Generalized Algorithms of Discrete Optimization and Their Applications

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Abstract: - Generalized algorithms for solving problems of discrete, integer, and Boolean programming are discussed. These algorithms are associated with the method of normalized functions, are based on a combination of formal and heuristic procedures, and allow one to obtain quasioptimal solutions after a small number of steps, that promotes overcoming the *NP*-completeness of discrete optimization problems. Questions of building so-called "duplicate" algorithms are considered to improve the quality of discrete optimization problem solutions. The subsequent development of the algorithms is related to using their modifications to solve optimization problems under conditions of uncertainty within the framework of a general approach to analyzing models with fuzzy coefficients in objective functions and constraints. In practical aspect, the algorithms are already being used to solve diverse problems of power engineering.

Key-Words: - Discrete Optimization, Method of Normalized Functions, Heuristic Procedures, Fuzzy Coefficients, Choice Functions, Fuzzy Preference Relations.

1 Introduction

In the general case, direct determination of discrete (integer, Boolean) solutions to problems of discrete (integer, Boolean) nature is necessary. This is explained by the fact that even though at the cost of ignoring parameter discreteness, with smoothing of functions, it is possible to replace an actual objective function by a convex function defined on a convex region, with such an approach the danger always exists that the objective function will be distorted (with a deviation from the optimum) or that the constraints will be violated [1]. Besides, the transition from the discrete model to its convex analog can lead to considerable "coarsening" the model that often makes vapid its essence [2]. Thus, the ability to solve discrete problems by discrete methods also makes it possible in the course of the solution to consider detailed situations and reflect individual forms of initial data reliably, thereby, to obtain solutions within the framework of more adequate models. Finally, with orientation to discrete methods it is possible to pose and solve problems of combinatorial nature, which had previously not be considered.

Theoretical and experimental evaluations [3,4] have revealed essential drawbacks of exact methods of discrete programming. Moreover, estimates of computational complexity in solving discrete problems [5] indicate that their *NP*-completeness does not permit one to develop general methods with polynomial dependence on the problem dimension.

Taking the above into account, algorithms of discrete optimization discussed in the paper are based on a combination of formal and heuristic procedures. They are close to the class of greedy methods [6]. Basically, these methods provide the best heuristic among possible heuristics with a priori estimates and can be the basis for fully polynomial approximate approaches [7]. They allow one to obtain quasioptimal solutions after a small number of steps, thus overcoming the problem *NP*-completeness. In addition, heuristic procedures allow one to consider the problem specificity. In particular, the algorithms do not require analytical specification of objective functions and constraints. Their specification may be tabular or algorithmic ensuring flexibility and the possibility to solve complex problems, for which adequate analytical descriptions are difficult.

In the process of posing and solving a wide range of problems related to the design and control of complex systems, one inevitably encounters different kinds of uncertainty [8]. Its consideration in shaping the models of complex systems serves as a means for increasing the adequacy of these models and, as a result, their credibility and the factual effectiveness of solutions based on their analysis. Considering this, the present paper includes an attempt to modify the generalized algorithms to use them in solving problems with fuzzy or interval coefficients.

2 Problem Formulation

It is possible to distinguish two classes of models of discrete optimization. The first class is related to the general problem of discrete programming, including the problems of integer, Boolean, and discrete programming proper. The problems with discrete variables may be reduced to integer or, in the general case, Boolean models [1,2]. However, such a reduction increases the problem dimension as regards the number of variables as well as constraints [2].

The second class of models is associated with problems of a combinatorial type. When solving them, an extremum of the objective function is defined on a given finite discrete set A. The totality of objects obtained from A may be considered as a combinatorial space D. The problem may be formulated as a search for a vector $x^0 = (x_1^0,...,x_n^0)$ from D or $G \subseteq D$ providing the extremum of the objective function, i.e., $F(x) \to \underset{x \in G \subseteq D}{\text{extr}}$.

The combinatorial problems are the most difficult from the computational standpoint [4]. Their solution is based on finiteness of $G \subseteq D$ and the problem specificity. Some of them may be reduced to the Boolean models, sometimes by accepting strong assumptions, sharp increasing the model dimension, and losing the possibility of effective considering combinatorial properties of the problems. Thus, when solving discrete problems, it is important that their formulation and corresponding solution algorithms should exploit those properties and peculiarities of the problems that promote their effective solution.

Taking the above into account, the desirability of allowing for constraints on the discreteness of variables in the form of discrete sequences

$$x_{s_i}, \alpha_{s_i}, \beta_{s_i}, ..., \quad s_i = 1,...,r_i \tag{1}$$

has been validated in [1,2]; here $\alpha_{s_i}.\beta_{s_i},...$ are characteristics (technical, economic, etc.) required for constructing the objective functions, constraints, and their increments that correspond to the *s*th standard value of the variable x_i

It is expedient to use the sequences (1) because $\alpha_{s_i}, \beta_{s_i},...$ cannot always be fitted closely to analytical relationships in terms of x_{s_i}, but in (1) these characteristics may be taken as exact. Besides, the flexible formulation of the combinatorial problems is possible on the basis of the sequences (1) because they can be different for different variables. Considering this, a maximization problem may be formulated as follows.

Assume we are given the discrete sequences of the type (1) (increasing or decreasing, depending on the problem formulation). From these sequences it is necessary to choose parameters that the objective

$$\text{maximize}\, F(x_{s_1}, \alpha_{s_1}, \beta_{s_i}, ..., ..., x_{s_n}, \alpha_{s_n}, \beta_{s_n}, ...) \tag{2}$$

is met while satisfying the constraints

$$g_j(x_{s_1}, \alpha_{s_1}, \beta_{s_1}, ..., ..., x_{s_n}, \alpha_{s_n}, \beta_{s_n}, ...) \le b_j, \quad j = 1,...,m. \tag{3}$$

The objective function (2) is interpreted as convex up and the constraints (3) are interpreted as convex down.

Given the maximization problem (1)-(3), we can formulate a problem of minimization:

$$\text{minimize}\, F(x_{s_1}, \alpha_{s_1}, \beta_{s_1}, ..., ..., x_{s_n}, \alpha_{s_n}, \beta_{s_n}, ...) \tag{4}$$

subject to the constraints

$$g_j(x_{s_1}, \alpha_{s_1}, \beta_{s_1}, ..., ..., x_{s_n}, \alpha_{s_n}, \beta_{s_n}, ...) \ge b_j, \quad j = 1,...,m. \tag{5}$$

The objective function (4) is interpreted as convex down and the constraints (5) are interpreted as convex up.

3 Solution Algorithms

Let us consider the Boolean problem of maximizing

$$F(x) = \sum_{i=1}^{n} c_i x_i \tag{6}$$

while satisfying the constraints

$$\sum_{i=1}^{n} a_{ji} x_i \le b_j, \quad j = 1,...,m, \tag{7}$$

where $c_i > 0$, $i = 1,...,n$ and $a_{ji} > 0$, $j = 1,...,m$, $i = 1,...,n$.

The idea of one of the most popular methods related to the class of heuristic methods [6,9] may be illustrated by considering the problem (6), (7) for $m = 1$. It is possible to assume (without generality loss) that $x_i, i = 1,...,n$ are arranged as follows:

$$\frac{c_1}{a_1} \ge \frac{c_2}{a_2} \ge ... \ge \frac{c_n}{a_n}. \tag{8}$$

It is possible to try to maximize (6) on the basis of the largest c_i/a_i, taking $x_1 = 1$, then $x_2 = 1$, and so on until (7) is observed. Similar methods are called greedy methods. In spite of their "naivety", in many cases [7] they represent the best heuristic among other heuristics with a priory estimates. In particular, they are the basis for fully polynomial approximate schemes for solving diverse versions of the problem (6), (7). However, a range of problems is not restricted by the case of $m=1$. Considering this, we discuss below ways of constructing algorithms for the general case ($m>1$) to solve problems (linear as well as nonlinear), which can cover not only Boolean, but integer and discrete variables as well.

When analyzing the model (6), (7) for $m=1$, maximization is reached by expending only one resource type. If $m>1$, the optimization process is stopped when a remaining amount of only one of resources is not sufficient for next incrementing any of x_i, $i=1,...,n$. This resource is the limiting one.

It is possible to speak about equivalentness of different types of resources from the standpoint of cessation of the process of maximizing (6). Thus, it is expedient to have a single measure for different resources. This consideration leads to the idea of normalization [1,10], which may be performed at each optimization step. For example, the constraints (7) are reduced to a single arbitrary resource b as

$$a_{ji}^{(t)} = a_{ji}\frac{b}{b_{ji}^{(t-1)}},\quad j=1,...,m,\quad i=1,...,n, \tag{9}$$

where t is the optimization step number.

Using (9), it is possible to convert the constraints (7) to equal conditions. For instance, before the first optimization step we have

$$\sum_{i=1}^{n} a_{ji}^{(0)} x_i \le b,\quad j=1,...,m. \tag{10}$$

The normalization (9) may also be useful for reducing the model dimension. If

$$a_{pi}^{(0)} \le a_{qi}^{(0)},\quad p\ne q,\quad i=1,...,n, \tag{11}$$

the qth constraint is disturbed earlier than the pth one. Thus, the pth constraint can be eliminated from consideration (the principle of explicit domination).

The algorithms, which generalize considerations given above, have been developed to solve the problems (1)-(3) and (1), (4), (5). The last problem is more difficult. In the case of maximization we cease changing the variable x_i when at least one of the constraints (3) is violated. In minimization the optimization is completed on any variable when all constraints (5) are obeyed. Thus, in the maximization case, there is usually only one "deficient" constraint during each step requiring particular attention, while in minimization we have to pay attention to each constraint because the optimization process cannot be completed until all constraints (5) have been obeyed.

Taking the above into account, we consider the algorithm for solving the problem (1), (4), (5).

It is assumed that the constraints (5) are already normalized

$$g_j^{(0)}(x_{s_1},\alpha_{s_1},\beta_{s_1},...,...,x_{s_n},\alpha_{s_n},\beta_{s_n},...)$$
$$= g_j(x_{s_1},\alpha_{s_1},\beta_{s_1},...,...,x_{s_n},\alpha_{s_n},\beta_{s_n},...)\frac{b}{b_j} \ge b_j,$$
$$j=1,...,m, \tag{12}$$

that permits us to present the algorithm as follows.

1. The components of the constraint increment vector $\{\Delta G_i^{(t)}\}$ are evaluated

$$\Delta G_i^{(t)} = \sum_j \Delta g_{ji}^{(t)},\quad i\in I^{(t)},\quad j\in J^{(t)}. \tag{13}$$

In (13),

$$\Delta g_{ji}^{(t)} = [g_j(x_{s_1}^{(t)},\alpha_{s_1}^{(t)},\beta_{s_1}^{(t)},...,...,x_{s_i+1}^{(t)},\alpha_{s_i+1}^{(t)},\beta_{s_i+1}^{(t)},...,$$
$$...,x_{s_n}^{(t)},\alpha_{s_n}^{(t)},\beta_{s_n}^{(t)},...) - g_j(x_{s_1}^{(t)},\alpha_{s_1}^{(t)},\beta_{s_1}^{(t)},...,$$
$$...,x_{s_i}^{(t)},\alpha_{s_i}^{(t)},\beta_{s_i}^{(t)},...,...,x_{s_n}^{(t)},\alpha_{s_n}^{(t)},\beta_{s_n}^{(t)},...)]\frac{b_j^{(t-1)}}{b},$$
$$j\in J^{(t)},\quad i\in I^{(t)}, \tag{14}$$

where $J^{(t)}$ is the set of the constraints (5) at the tth step (for t=1 we have $j\in J_m$, J_m is the initial set of constraints); $I^{(t)}$ is the set of variables at the tth step (for t=1 we have $i\in I_n$, I_n is the initial set of variables); $b_j^{(t-1)} = b_j^{(0)} = b_j$.

2. The components of the increment vector of the objective function $\{\Delta F_i^{(t)}\}$ are calculated as

$$\Delta F_i^{(t)} = F(x_{s_1}^{(t)},\alpha_{s_1}^{(t)},\beta_{s_1}^{(t)},...,...,x_{s_i+1}^{(t)},\alpha_{s_i+1}^{(t)},\beta_{s_i+1}^{(t)},...,$$
$$...,x_{s_n}^{(t)},\alpha_{s_n}^{(t)},\beta_{s_n}^{(t)},...) - F(x_{s_1}^{(t)},\alpha_{s_1}^{(t)},\beta_{s_1}^{(t)},...,$$
$$...,x_{s_i}^{(t)},\alpha_{s_i}^{(t)},\beta_{s_i}^{(t)},...,...,x_{s_n}^{(t)},\alpha_{s_n}^{(t)},\beta_{s_n}^{(t)},...),\quad i\in I^{(t)}. \tag{15}$$

3. The components of the vector $\{V_i^{(t)}\}$ are calculated as

$$V_i^{(t)} = \frac{\Delta F_i^{(t)}}{\Delta G_i^{(t)}},\quad i\in I^{(t)}. \tag{16}$$

4. The index $i=l_t$ of the variable to be incremented is determined from

$$V_{l_t}^{(t)} = \min_i V_i^{(t)},\quad i\in I^{(t)}. \tag{17}$$

5. We recalculate the values of the quantities:

$$x_{s_i}^{(t)} = \begin{cases} x_{s_i}^{(t)} & \text{if } i\ne l_t,\ i\in I^{(t)}, \\ x_{s_i+1}^{(t)} & \text{if } i=l_t, \end{cases} \tag{18}$$

$$b_j^{(t)} = b_j^{(t-1)} - \Delta g_{jl_t}^{(t)}\frac{b}{b_j^{(t-1)}},\quad j\in J^{(t)}. \tag{19}$$

6. We refine the set $J^{(t)}$

$$J^{(t)} = \{j \mid b_j^{(t)} > 0,\quad j\in J^{(t)}\}. \tag{20}$$

7. We make a check for nonemptiness of the set $J^{(t)}$. If $J^{(t)} \ne \varnothing$, then go to operation 8; otherwise go to operation 11.

8. We refine the set $I^{(t)}$

$$I^{(t)} = \{i \mid s_i < r_i, i \in I^{(t)}\}. \quad (21)$$

9. We make a check for nonemptiness of the set $I^{(t)}$. If $I^{(t)} \neq \varnothing$, then go to operation 1, taking t=t+1; otherwise go to operation 10.

10. The calculations are completed because the problem has no solution.

11. The calculations are completed because the solution is obtained.

The algorithm is directly related to minimizing the objective function interpreted as convex down. However, this does not narrow a field of its applications because prior to using the algorithm it is possible to carry out simple minimizing the objective function (4) without considering the constraints (5).

A large body of comparisons of solutions for diverse types of discrete problems, based on the paper results and exact methods, shows their convincing agreement. However, considering difficulties in predicting a priori effectiveness of the approximate algorithms, it is expedient to have not only one, but several algorithms realizing different strategies. Considering this, so-called "duplicate" algorithms have been developed on the basis of a qualitative analysis of the problem statement. One of them is based on evaluating the components of the vector $\{\Delta G_i^{(t)}\}$ (operation 1) as follows:

$$\Delta G_i^{(t)} = \min_j \Delta g_{ji}^{(t)}, \quad i \in I^{(t)}, \quad j \in J^{(t)}, \quad (22)$$

where $\Delta g_{ji}^{(t)}, i \in I^{(t)}, j \in J^{(t)}$ are calculated as (14).

Am alternative "duplicate" algorithm is associated with the results of [9] and is based on calculating $\Delta g_{ji}^{(t)}, i \in I^{(t)}, j \in J^{(t)}$ (operation 1) as

$$\Delta g_{ji}^{(t)} = \min\{[g_j(x_{s_1}^{(t)}, \alpha_{s_1}^{(t)}, \beta_{s_1}^{(t)}, \ldots, \ldots, x_{s_i+1}^{(t)}, \alpha_{s_i+1}^{(t)}, \beta_{s_i+1}^{(t)}, \ldots,$$
$$\ldots, x_{s_n}^{(t)}, \alpha_{s_n}^{(t)}, \beta_{s_n}^{(t)}, \ldots) - g_j(x_{s_1}^{(t)}, \alpha_{s_1}^{(t)}, \beta_{s_1}^{(t)}, \ldots,$$
$$\ldots, x_{s_i}^{(t)}, \alpha_{s_i}^{(t)}, \beta_{s_i}^{(t)}, \ldots, \ldots, x_{s_n}^{(t)}, \alpha_{s_n}^{(t)}, \beta_{s_n}^{(t)}, \ldots), b_j^{(t-1)}]\},$$
$$j \in J^{(t)}, \quad i \in I^{(t)} \quad (23)$$

with recalculating $b_j^{(t)}, j \in J^{(t)}$ (operation 5) as

$$b_j^{(t)} = b_j^{(t-1)} - [g_j(x_{s_1}^{(t)}, \alpha_{s_1}^{(t)}, \beta_{s_1}^{(t)}, \ldots, \ldots, x_{s_{l_t}+1}^{(t)}, \alpha_{s_{l_t}+1}^{(t)}, \beta_{s_{l_t}+1}^{(t)}, \ldots,$$
$$\ldots, x_{s_n}^{(t)}, \alpha_{s_n}^{(t)}, \beta_{s_n}^{(t)}, \ldots) - g_j(x_{s_1}^{(t)}, \alpha_{s_1}^{(t)}, \beta_{s_1}^{(t)}, \ldots,$$
$$\ldots, x_{s_{l_t}}^{(t)}, \alpha_{s_{l_t}}^{(t)}, \beta_{s_{l_t}}^{(t)}, \ldots, \ldots, x_{s_n}^{(t)}, \alpha_{s_n}^{(t)}, \beta_{s_n}^{(t)}, \ldots)], \ j \in J^{(t)}. \quad (24)$$

The availability of the "duplicate" algorithms can be considered, in a certain measure, as an assurance of obtaining optimal solutions. Besides, the analysis of one and the same problem on the basis of several algorithms permits one to reveal a series of solutions of equal worth, that is important as well.

The described results have a high degree of generality and are used in solving power engineering problems: optimization in the design and development (selecting elements of power systems and subsystems, allocating reactive power sources, selecting means for increasing reliability, etc.), load management, and voltage and reactive power control.

However, the discussed algorithms have been constructed without sufficient formal justifications. This circumstance, in spite of the considerations given above, forces to look for additional means for possible improving the solution performance. As such a means may serve formulating and solving one and the same problem within the framework of so-called mutually interrelated models (1)-(3) and (1), (4), (5) using the algorithms of maximization and minimization, respectively. It is natural that a good agreement of solution results for these interrelated problems is a convincing indication of the proximity to the optimum. Using this approach, if we have the increasing (decreasing) sequences (1) for (1)-(3), the sequences (1) must be decreasing (increasing) for (1), (4), (5). Thus, it is possible to solve one and the same problem from above and from below as well. This approach is fruitful and also serves for solving problems with fuzzy (or interval) coefficients.

3 Discrete Optimization Problems with Fuzzy Coefficients

Numerous optimization problems related to the design and control of complex system [8,11] may be formulated as follows.

From the discrete sequences (1) it is necessary to choose standard parameters that the objective

$$\text{maximize}\, \widetilde{F}(x_{s_1}, \alpha_{s_1}, \beta_{s_1}, \ldots, \ldots, x_{s_n}, \alpha_{s_n}, \beta_{s_n}, \ldots) \quad (25)$$

is met while satisfying the constraints

$$\widetilde{g}_j(x_{s_1}, \alpha_{s_1}, \beta_{s_1}, \ldots, \ldots, x_{s_n}, \alpha_{s_n}, \beta_{s_n}, \ldots) \subseteq \widetilde{b}_j, \quad j = 1, \ldots, p \quad (26)$$

The objective function (25) and constraint (26) include fuzzy coefficients, as indicated by the ~ symbol.

A fuzzy analog to the problem (1), (4), (5) may be presented in the following form:

$$\text{minimize}\, \widetilde{F}(x_{s_1}, \alpha_{s_1}, \beta_{s_1}, \ldots, \ldots, x_{s_n}, \alpha_{s_n}, \beta_{s_n}, \ldots) \quad (27)$$

subject to the constraints (26).

An approach [8] to handling the constraints (26) involves approximate replacement of each of the constraints by a finite set of nonfuzzy constraints. This allows one to change from (1), (25), (26) or (1), (27), (26) to (1), (25), (3) or (1), (27), (4) with fuzzy coefficients in the objective function alone. The solution, for example, of (1), (27), (4) is possible on the basis of modification of the algorithm of minimization or "duplicate" algorithms. In particular, the expressions (15) and (16) are to be related to algebraic operations on fuzzy numbers [12].

The comparison (17) (in essence, the comparison of fuzzy numbers $\widetilde{V}_i^{(t)}, i \in I^{(t)}$) can be done using the idea of a membership function of a generalized preference relation [13].

If the membership functions corresponding to the values $\widetilde{V}_1$ and $\widetilde{V}_2$ are $\mu(v_1)$ and $\mu(v_2)$, the quantity $\eta\{\mu(v_1), \mu(v_2)\}$ is the degree $\mu(v_1) \succcurlyeq \mu(v_2)$, while $\eta\{\mu(v_2), \mu(v_1)\}$ is the degree $\mu(v_2) \succcurlyeq \mu(v_1)$. Then, if V is the numerical axis on which the values of $\widetilde{V}_i^{(t)}, i \in I^{(t)}$ are plotted, the membership functions of the generalized preference relations take the forms:

$$\eta\{\mu(v_1), \mu(v_2)\} = \sup_{\substack{v_1, v_2 \in V \\ v_1 \le v_2}} \min\{\mu(v_1), \mu(v_2)\}, \qquad (28)$$

$$\eta\{\mu(v_2), \mu(v_1)\} = \sup_{\substack{v_1, v_2 \in V \\ v_2 \le v_1}} \min\{\mu(v_1), \mu(v_2)\}, \qquad (29)$$

which agree with some important choice functions (for example, fuzzy number ranking indices of Baas-Kwakernaak, Baldwin-Guild, and one of the indices of Dubois-Prade) [8].

On the basis of the relation between (28) and (29) it is possible to judge the preference of any of the alternatives. Although this approach seems justified [14], experience shows that in many cases $\mu(v_1)$ and $\mu(v_2)$ correspond to flat fuzzy numbers [12]. In view of this, using (28) and (29), we can say that the alternatives $\widetilde{V}_1$ and $\widetilde{V}_2$ are indistinguishable if

$$\eta\{\mu(v_1), \mu(v_2)\} = \eta\{\mu(v_2), \mu(v_1)\}. \qquad (30)$$

In such situations the discussed algorithms do not allow one to obtain unique solutions: they "stop" when conditions like (30) arise. This also occurs with modifications of other optimization methods because combination of the uncertainty and relative stability of optimal solutions produces the decision uncertainty regions. In this connection, other choice functions (for example, [15]) may be used as additional means for the ranking of fuzzy numbers. However, these indices occasionally result in choices, which appear inconsistent with intuition, and their application does not permit one to close the question of building an order on a set of fuzzy numbers [8,11]. There is another approach, which is better validated for the practice of decision making. It is associated with transition to multicriteria selection of alternatives because the use of additional criteria (of quantitative as well as qualitative character) serves as convincing means to contract the decision uncertainty regions.

Before starting to discuss multicriteria decision making in a fuzzy environment, it is necessary to note that the maximal contraction of the decision uncertainty region may be obtained by formulating and solving one and the same problem within the framework of mutually interrelated models [8,11]:

(a) the model of maximization (25) with the constraints (26) approximated by (3) with the decreasing (increasing) discrete sequences (1);

(b) the model of minimization (27) with the constraints (26) approximated by (4) with the increasing (decreasing) discrete sequences (1).

Assume we are given a set X of alternatives (from the decision uncertainty region) that are to be examined by q criteria. That is, indices $\widetilde{F}_p(X_k)$, $p = 1,...,q$, $X_k \in X$ with the membership functions $\mu[f_p(X_k)]$, $p = 1,...,q, X_k \in X$ are to be compared to make a selection among alternatives. The problem is presented by $<X, R>$, where $R = \{R_1,..., R_q\}$ is a vector fuzzy preference relation. Thus, we have $R_p = [X \times X, \mu_{R_p}(X_k, X_l)]$, $p = 1,...,q$, $X_k, X_l \in X$, where $\mu_{R_p}(X_k, X_l)$ is a membership function of fuzzy preference relation.

The relations $R_p, p = 1,...,q$ may be constructed on the basis of $\mu[f_p(X_k)], p = 1,...,q, X_k \in X$ with the use of expressions [11] similar to (28) and (29).

If we have a single fuzzy preference relation, it can be put in correspondence [13] with a strict fuzzy preference relation with

$$\mu_R^s(X_k, X_l) = \max\{\mu_R(X_k, X_l) - \mu_R(X_l, X_k), 0\}. (31)$$

The membership function of a subset of nondominated alternatives can be presented as

$$\mu_R^n(X_k) = 1 - \sup_{X_l \in X} \mu_R^s(X_l, X_k). \qquad (32)$$

Since $\mu_R^n(X_k)$ is the degree of nondominance of alternatives, it is natural to define

$$X' = \{X'_k \mid X'_k \in X, \mu_R^n(X'_k) = \sup_{X_l \in X} \mu_R^n(X_k)\}. \quad (33)$$

When R is a vector fuzzy preference relation, (31)-(33) are applicable to use two techniques of multicriteria selection of alternatives [8,11]. The first technique is related to constructing and analyzing the membership functions of a subset of nondominated alternatives with simultaneous considering all criteria. The second technique is based on a lexicographic procedure that consists in step by step comparison of alternatives, as a result of which we can obtain a sequence $X^1, X^2,..., X^q$ so that $X \supseteq X^1 \supseteq X^2 \supseteq ... \supseteq X^q$.

Finally, it is possible to propose the third approach to contract the decision uncertainty region. Using (32), we can build the membership functions of a subset of nondominated alternatives $\mu^n_{R_p}(X_k)$ for all fuzzy preference relations. Their intersection

$$\mu^n(X_k) = \min_{1 \le p \le q} \mu^n_{R_p}(X_k), \quad X_k \in X \tag{34}$$

permits one to obtain X^n in accordance with (33).

Thus, three techniques for selecting alternatives can be applied. They may lead to different solutions and this is natural: the choice of the technique is a prerogative of the decision making person.

The described results are of a universal character and can be applied to the design and control of systems and processes of different nature as well as enhancement of corresponding CAD/CAM systems and intelligent decision support systems. These results are used in solving power engineering problems: optimization in the design and development (selecting elements of power systems and subsystems and means for increasing reliability), load management, and energy market planning.

4 Conclusion

In this paper, generalized algorithms for solving problems of discrete, integer, and Boolean programming have been described. The algorithms are associated with the method of normalized functions, are based on a combination of formal and heuristic procedures, and allow one to obtain quasioptimal solutions after a small number of steps, that promotes overcoming the *NP*-completeness of the problems. Questions of constructing "duplicate" algorithms have been considered to improve the problem solution quality. The subsequent development of the algorithms is related to using their modifications to solve problems under conditions of uncertainty within the framework of a general approach to analyzing models with fuzzy coefficients. In practical aspect, the results of the paper are already being used to solve problems of power engineering.

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References:

[1] V.V. Zorin and P.Ya. Ekel, Discrete-optimization methods for electrical supply systems, *Power Engineering*, Vol.18, No.5, 1980, pp.19-30.

[2] P.Ya. Ekel, *Models and Methods of Discrete Optimization of Power Supply Systems*, KPI, 1990 (in Russian).

[3] R.S. Jeroslow, Trivial integer programs unsolvable by branch-and-bound, *Mathematical Programming*, Vol.6, No. 1, 1974, pp. 105-109.

[4] I.V. Sergienko, *Mathematical Models and Methods in Solving Discrete Optimization Problems,* Naukova Dumka, 1985 (in Russian).

[5] M.R. Garey and D.S. Johnson, *Computers and Intractability: A Guide to the Theory of NP-Completeness*, W.H. Freemaan, 1979.

[6] T.H. Cormen, C.E. Leiserson, and R.L. Rivest, *Introduction to Algorithms*, McGraw-Hill, 1990.

[7] A.A. Korbut and Yu.Yu. Finkelshtein, Approximate methods of discrete programming, *Soviet Journal of Computer and System Sciences*, Vol.21, No.1, 1983, pp.147-157 (printed in the USA).

[8] P. Ekel, W. Pedrycz, and R. Schinzinger, A general approach to solving a wide class of fuzzy optimization problems, *Fuzzy Sets and Systems*, Vol.97, No.1, 1998, pp.49-66.

[9] G. Dobson, Worst-case analysis of greedy heuristics for integer programming with nonnegative data, *Mathematics of Operations Research*, Vol.7, No.4, 1983, pp.515-531.

[10] E.A. Berzin, *Optimal Resource Allocation and Elements of System Synthesis*, Sovetskoe Radio, 1974 (in Russian).

[11] P.Ya. Ekel, Methods of decision making in fuzzy environment and their applications, *Nonlinear Analysis: Theory, Methods and Applications*, Vol.47, No.5, 2001, pp. 979-990.

[12] D. Dubois and H. Prade, *Fuzzy Sets and Systems. Theory and Applications*, Academic Press, 1980.

[13] S.A. Orlovski, Decision-making with a fuzzy preference relation, *Fuzzy Sets and Systems*, Vol.1, No.3, 1978, pp.155-167.

[14] C.R. Barrett, P.K. Patanalk, and M. Salles, On choosing rationally when preferences are fuzzy, *Fuzzy Sets and Systems*, Vol.34, No.2, 1990, pp.197-212.

[15] H. Lee-Kwang, A method for ranking fuzzy numbers and its application to decision making, *IEEE Transactions on Fuzzy Systems*, Vol.7, No.6, 1999, pp.677-685.

[16] S.A. Orlovski, *Problems of Decision Making with Fuzzy Information*, Nauka, 1981 (in Russian).