied Mathematics and Computational Intelligence,* ISBN: 960-8052-30-0, 2001, pp. 12-16.

[3] Bhargava Amit B., Kurose James, Towsley Don, Vanleemput Guy Performance comparison of error control schemes in high-speed computer communication networks // *IEEE J.Select. Aras Commun.*-1988.-Vol.6,-№ 9, - P.1565-1575.

[4] Fu F.W., Klove T., Wei V.K. On the Undetected Error Probability for Binary Codes. // *IEEE Transaction on Information Theory*, Vol. 49, No. 2, 2003, pp.382-391.

[5] Klove T., Korzhik V. Error Detecting Codes: General Theory and Their Application in Feedback Communication Systems. *Norwell,* MA: Kluwer, 1995.- 433 p.

[6] M.Mitrouli, A.P.Markovskyy, Method for design of balanced Boolean functions satisfying strict avalanche criterion (SAC), *WSES Press, Recent Advances in Communications and Computer Science*, ISBN: 960-8052-86-6, 2003, pp.148-154

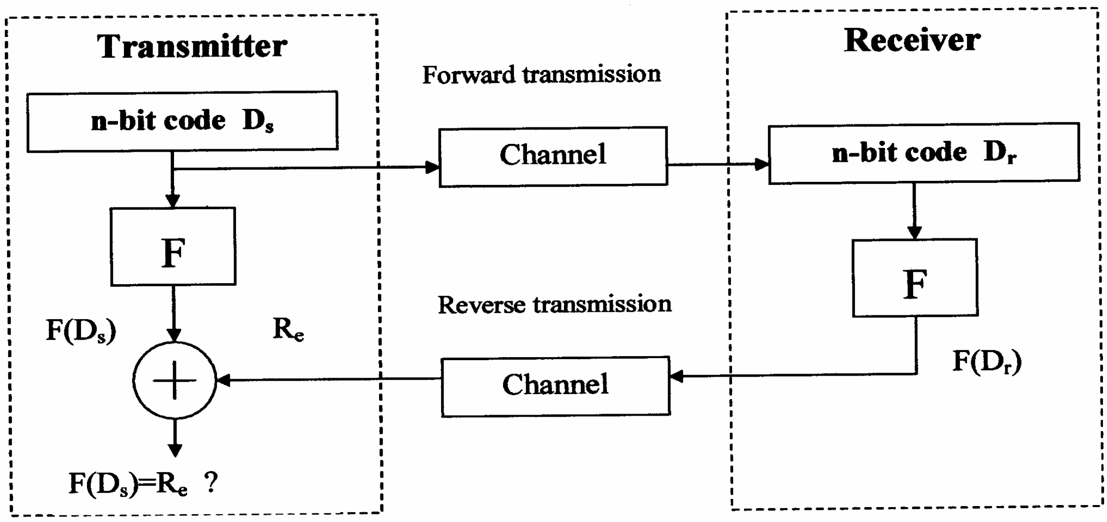

Fig. 1.