Adaptive Fuzzy-Sliding Mode Controller Design for Uncertain Time-Delayed Systems with Nonlinear Input

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Abstract: - In this paper, the principle of sliding mode control is used as a basis to develop an adaptive fuzzy controller for uncertain dynamic time-delayed systems with series nonlinearities. The control method provides a simple way to achieve asymptotic stability of the uncertain time-delayed system. Other attractive features of the method include a minimal realization of the adaptive fuzzy controller and insensitivity to the uncertainties and disturbances. In addition, the method is capable of handling the chattering problem inherent to the sliding mode control easily and effectively. Simulation results are presented to demonstrate these features of the method.

Key-Words: - adaptive fuzzy control, sliding mode control, series nonlinearity, time-delayed system

1 Introduction
In recent years, fuzzy control has made considerable achievements in theory and applications. Fuzzy logic provides a novel methodology which differs from conventional modeling methods. It provides an effective way for qualitative knowledge to be quantitatively denoted. But for a mathematical foundation of fuzzy control, fuzzy set theory itself has yet to be perfected. Combining theory with practice has yet to be deeply explored. At present, one of the major developments of fuzzy control is from past single science research to multiscience hybrid research. There is no doubt that a new way is provided for fuzzy control to be deeply studied. There are many results [3-5] in the aspect. Fuzzy control is essentially nonlinear control. In fact, fuzzy systems are universal approximators [3]. Usually, fuzzy control can be applied to ill-defined and complex processes [4]. Most work of robust control has concentrated on the systems with uncertainties and/or external disturbances. From the viewpoint of design, the uncertain system with linear input is more preferred than that with nonlinear input owing to simplicity. The assumption of linear input is that the system model is indeed linearizable. However, in practice, due to physical limitation, there do exist nonlinearities in the control input and their effect can not be ignored in realization. In addition, to obtain a more practical system model, the information of delay time should not be discarded.

In this study, based on an uncertain time-delayed system with nonlinear input, a robust controller is derived through variable structure control (VSC) [2]. Although, the method of variable structure control can solve the above-mentioned system, the drawback of the variable structure control is their chattering owing to the sliding control law that has to be discontinuous across the sliding surface. Chattering is undesirable because it involves high control activity and may excite high-frequency dynamics. So, we apply the adaptive fuzzy control principle to overcome the drawback. Therefore, the combination of the two control principles, which is called adaptive fuzzy sliding mode control (FSMC).

The FSMC method is applied in the nonlinear system such as the inverted pendulum system [4] and tracking control of a nonlinear system [5]. In [13] employed fuzzy control to substitute the boundary layer term of a sliding mode in order to improve the chattering behavior of a sliding mode control. In [14] employed a switch function as the fuzzy input variables and proposed a fuzzy sliding mode controller.

Although the FSMC method can be used to control above-mentioned systems (the inverted pendulum system, tracking control system) very well, those are SISO systems with a linear input not a nonlinear input which can not be ignored in reality and depend on a certain model for a sliding mode to operate. The number of rules was increased exponentially with respect to the number of fuzzy subsets. In this paper, we adopt the adaptive fuzzy sliding mode controller (AFSMC) method to design a robust controller for the uncertain time-delayed system with a nonlinear input.

The rest of the paper is divided into five sections. In Section 2, some properties of the uncertain nonlinear time-delayed system with nonlinearities are reviewed. In Section 3, a robust variable structure
controller is derived from those properties. In Section 4, the adaptive fuzzy sliding mode control is developed and presented. In Section 5, a simulation example is illustrated to demonstrate the features of the AFSMC. Finally, we conclude with Section 6.

2 System Statement

A general description of uncertain time-delayed dynamical systems with nonlinear input [2] is given in the form of

\[
x(t) = (A + \Delta A)x(t) + b\Phi(u) + A_d x(t - \tau_d) + f(t)
\]

where \( x(t) \in \mathbb{R}^n \) and \( u \in \mathbb{R} \) is the state variable and control input of the system, respectively. \( A \in \mathbb{R}^{n \times n} \) is the state matrix, \( b \in \mathbb{R}^n \) is the input vector, \( A_d \) is the delay term matrix including the uncertainty, and \( \Phi(u) : \mathbb{R} \rightarrow \mathbb{R} \) is a continuous function and \( \Phi(0) = 0 \). \( \Delta A \) is the bounded uncertainty matrix of \( A \), \( \tau_d \) represents a nonzero time delay, \( \theta(t) \) is a continuous vector-valued initial function, and \( f(t) \in \mathbb{R}^n \) stands for the disturbance vector.

Assumption 1: For the nominal part of the uncertain time-delayed system with series nonlinearities indicated in eq. (1), matrix pair \((A, b)\) is completely controllable.

Assumption 2: The matrix \( A_d \) and uncertainty matrices \( \Delta A \), \( f \) satisfy the following rank conditions.

\[
\begin{align*}
\text{rank}[\Delta A] &= \text{rank}[\Delta A \ M \ b] \\
\text{rank}[A_d] &= \text{rank}[A_d \ : \ b] \\
\text{rank}[f] &= \text{rank}[f : b]
\end{align*}
\]

Based on Assumption 2, there exist vectors \( H, G \), and scalar \( d \) such that the following matching conditions hold.

\[
\begin{align*}
\Delta A &= bH & \|H\| &\leq \beta_1 \\
A_d &= bG & \|G\| &\leq \beta_2 \\
f &= bd & \|d\| &\leq \beta_3
\end{align*}
\]

Assumption 3: The nonlinear input \( \Phi(u) \) applied to the system satisfies the following property:

\[
u \cdot \Phi(u) \geq \alpha \cdot u^2 \tag{4}
\]

where \( \alpha \) is a nonzero positive constant, and \( \Phi(u) \) satisfies \( \Phi(0) = 0 \).

Remark 1: In general, if the series nonlinearity is inside sector \([\alpha_1, \alpha_2]\), i.e.,

\[
\alpha_2 \cdot u \cdot \Phi(u) \geq u \cdot \Phi(u) \geq \alpha_1 \cdot u^2,
\]

where \( \alpha_2 \) is often called the gain margin, and \( \alpha_1 \) is called the gain reduction tolerance. So, we have \( \alpha_2 \rightarrow \infty \) and \( \alpha_1 = \alpha \) in Assumption 3. It is seen that when \( u \) is increasing, then \( \Phi(u) \) is increasing and vice versa. An example of a scalar nonlinear function is shown in Fig. 1.

In consequence, to design variable structure control is, firstly, to choose a proper switching surface for the uncertain nonlinear system with time delay, so that the sliding motion on that switch surface has the desired properties. Then, choose a discontinuous control law, which enforces the motion of the sliding mode to be asymptotically stable.

3 Variable Structure Control Design

First, the following lemmas will be employed to derive the variable structure controller [2].

Lemma 1: If the nonlinear input satisfies the property as indicated in eq. (4), there exists a continuous function \( \phi(\cdot) : \mathbb{R} \rightarrow \mathbb{R} \), \( \phi(0) = 0 \), and \( \phi(p) > 0 \) for \( p > 0 \). Therefore, if 

\[
|u(t)| = \phi(q),
\]

then

\[
\alpha \cdot u^2 \geq q \cdot \phi(q).
\]

Proof: The proof is evidently straightforward if one chooses \( \phi(q) = \rho \cdot q \), \( \rho > \frac{1}{\alpha} \) then

\[
\alpha \cdot u^2 = \alpha \cdot |u|^2 = \alpha \cdot \phi^2(q) \geq q \cdot \phi(q)
\]

Usually the VSC design is a two-phase process. Phase one is to choose a switching surface so that the original system, restricted to the intersection of the switching surface (sliding mode), results in the desired behavior. Phase two is to determine a switching control that is able to force the trajectory of the system approaching to and staying on the sliding surface. Hereby, the switching surface is defined as

\[
s(t) = c^T x(t) = 0
\]

where \( c \in \mathbb{R}^n \) is a constant vector. It is necessary to examine if the property in sliding mode, described in...
eq. (7), is valid for the system with nonlinear input, shown in eq. (1). A necessary condition for the system state trajectory to remain on the sliding surface \( s = 0 \), to yield
\[
s(t) = c^T \dot{x}(t)
\]
\[
= c^T [(A + \Delta A)x(t) + b\Phi(u) + A_dx_d + f]
\]
\[
= 0,
\]
where \( x_d \) denotes \( x(t - \tau_d) \). Therefore, the equivalent control \( \Phi_{eq} \) in the sliding mode \( s = 0 \) is given by
\[
\Phi_{eq} = -(c^T b)^{-1} c^T [(A + \Delta A)x(t) + A_dx_d + f]
\]
(9)

Introducing equations (3) and (9) into eq. (1) produces the equivalent dynamic system with nonlinear input in the sliding mode as:
\[
s(t) = 0
\]
\[
x(t) = (A + \Delta A)x(t) + b\Phi(u) + A_dx_d + f
\]
\[
= (A + bH)x - b(c^T b)^{-1} c^T [(A + bH)x(t) + bGx_d + bd] + bGx_d + bHx + Gx_d + d)
\]
(10)

Remark 2 From the above analysis, it can be concluded that the uncertain time-delayed system with “nonlinear” input in the sliding mode possesses the same property as those with “linear” input in the sliding mode. Accordingly, the design of the switching surfaces can be achieved through same design tool for the systems with “linear” input.

LEMMA 2 The motion of the sliding mode (7) is asymptotically, if the following condition is held
\[
s \dot{s} < 0, \quad \forall t > 0
\]
(11)

To fulfill the condition stated in eq.(11), the desired switching control is suggested by
\[
u(t) = \frac{sc^T b}{\|sc^T b\|}(x(t))
\]
(12)

where
\[
\phi(x, t) = \frac{r}{\alpha} \left\{ \| (c^T b)^{-1} c^T A \| x + \beta_1 \| x_d \| + \beta_2 \right\}, \quad r > 1
\]
(13)

The following theorem shows that the proposed control in eq.(13) drives the uncertain system with nonlinear input onto the sliding mode.

**Theorem 1** Consider the uncertain system (1) subjected to Assumptions 1, 2, and inequality of (3). If the input \( u(t) \) in equation (1) is given as that indicated by (13), then the system trajectories asymptotically converge to the sliding mode (7).

**Proof:** Consider the reaching condition of the sliding mode (7). Let \( V(t) = \frac{1}{2}s^2 \) be the Lyapunov function of the system described in (1). If equation (1) is substituted into the derivative of the states in (11), one can obtain
\[
\dot{V}(t) = s \dot{s} = sc^T \dot{x}
\]
\[
= sc^T [(A + \Delta A)x(t) + A_tx(t - \tau_d) + f(t)]
\]
\[
= sc^T [Ax + b\Phi(u) + b(Hx + Gx_d + d)]
\]
(14)

Then eq. (23) is applied to the above equation to yield the following inequality expression:
\[
s \dot{s} < \|sc^T b\| (c^T b)^{-1} c^T A \| x + \| sc^T b\| \| \Phi(u) \|
\]
\[
+ \|sc^T b\| (\beta_1 \| x \| + \beta_2 \| x_d \| + \beta_3 )
\]
(15)

From equations (12) and (4), we have
\[
u(\Phi(u)) = -\frac{sc^T b}{\|sc^T b\|}(\Phi(x, t))\Phi(u) \geq \alpha u^2 = \alpha \phi^2
\]

Therefore, the above expression can be rearranged as
\[
s c^T b \cdot \Phi(u) \leq -\alpha \dot{\phi}^2(x, t) \frac{sc^T b}{\phi(x, t)}
\]
(16)

Inserting equation (16) into the right hand side of the inequality in eq. (15), it yields
\[
s \dot{s} < \|sc^T b\| \left\{ \| (c^T b)^{-1} c^T A \| x \| - \alpha \phi + \beta_1 \| x \| + \beta_2 \| x_d \| + \beta_3 \right\}
\]
(17)

Thus, once the expression of \( \phi(x, t) \) in (13) is held, it is conspicuous to result in
\[
s \dot{s} \leq (1 - r) \|sc^T b\| \left\{ \| (c^T b)^{-1} c^T A \| x \| + \beta_1 \| x \| + \beta_2 \| x_d \| + \beta_3 \right\}
\]
(18)
Since $r > 1$, it is easy to confirm that
\[ s \dot{s} < 0 \quad \text{or} \quad V(t) < 0 \] (19)
According to the Lyapunov stability theorem, condition (19) ensures that $S(t)$ is toward the switching surface and the sliding mode is asymptotically stable. Then the proof is completed.

In summary, variable structure control method has been derived to stabilize the uncertain time-delayed system with nonlinear input. It has been shown that the proposed control can drive the uncertain system trajectories onto the sliding mode even through the time-delayed systems are with “nonlinear” input. Moreover, the uncertain system can also preserve the invariance condition even though the uncertain systems are with delay time and subjected to the “nonlinear” input.

4 Adaptive fuzzy Sliding Mode Control

As described in the last section, the objective of the SMC is to design a control law (11) so that the reaching condition (12) is satisfied. In this section, we will develop an adaptive fuzzy logic control law [15] in an attempt to accomplish this object. The proposed adaptive fuzzy control is called the adaptive fuzzy sliding mode control because it is based on the principle of SMC. The proposed AFSMC has on line tuning fuzzy rule without the trial-and-error process for find appropriate fuzzy rules where $s$ and $\dot{s}$ are the inputs and $u$ denote the output. The sliding surface variable $s$ is employed is employed as the one dimensional fuzzy input variable. The control law is derived from the fuzzy inference decision and defuzzification operation
\[
\sum_{i=1}^{n} \mu^i U^i = \sum_{i=1}^{n} \mu^i \lambda^i \equiv \sum_{i=1}^{n} \varphi^i \lambda^i
\] (20)
where $n$ is the number of rules and $\lambda^i$ is the consequent parameter. $\mu^i$ is the weight for the corresponding rule which has activated. $\varphi^i$ is the weight of each singleton fuzzy subset for constituting the control law $u$. In the implementation, $S$ is approximated with $S_i$ where $T$ is the sampling period. For simplicity, a triangular type membership function is chosen for the aforementioned adaptive fuzzy variables. The reasoning behind the adaptive fuzzy control rules for the AFSMC, which will be presented later, is explained by the following analysis:

By taking the time derivative of both sides of (7), we obtain
\[
\dot{s} = \begin{bmatrix} c^T \end{bmatrix} x
\]
\[
= b x + c^T b \Phi(u) + c^T b (Hx + Gx_d + d) \] (21)
Then, multiplying both sides of the above equation by $s$ gives
\[
ss = sc^T x
\]
\[
= sc^T Ax + sc^T b \Phi(u) + sc^T b (Hx + Gx_d + d) \] (22)
Here, we assume that $c^T b > 0$.

It can also be found that $s$ increases as $u$ decreases and vice versa in (21). If the situation is $s > 0$, then the increasing of $u$ will result in $ss$ decreasing. When the condition is $s < 0$, $ss$ will decrease with the decreasing of $u$. Based on this qualitative analysis, the control input $u$ can be designed in an attempt to satisfy the inequality $ss < 0$.

5 Simulation

Consider an uncertain time-delayed system with series nonlinearity, which is described by equation (1) [2]. It is shown below
\[
\dot{x}(t) = (A + \Delta A)x(t) + b\Phi(u) + A_d x(t - \tau_d) + f(t)
\]
\[
= Ax(t) + b\Phi(u) + b(Hx(t) + Gx_d + d)
\]
where \( x_d = x(t - \tau_d) \), \( ||H|| \leq \beta_1 \), \( ||G|| \leq \beta_2 \) and \( ||d|| \leq \beta_3 \). The corresponding parameter of the illustrated system are given as follows:
\[
A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ 1 \end{bmatrix}
\]
\[
x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad x_d = \begin{bmatrix} x_1(t - \tau_d) \\ x_2(t - \tau_d) \end{bmatrix}
\]
\[
\Phi(u) = (\delta e^{\sin s_d} + \gamma \cos s_d) u \quad , \quad \delta > \gamma > 0
\]
\[
Hx + Gx_d + d = l_1 e^{(1+\sin s_2)} \sqrt{x_1^2 + x_2^2} + l_2 \cos x_2
\]
\[
+ l_3 \cos x_2 d + x_1^2 + x_2^2 + l_1 \cos x_2
\]
For both VSC and AFSMC controllers, the switching surface is taken as
\[
s(t) = 2x_1 + x_2
\]
Obviously, $c^T b > 0$. This conforms to the assumption $c^T b > 0$ in (22). For this studied system, the following parameter are given: $\delta = 1.0$, $l_1 = 0.04$, $l_2 = 0.3$, $l_3 = -0.6$, $\gamma = 0.3$. In addition, the following initial values are arbitrarily chosen, that is,
\[ x_{10} = -1, \ x_{20} = 1 \]

5.1 VSC method

The computer simulation for this uncertain time-delayed system with series nonlinearity is carried out with sampling step of 0.01 second. For computer simulation the corresponding parameters are \( r = 1.1, \ \alpha = 0.7, \ \beta_1 = 0.3, \ \beta_2 = 0.3, \ \beta_3 = 0.3 \). The delay time constants are taken as \( \tau_d = 0.2 \). Fig. 2 shows the transient response of \( x_1 \) and \( x_2 \) under the VSC controller. Fig. 3 shows the phase plane between \( x_1 \) and \( x_2 \) under the VSC. From the simulation result, it shows that the VSC works well for the uncertain time-delayed system with nonlinear input. But there exists chattering phenomenon that is not desired.

5.2 AFSMC method

For this uncertain time-delayed system with series nonlinearity, the sampling step and delay time is the same for VSC method. Since the AFSMC has online tuning algorithm for fuzzy rules adjustment, the number of rules is not critical. Here eleven fuzzy rules are employed in this control system to obtain appropriate dynamic response and control accuracy. The input membership functions are scaled into the range of \(-5\) and \(+5\) with equal span. Fig. 4 and Fig. 5 shows the phase plane between \( x_1 \) and \( x_2 \) under the AFSMC. From the simulation result, it is shown that the FSMC not only works well for the uncertain time-delayed system with nonlinear input, but also eliminates the chattering phenomenon.

6. Conclusions

We have shown that the AFSMC developed in this paper can be easily applied to uncertain time-delayed dynamical systems with nonlinear input that can be described by state equations of the form (1). Several attractive features of adaptive fuzzy-sliding mode regime are summarized in the following:

(1) No exact mathematical model for the nonlinear systems is required for the controller design. The only required information to design the controller is the qualitative influences of \( u \) on \( s \).
(2) The chattering problem inherent with sliding mode control can be handled effectively without involving sophisticated mathematics.
(3) We only need to adjust the control input \( u \) to satisfy \( s < 0 \), the construction of rule base is simple and clear. By doing so, the asymptotic system stability can be guaranteed.

(4) The AFSMC method without trial-and-error process for find appropriate fuzzy rules in fuzzy control implementation.
(5) From the simulation result, it is shown that the AFSMC indeed can be applied to the uncertain time-delayed system with a nonlinear input. The robustness for the uncertainty and perturbation is still very satisfactory. The effect is the same as that of systems with a linear input.

References:


Fig. 1. Series nonlinearity $\Phi(u)$ in a single input case.

Fig. 2. State variable dynamics for the system under VSC: $x_1$ and $x_2$.

Fig. 3 The phase plane between $x_1$ and $x_2$: $x_2 (x_1)$ under the VSC.

Fig. 4. State variable dynamics for the system under AFSMC: $x_1$ and $x_2$.

Fig. 5 The phase plane between $x_1$ and $x_2$: $x_2 (x_1)$ under the AFSMC.