Numerical Simulation of a High Voltage Circuit for Laser Applications

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Abstract: A numerical model of a typical Nitrogen laser set-up is presented. The proposed model is based on a fluid description of the discharge plasma and on Poisson's law for the electric field calculation, coupled with Kirchhoff' s equations that describe the high voltage circuit. Remarks are made on the influence of the external H. V. circuit parameters on the electrical discharge and consequently on the output laser pulse. Namely the role of the spark gap and the applied high voltage are explored. Based on simulation results, optimal working conditions are proposed.

Keywords: Numerical Simulation, H.V impulse generator, Electrical Discharge, Nitrogen Laser, Spark Gap.

1 Introduction

As it is well known, the active medium of many lasers commonly used nowadays is a plasma, produced by an electrical discharge. Such lasers, as CO_2, N_2, Excimer for example, are thoroughly used in a variety of medical and industrial applications as well as research activities.

Transverse electrical discharges that are used to excite such lasers, very rapidly create population inversion and thus laser emission. Experimental studies [1] have shown that population inversion is optimal when the High Voltage (H.V.) circuit parameters and those of the electrical discharge are also optimized.

In order to investigate the influence of the external H. V. circuit on the electrical discharge and thus on laser performance, a 1D numerical model for a Nitrogen Laser has been developed. The model consists of a hydrodynamic description of the discharge, coupled with Kirchhoff's equations that incorporate the external circuit. The laser effect is then accounted for by the rate equations that yield the spatio-temporal evolution of the excited species involved. Population inversion in the Nitrogen laser takes place between the $N_2(C^3\Pi_u)$ –higher laser level and the $N_2(B^3\Sigma_g)$ –lower laser level, electronic states.

In the following part a thorough description of the model is given, underlining it's physical reasoning and the numerical scheme's details. Then, numerical results are presented, pointing out the influence of various macroscopical parameters in the laser's output. Optimal working conditions obtained numerically are proposed. Finally, a brief discussion is carried out, followed by useful suggestions for future work.

2 Model's Description

2.1 Electric Circuit

Figure 1 shows the electrical equivalent of the circuit used in our model. It consists of two distinctive parts. The first one, serves as an energy storage bank (Capacitor array C_S) from which energy is transferred to the second part. Energy transfer is controlled by a spark gap switch, usually modeled by a resistance R_S and an inductance L_S, which triggers the fast discharge breakdown.

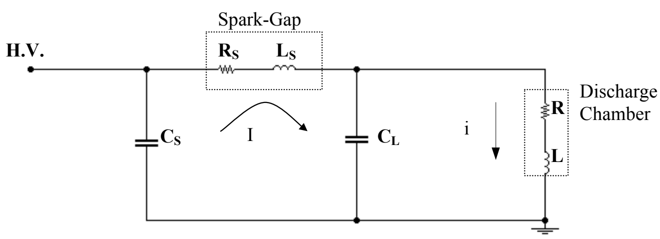

Figure 1: A typical electrical circuit used in a Nitrogen laser.

The second part includes the laser discharge chamber which is also modeled by a resistiveinductive (R-L) series component. It is obvious that the C_S-Spark Gap-C_L set-up forms a classical high voltage impulse generator, that is purposed to deliver fast H.V. pulses to the discharge gap that are essential to achieve the laser effect.

A DC High Voltage, is used to charge the capacitors C_S. When the spark gap is triggered, the energy stored in the C_S is transferred to the C_L applying a gradually increasing voltage to the discharge gap. Then the laser discharge is auto-triggered when voltage across the gap becomes greater than the Paschen breakdown threshold.

A rather straightforward application of circuit theory yields the following equations, according to Fig. 1:

$$\frac{dI}{dt} = -\frac{R_S}{L_S} I + \frac{1}{L_S} V_{CS} - \frac{1}{L_S} V_{CL} \quad \textbf{(1)}$$

$$\frac{dV_{c_S}}{dt} = -\frac{1}{C_S} I \quad \textbf{(2)}$$

$$\frac{dV_{c_L}}{dt} = -\frac{1}{C_L}(I - i) \quad \textbf{(3)}$$

2.2 Electrical Discharge

The electrical discharge is produced in a transverse structure (the direction of the field lines is orthogonal to the laser beam direction) by high voltage pulse which is applied between the electrodes at a gas pressure not greater than 70 Torr. The gap geometry (plane-plane configuration) initially ensures a homogenous field distribution. When the applied voltage exceeds the Paschen breakdown one, a discharge is initiated and a very fast (rise time < 5 ns) high peek impulse current (I >10kA) crosses the gap.

The discharge plasma behavior is described by a set of fluid equations written for electron and positive ion densities n_e and n_i, along the discharge axis [2]. Thus:

$$\frac{\partial n_e}{\partial t} + \frac{\partial}{\partial x}(n_e u_e) = S(n_e) - L(n_e) \quad \textbf{(4)}$$

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i u_e) = S(n_e) - L(n_e) \quad \textbf{(5)}$$

where u_e and u_i are electron and ion velocities. Right hand side terms account for ionization and recombination processes:

$$S(n_e) = n_e |u_e| \alpha\left(\frac{E}{p}(x,t)\right) \quad \textbf{(6)}$$

$$L(n_e) = n_e n_i \beta\left(\frac{E}{p}(x,t)\right) \quad \textbf{(7)}$$

In a convenient form, swarm parameters are expressed as functions of the reduced electric field E/p. The values adopted in this work are :

$$\frac{a}{p}(cm \cdot Torr)^{-1} = 7.4326 \exp\left(-270.5 \frac{p}{E}\right) \quad \textbf{(8)}$$

$$\beta(cm^3 \sec^{-1}) = 6.3636 \times 10^{-12} \left(\frac{p}{E}\right)^{0.8} \quad \textbf{(9)}$$

$$u_e(cm \sec^{-1}) = 2.9 \times 10^5 \left(\frac{E}{p}\right) \quad \textbf{(10)}$$

$$u_i(cm \sec^{-1}) = 1.67 \times 10^3 \left(\frac{E}{p}\right) \quad \textbf{(11)}$$

To determine the electrical field distribution, Poisson's equation is employed:

$$\nabla \cdot \mathrm{E} = -\frac{\mathrm{e}}{\varepsilon_0} \cdot (\mathrm{n_i} - \mathrm{n_e}) \quad \textbf{(12)}$$

where e is the electron charge and e_0 the permittivity of free space.

Secondary electron emission due to ion bombardment of the cathode implies the following boundary condition:

$$J_e(0,t) = \gamma J_i(0,t) = \gamma n_i u_i \quad \textbf{(13)}$$

where the generally used γ=0.01 value has been adopted [3].

The discharge current is calculated according to Sato's formula [4]:

$$i(t) = \frac{sE_0}{V(t)} \int_0^d \left(J_e(x,t) + J_i(x,t)\right) dx \quad \textbf{(14)}$$

where s is the discharge cross section, E_0 the static field, V(t) the gap voltage, d is the gap length and $J_e(x,t)$ and $J_i(x,t)$ are the electron and ion current densities.

A simple observation of equations 1-3 that electrically describe the problem as well as equations 4-14 that model the electrical discharge shows that the physical quantity that couples the two sets is the discharge current.

2.3 Nitrogen Laser

The Nitrogen laser is a typical example of a three level laser system. Population inversion occurs between the $N_2(C^3\Pi_u)$ and $N_2(B^3\Sigma_g)$ electronic states whereas repletion of these states is achieved by excitation of the $N_2(X^1\Sigma_g^+)$ ground state via electron impacts:

$$e + N_2(X^1\Sigma_g^+) \rightarrow e + N_2(B^3\Sigma_g) \qquad \textbf{(R1)}$$

$$e + N_2(X^1\Sigma_g^+) \rightarrow e + N_2(C^3\Pi_u) \qquad \textbf{(R2)}$$

Stimulated photons are emitted, provided that population inversion has been attained, with a wavelength of 337.1nm. Thus :

$$h\nu + N_2(C^3\Pi_u) \rightarrow 2 \cdot h\nu + N_2(B^3\Sigma_g) \qquad \textbf{(R3)}$$

Reactions (R1-R3) are mathematically formalized to yield the spatio-temporal evolution of the excited species, as well as the stimulated photon density. Analytically:

$$\frac{\partial N_{CB}(x,t)}{\partial t} = N_0 N_e(x,t) R_{X\rightarrow C}(x,t) - N_0 N_e(x,t) R_{X\rightarrow B}(x,t)$$
$$-2 \cdot \frac{N_c(x,t)}{\tau_c} - 2 \cdot N_{ph}(x,t) \cdot \sigma_{stim} \cdot c \cdot N_{CB}(x,t) \qquad \textbf{(15)}$$

$$\frac{\partial N_{ph}}{\partial t} = N_{ph} \cdot \sigma_{stim} \cdot c \cdot \left(N_C - N_B\right) \qquad \textbf{(16)}$$

where N_C, N_B, N_{ph} are the $C^3\Pi_u$, $B^3\Sigma_g$ and stimulated photon densities. σ_{stim} and c are the stimulated photon emission cross section and the velocity of light respectively. The term $N_{CB}=N_C-N_B$, corresponds to population inversion, whereas τ_C is the radiative lifetime of the $C^3\Pi_u$ state. The terms $R_{x\rightarrow C}$ and $R_{x\rightarrow B}$ correspond to the coefficients of the reactions R1 and R2 accordingly. In general, reaction coefficients, are a function of the electron energy distribution *f(ε)*, the electron energy *ε* and on the cross section *$Q_i(\varepsilon)$* corresponding to each reaction:

$$R_{X\rightarrow i} = \delta \cdot \int_0^\infty f(\varepsilon) \cdot \varepsilon \cdot Q_i(\varepsilon) d\varepsilon \qquad \textbf{(18)}$$

where δ is a constant. Cross sectional data have been adopted by experimental results from [5].

2.4 Numerical Scheme

The solution of the external circuit equations by fourth order Runge-Kutta method determines the voltage gap. Fluid equations are numerically treated by the FCT technique [6] applied in an explicit 1D finite differences discretizasion scheme. Rate equations are solved by means of a Milne-Simpson sixth order method belonging to the predictor-corrector family.

3 Numerical Results

In this section results obtained by the numerical model will be presented. The results are given for a 1cm, plane-to-plane discharge gap, with a gas pressure of 40Torr. The C_L and C_S capacitances were fixed at 0.06 μF whereas the R_S value was 0.1Ω. In order to investigate the influence of the external circuit on the laser performance (output power, laser pulse duration) a parametric analysis has been carried out, varying the applied H.V.'s value as well as the spark gap's inductance L_S.

3.1 Applied Voltage

The High Voltage value used to charge the energy storage capacitor C_S designates the amount of energy transferred to the discharge gap. Fig. 2 shows the discharge current curves for three different H.V. values and for L_S=10nH. These curves clearly show that peak current values, follow the applied voltage maxima. This should be attributed to the energy surplus supplied to the discharge gap. On the other hand, the time shift observed is due to the fact that higher H.V. values result in a faster breakdown, as Paschen voltage is more rapidly attained.

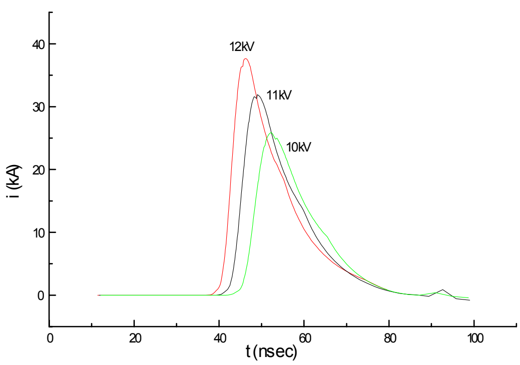

Figure 2: Discharge Current for 8, 10 and 12 kV of applied Voltage

The influence of the applied voltage on the stimulated photon emission is shown in fig. 3.

Maximum photon density occurs for 12 kV, whereas the value of 10 kV is not sufficient to create a notable photon density, mainly due to the time lag that it introduces with respect to the other curves.

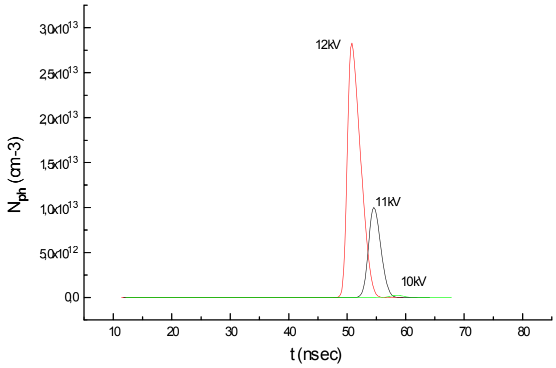

Figure 3: Stimulated Photon density in the middle of the discharge gap for 8, 10 and 12 kV of applied Voltage

3.2 Spark Gap Inductance

Fig. 4 presents the discharge current curves for three distinct L_S values and for an applied voltage equal to 12 kV. The maximum pumping current occurs for the 5nH value, implying best energy transfer conditions. For this inductance value, optimal matching between the energy storage circuit and the discharge gap can be claimed. Oscillations appearing on the tail of the 2.5 nH curve denote energy ringing between the two circuits.

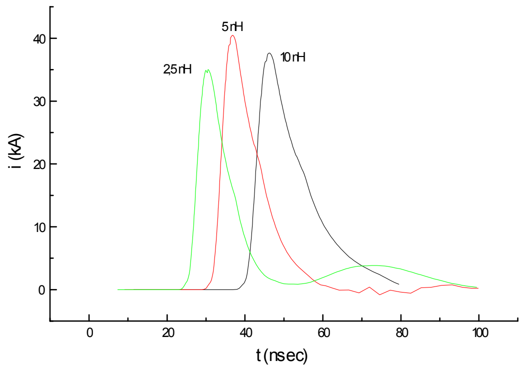

Figure 4: Discharge Current for 2.5, 5 and 10 nH of spark gap inductance.

As expected, maximum stimulated photon density occurs for L_S=5nH, following the current maximum. This fact demonstrates the correlation between discharge current and population inversion. Fig. 5 shows the respective stimulated photon densities, resulting from population inversion, at the middle of the discharge gap.

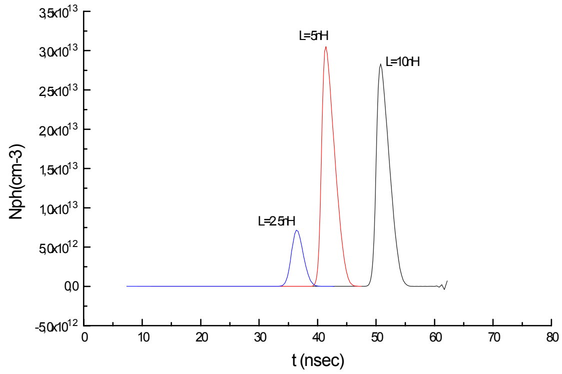

Figure 5: Stimulated Photon density in the middle of the discharge gap for 2.5, 5 and 10 nH of Spark Gap Inductance

4 Discussion

In the present work, a one-dimensional numerical model for the investigation of the performance of a Nitrogen Laser has been developed. The correlation between macroscopical parameters and stimulated photon emission, which is directly related with laser output, has been presented.
In particular, the parametric study conducted has pointed out the influence of the high voltage circuit on laser performance. The need for efficient laser systems imposes optimized matching conditions between the external circuit and electrical discharge. Based on the results presented here, optimal working conditions have been obtained for 5nH spark gap inductance and 12 kV of applied High Voltage.
According to the aforementioned analysis, the peak stimulated photon density, as a function of spark gap inductance is a monotonous curve, acquiring an overall maximum value.
In conclusion, it should be pointed out that a 2D description of the problem would yield more accurate results. Such a description would also enable the exact calculation of the laser's output power, as the photon optical path would be accounted for (orthogonal to the discharge's axis). A more rigorous analysis should also require the modification of Sato's formula [7] in order to include the displacement current due to the time variation of the applied electric field. Such work is currently under way.

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