A Hydrothermal Optimal Power Flow Problem Solved by Lagrangian Relaxation Approach

LEONARDO NEPOMUCENO †, SECUNDINO SOARES FILHO ‡, TAKAAKI OHISHI ‡
† Electrical Engineering Department
State University of São Paulo – UNESP – Bauru - SP
‡ Electrical Engineering Department
State University of Campinas – UNICAMP – Campinas - SP
BRAZIL
leo@feb.unesp.br, dino@densis.fee.unicamp.br, taka@densis.fee.unicamp.br

Abstract: - Short-term Generation Scheduling models (STGS) are concerned with the calculation of an optimal generation policy while taking into account various operational limits in transmission and generation systems. Most STGS models described in the literature totally ignore representation of reactive portions of the transmission system. Such purely active models tend to calculate generation policies that may lead the system to operating points presenting security problems associated with reactive aspects (such as voltage instability, etc.). A class of problems denominated Hydrothermal Optimal Power Flow (HOPF) models have been proposed to represent active/reactive STGS studies. This paper proposes a HOPF model concerned with detailed representation of active and reactive aspects of the transmission system. Lagrangian Relaxation is applied to solve the proposed HOPF model. This technique is based on relaxation of dynamic constraints together with the coordination of such constraints using Lagrange multipliers. HOPF and the proposed solution technique are applied to IEEE 30 bus test system. Solutions obtained by the HOPF model are compared with those obtained by purely active dispatch models. Results point-out the importance of the representation of reactive aspects in dispatch studies. Results also confirm robustness of the proposed solution methodology.

Key-Words: - Hydrothermal Optimal Power Flow, Lagrangian Relaxation, Optimal Power Flow

1 Introduction

Short-term Generation Scheduling (STGS) models are concerned with the calculation of an active generation dispatch policy (generally on an hourly basis and for one day ahead) for both hydraulic and thermal units, while taking into account various constraints involving transmission and generation systems. For dominantly hydraulic systems, such as the Brazilian system, STGS models are integrated in a hierarchy of optimization models concerned with long, medium and short-term generation planning. In such a case the solution calculated by the STGS model must take into account additional constraints involving generation targets established by medium-term models. The inclusion of such constraints turns STGS into a dynamic problem. This means that generation decisions for a certain time interval are dependent on the decisions calculated for all other time intervals.

Most STGS models presented in the literature totally ignore operational constraints associated with reactive representation of the transmission system. Such modeling approaches adopts linear dispatch models in which purely active representation is used for the transmission system [1][2][3][4]. Recent studies have pointed-out the need to enhance the transmission system representation for short-term optimal dispatch calculations. It is shown in reference [5] that purely active dispatch models tend to calculate very inaccurate generation dispatches. Purely active dispatch models may also produce dispatch policies that are more susceptible to security problems associated with the reactive representation, such as voltage instabilities, etc.

Some STGS problems involving reactive representation have been proposed in the literature [6][7][8][9][10]. In references [10] the approach adopted is to decouple STGS problem in two dispatch problems: a purely active and an active/reactive dispatch model. The solution to STGS is obtained by the coordination of such models. In [8] a class of problems denominated Hydrothermal Optimal Power Flow (HOPF) is proposed. These models may be faced as active/reactive Optimal Power Flow models (OPF) in which additional constraints associated with the hydraulic generation system are incorporated. The basic difference between HOPF and OPF models is that the former is a dynamic problem (due to the inclusion of generation target constraints for hydraulic units), where decisions at an early time in the optimization interval influence decisions at later times. Various HOPF modeling approaches, together with specific solution methodologies, have been proposed [8][9][10]. The HOPF presented in [9] is probably the most representative, including constraints such as water balance, water delay, etc. This model is solved by interior-point methods.

This work proposes a HOPF model that fully represents active and reactive aspects associated with generation and transmission systems. The following constraints are incorporated: limits on voltage levels, on transformer taps, on capacitor/reactor banks, and on reactive generation. The generation targets are also taken into account. The HOPF model proposed calculates the active generation dispatch together with a reactive dispatch for reactive controls (voltage in controllable buses, taps and capacitor/reactor banks). A multi-objective problem is adopted aiming to calculate a dispatch policy that minimizes a linear combination of active power losses in the transmission system and generation costs in the generation system. The model here presented is based on the active model described in [4], but generalizing this problem such that active and reactive aspects are included.

The methodology here proposed to solve HOPF problem is based on Lagrangian Relaxation. This technique consists on relaxing dynamic constraints which are taken into account in a coordination process through the associated Lagrange multipliers. This methodology has been extensively used to solve unit commitment problems [11] [12]. In such problems Lagrangian Relaxation is used to coordinate constraints associated with load balance. The STGS model here proposed and the solution methodology adopted are applied to IEEE 30 test system. The results highlight the comparison between the proposed model and purely active dispatch approaches. The results also points-out the robustness of the proposed solution algorithm.

This work is organized as follows. In section 2 the proposed HOPF model is described. In section 3 the solution technique adopted, based on Lagrangian relaxation, is discussed and the solution algorithm is presented. Case studies involving IEEE 30 bus test system are presented in section 4. Final conclusions are drawn in section 5.

2 HOPF Formulation

The basic difference between HOPF, here proposed, and traditional OPF models is the inclusion of generation target constraints for hydraulic units. Such constraint turns HOPF into a dynamic problem, where decisions at an early time in the optimization interval influence decisions at later times. HOPF is mathematically formulated as follows.

$$\begin{cases} Min\ C(\mathbf{x}) \\ s.a: \\ \Delta Q_i^t(\mathbf{x}) = 0 \quad i \in \Omega_{load} \quad t \in \mathbf{T} \quad (1.1) \\ \Delta P_i^t(\mathbf{x}) = 0 \quad i \in \Omega_{all} \quad t \in \mathbf{T} \quad (1.2) \\ \underline{\mathbf{x}} \le \mathbf{x}^t \le \overline{\mathbf{x}} \quad t \in \mathbf{T} \quad (1.3) \\ \underline{Q}_i \le h_i^t(\mathbf{x}) \le \overline{Q_i} \quad i \in \Omega_{gen} \quad t \in \mathbf{T} \quad (1.4) \\ \sum_{t=1}^{\mathbf{T}} P_i^t = M_i \quad i \in \Omega_{hyd} \quad (1.5) \end{cases}$$

where:

$\mathbf{x} = [\mathbf{\Theta}\ ,\ \mathbf{V}\ ,\ \mathbf{tp}\ ,\ \mathbf{Bsh}, \mathbf{P}]$

$\mathbf{\Theta}$: vector of voltage angles;

$\mathbf{V}$: vector of voltage magnitudes;

$\mathbf{tp}$: vector of transformer taps;

$\mathbf{Bsh}$: vector of capacitor/reactor banks

$\mathbf{P}$: vector of active generation for generating units;

$\Delta Q_i^t(x) = 0$: reactive power mismatch at bus *i* for time interval *t*;

$\Delta P_i^t(x) = 0$: reactive power mismatch at bus *i* for time interval *t*;

$\underline{Q}_i, \overline{Q_i}$: minimum and maximum reactive generation limits at bus *i*;

$h_i^t(x)$: reactive power generation function at bus *i* for time interval *t*;

P_i^t : active power generation at bus *i* for time interval *t*;

M_i : generation target for hydraulic generating unit *i*;

Ω_{load} : set defined by load buses;

Ω_{all} : set defined by all system buses;

Ω_{gen} : set defined by all generating units;

Ω_{hyd} : set defined by hydraulic generating units;

$\mathbf{T}$: set defined by the time intervals of the problem;

The decision variables of the proposed HOPF problem include voltage magnitudes (at controllable buses), transformer taps, capacitor/reactor banks, and active power generation. The transmission system is fully represented through non-linear load flow equations 1.1 and 1.2. Operational limits on the variables are represented by 1.3. Reactive generation limits, represented by 1.4, are nonlinear functional constraints. Equations 1.1 through 1.4 are written for each time interval *t*. Equation 1.5 establishes the generation targets for each hydraulic unit *i*. In [8] the generation target equation is formulated in terms of a predefined volume of water. In the proposed HOPF the generation target constraint is formulated in terms of a predefined active power generation. This formulation avoids the use of hydraulic variables in the dispatch problem. However, important hydraulic features, such as turbine-generator efficiency and effective water head, are incorporated in the problem by means of the generation loss function (defined in [4]) introduced in the objective function.

The objective function here proposed is a trade-off between two conflicting objectives: i) the minimization of non-linear losses in the transmission system ii) the optimization of hydraulic and thermal generation resources. The mathematical formulation is as follows:

$$C(\mathbf{x}) = \sum_{t=1}^{\mathbf{T}} \left(\alpha P_{tr}^{t}(\mathbf{x}) + \beta\, \Pi(\mathbf{x}) \right) \qquad (2)$$

where:

$P_{tr}^{t}(\mathbf{x})$: nonlinear transmission power losses; $\Pi(\mathbf{x}) = \sum_{i \in \Omega_{hid}} PH_i^t(\mathbf{x}) + \sum_{i \in \Omega_{ter}} CT_i^t(\mathbf{x})$ is a function associated with hydraulic and thermal resources.

$CT_i^t(\mathbf{x})$: quadratic function representing thermal generating costs for unit *i* at time interval *t*.

$PH_i^t(\mathbf{x})$: quadratic function representing hydraulic generation costs for unit *i* at time interval *t*.

α, β: are weighting factors associated with the objective functions.

The function $PH_i^t(\mathbf{x})$ cited above is proposed in reference [4]. This function aims to improve the utilization of hydraulic generation resources. Such function incorporates hydraulic losses associated with turbine-generator efficiency and effective water head.

The incorporation of the generation target constraint couples, dynamically in time, the solution for HOPF. The strategy proposed to decompose the problem into independent sub-problems is discussed as follows.

3 Problem Solution

Only the constraints associated with generation targets couple the proposed HOPF problem in time domain. All other constraints are time-independent. Lagrangian Relaxation is proposed in this section to handle such dynamic constraint. This methodology is described as follows.

Associating Lagrange multipliers λ_i with the generation target constraint for each hydraulic unit *i*, the following Lagrangian function may be written:

$$L(\mathbf{x}, \boldsymbol{\lambda}) = C(\mathbf{x}) + \sum_{i \in \Omega_{ger}} \lambda_i \left(M_i - \sum_{t \in \mathbf{T}} P_i^t \right) \qquad (3)$$

The Lagrangian Relaxation approach consists on solving the corresponding dual problem associated with the primal HOPF problem. The dual problem consists on the unconstrained maximization of the $\mathbf{H}(\boldsymbol{\lambda})$ function, as follows:

$$\text{Dual} \quad \left\{ \underset{\{\boldsymbol{\lambda}\}}{Max}\ \mathbf{H}(\boldsymbol{\lambda}) \right. \qquad (4)$$

where $\mathbf{H}(\boldsymbol{\lambda})$ is written as:

$$\mathbf{H}(\boldsymbol{\lambda}) = \begin{cases} Min\ L(\mathbf{x}, \boldsymbol{\lambda}) & \\ s.t: & \\ \quad \Delta Q_i^t(\mathbf{x}) = 0 & i \in \Omega_{load} \quad t \in \mathbf{T} \\ \quad \Delta P_i^t(\mathbf{x}) = 0 & i \in \Omega_{all} \quad t \in \mathbf{T} \\ \quad \underline{\mathbf{x}} \le \mathbf{x}^t \le \overline{\mathbf{x}} & t \in \mathbf{T} \\ \quad \underline{Q}_i \le h_i^t(\mathbf{x}) \le \overline{Q_i} & i \in \Omega_{gen} \quad t \in \mathbf{T} \end{cases}$$

The optimization problem $\mathbf{H}(\boldsymbol{\lambda})$ may be partitioned in a series of $\mathbf{T}$ time-independent problems, one for each time interval. Actually, each problem is very similar to a traditional active/reactive Optimal Power Flow (OPF). The basic difference is the introduction of linear terms (associated with generation target constraints), in the objective function $L(\mathbf{x}, \boldsymbol{\lambda})$. Thus, given the Lagrangian multiplier vector $\boldsymbol{\lambda}$ the solution to $\mathbf{H}(\boldsymbol{\lambda})$ is obtained, in a decomposed way, solving one OPF for each time interval. The solution to the dual problem is not a hard task once that the problem is unconstrained.

The problem decomposition is described in two levels as shown in Fig. 1. In the highest level (coordinator) the solution for the dual problem is obtained through unconstrained maximization of the dual function. In the lower level independent OPF sub-problems are solved, one for each time interval, minimizing the decomposed Lagrangian function.

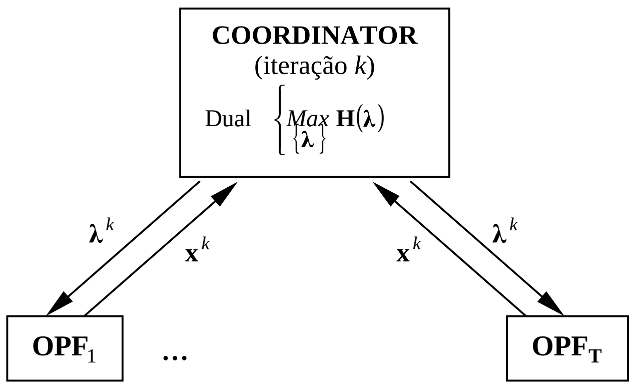

Fig.1 – HOPF Decomposition

The dual problem is solved in this work by a traditional gradient method using false position line search as described in [13]. The solution for such problem provides a means for updating the Lagrange multipliers λ. OPF independent sub-problems are solved using Newton method as described in [14]. Inequality constraints on optimization variables are handled by quadratic penalty functions (as described in [15]). Functional constraints on reactive power generation are handled through parameterization techniques [16]. The solution for the OPF problems provides optimal active and reactive controls. Active controls include the active generation and reactive controls include controllable voltages, transformer taps and capacitor/reactor banks. Active and reactive controls are calculated for each time interval. Although independent, the OPF problems are solved in a concatenated manner. The solution for a certain time interval *t* is used as a hot start condition for the next interval *t*+1. This strategy has considerably reduced the overall number of iteration to solve the whole $\mathbf{H}(\lambda)$ problem, as presented in the results as follows.

4 Simulation Results

Simulation results are carried-out on the IEEE 30 bus test system. The basic load profile provided for the system is multiplied by load factors such that 24 load profiles (one for each time interval) are obtained. All six generator are modeled as hydraulic units. Loss generation functions as in [4] are stipulated for each generator. Generation targets are imposed based on an initial load flow solution and are shown in Table 1 as follows.

Table 1. Generation Targets

	Gen1	Gen2	Gen5	Gen8	Gen11	Gen13
Targets (Mw)	1224	1081	1081	1620	1358	1358

Solutions for an Active Dispatch (AD) model are compared with the proposed HOPF problem to highlight discrepancies between a purely active dispatch and a general active/reactive formulation for the transmission system. The AD model used in the results is a particular formulation for the HOPF in which all the constraints associated with reactive aspects (such as limits on voltage magnitude, transformer taps, capacitor/reactor banks, and reactive generation) are relaxed. The dispatch calculated by such AD model is shown in Table 2. The lower line in the table shows the generation targets for each unit. Reactive aspects of dispatch provided in Table 2 are evaluated through an active/reactive OPF study for each time interval. Such reactive evaluation is depicted in Table 3. The table shows the violated and the biding constraints for each time interval t. The violated variables plotted include, active power generation (Mw), voltage magnitude (V), capacitor/reactor banks (C) and transformer taps (Tp). No violation was obtained for reactive generation. The binding constraints include all previously cited variables and also the reactive generation (Mvar).

Table 2. Generation Dispatch calculated by AD

t	Gen1	Gen2	Gen5	Gen8	Gen11	Gen13
1	42,79	33,25	21,81	51,06	42,06	46,94
2	42,24	32,44	20,22	49,95	41,07	46,24
3	40,03	29,2	13,87	45,51	37,13	43,45
4	37,82	25,97	7,54	41,11	33,2	40,62
5	42,79	33,25	21,81	51,06	42,06	46,94
6	47,98	40,97	36,97	61,74	51,47	53,42
7	50,73	45,04	44,99	67,43	56,48	56,76
8	52,42	47,49	49,81	70,86	59,5	58,73
9	52,71	47,9	50,61	71,43	60,01	59,06
10	53,29	48,71	52,22	72,58	61,03	59,71
11	53,89	49,53	53,84	73,73	62,04	60,36
12	53,59	49,12	53,03	73,16	61,53	60,03
13	54,17	49,94	54,64	74,31	62,55	60,68
14	54,72	50,75	56,25	75,46	63,57	61,33
15	55,58	51,98	58,68	77,19	65,11	62,29
16	56,47	53,21	61,1	78,93	66,65	63,25
17	57,43	54,43	63,52	80,67	68,21	64,2
18	59,34	56,48	67,58	83,59	70,81	65,78
19	61,26	58,58	71,77	86,63	73,44	67,32
20	55,54	51,98	58,68	77,2	65,11	62,28
21	54,17	49,94	54,64	74,31	62,55	60,68
22	50,59	44,23	43,38	66,29	55,47	56,1
23	47,98	40,97	36,97	61,74	51,47	53,42
24	44,49	35,68	26,59	54,42	45,02	49,01
	1222	1081	1080,6	1620,4	1357,6	1358,6

Heavy load profiles (for time intervals 18:00 and 19:00) are highlighted in the table. In the OPF solution for such intervals there has been some violated constraints on voltage magnitude and on transformer taps. Thus, if the dispatch described by Table 2 is adopted, the reactive representation of the transmission system will not be fully respected. This result confirms [5] which asserts that purely active generation dispatches may compromise the reactive representation of the solution. The example also shows that specific active/reactive dispatch models, such as HOPF here proposed, are required.

The same problem was solved by the proposed HOPF approach and the results are shown in Tables 4 and 5. Active generation dispatch is shown in Table 4. Violated and binding constraints are shown in Table 5. It is clear from tables 2 and 4 that the dispatch policies calculated respectively by a purely active model and an active/reactive methodology (HOPF) are significantly different, specially for heavy loaded profiles. As seen from table 5, the reactive unfeasibility problems were solved in the new dispatch policy calculated by HOPF model. This result also shows that convenient active re-dispatches may help solving critical reactive aspects.

The solution calculated by the HOPF is completely feasible with respect to active and reactive aspects. The generation target constraint is also satisfied as indicated in the lower line of table 4.

Table 3 Reactive Analysis for purely Active Dispatch

	Violation				Binding Constraints				
t	Mw	V	C	Tp	Mw	V	C	Tp	Mvar
1	0	0	0	0	0	0	2	0	0
2	0	0	0	0	0	0	2	0	0
3	0	0	0	0	0	0	1	0	0
4	0	0	0	0	0	0	1	0	0
5	0	0	0	0	0	0	2	0	0
6	0	0	0	0	0	0	2	0	1
7	0	0	0	0	0	0	2	0	2
8	0	0	0	0	0	0	2	0	2
9	0	0	0	0	0	0	2	0	2
10	0	0	0	0	0	0	2	1	3
11	0	0	0	0	0	0	2	1	3
12	0	0	0	0	0	0	2	1	3
13	0	0	0	0	0	0	2	1	4
14	0	0	0	0	0	0	2	1	4
15	0	0	0	0	0	0	2	1	4
16	0	0	0	0	0	0	2	1	4
17	0	0	0	0	0	1	2	1	4
18	0	0	0	1	0	3	2	3	5
19	0	4	2	3	0	4	2	3	5
20	0	0	0	0	0	0	2	1	4
21	0	0	0	0	0	0	2	1	4
22	0	0	0	0	0	0	2	0	1
23	0	0	0	0	0	0	2	0	1
24	0	0	0	0	0	0	2	0	0

Table 4. Dispatch calculated by HOPF

t	Gen1	Gen2	Gen5	Gen8	Gen11	Gen13
1	50,59	37,33	11,33	37,8	46,95	54,52
2	50,36	36,67	9,38	36,61	45,74	54,04
3	49,5	34,09	1,54	32,08	40,87	51,84
4	48,09	28,15	0	26,92	34,57	49,04
5	50,59	37,33	11,33	37,8	46,95	54,52
6	53	44,13	29,76	51,53	56,91	57,61
7	54,26	47,83	39,43	60,94	60,42	58,81
8	54,84	49,56	46,33	66,58	62,12	59,51
9	54,97	49,87	47,49	67,61	62,33	59,54
10	55,25	50,56	49,8	69,81	62,67	59,51
11	55,56	51,31	52,05	72,33	62,82	59,31
12	55,4	50,93	50,93	71,05	62,76	59,41
13	55,71	51,68	53,17	73,6	62,9	59,21
14	56,01	52,43	55,41	76,12	63,06	59,04
15	56,44	53,53	58,76	79,86	63,36	58,84
16	56,51	54,56	62,85	83,91	63,22	58,43
17	56,49	55,64	67,2	88,57	62,53	57,81
18	40,58	60,13	83,42	104,7	57,96	55,74
19	0	0	154,7	180,19	43,81	39,18
20	56,47	53,54	58,71	79,84	63,38	58,86
21	55,71	51,68	53,17	73,6	62,9	59,21
22	53,64	46,73	37,11	57,53	60,94	60,01
23	53	44,13	29,76	51,53	56,91	57,61
24	51,39	39,42	17,2	39,6	51,81	56,32
	1224,3	1081,2	1080,8	1620,1	1357,9	1357,9

Four iteration were necessary so that the generation target constraints were gradually corrected (through Lagrange multiplier updating), and the dual problem solved.

Some important features of the problem $\mathbf{H}(\boldsymbol{\lambda})$ are depicted as follows. As already discussed $\mathbf{H}(\boldsymbol{\lambda})$ may be decomposed in **T** OPF independent problems. In the solution methodology adopted here such problems are solved in a concatenated manner. The solution for a certain time interval *t* is used as a hot start condition for the next interval *t*+1. OPF solution for the first time interval took 27 Newton iteration (as depicted in Fig 2); taking this solution as a start for the next time interval, only one more iteration was necessary so that the next OPF study reaches the solution. This strategy has considerably reduced the overall number of iteration to solve the whole $\mathbf{H}(\boldsymbol{\lambda})$ problem, as shown in Fig. 2. Heavy load profiles have taken larger number of iteration. This feature is explained by increase in load ramp rate and also by increase on the active set in the OPF problems for such intervals.

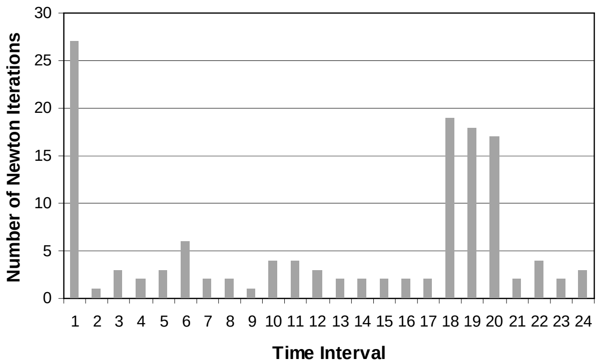

Fig. 2 - Solution Process for $\mathbf{H}(\boldsymbol{\lambda})$

Table 5. Reactive Solution for HOPF

	Violation				Binding Constraints				
t	Mw	V	C	Tp	Mw	V	C	Tp	Mvar
1	0	0	0	0	0	0	2	0	0
2	0	0	0	0	0	0	1	0	0
3	0	0	0	0	0	0	1	0	0
4	0	0	0	0	1	0	1	0	0
5	0	0	0	0	0	0	2	0	0
6	0	0	0	0	0	0	2	0	1
7	0	0	0	0	0	0	2	0	1
8	0	0	0	0	0	0	2	0	2
9	0	0	0	0	0	0	2	0	3
10	0	0	0	0	0	0	2	0	3
11	0	0	0	0	0	0	2	1	3
12	0	0	0	0	0	0	2	1	3
13	0	0	0	0	0	0	2	1	3
14	0	0	0	0	0	0	2	1	3
15	0	0	0	0	0	0	2	1	3
16	0	0	0	0	0	1	2	1	4
17	0	0	0	0	0	1	2	1	4
18	0	0	0	0	0	2	2	1	5
19	0	0	0	0	2	3	2	3	4
20	0	0	0	0	0	0	2	1	3
21	0	0	0	0	0	0	2	1	3
22	0	0	0	0	0	0	2	0	1
23	0	0	0	0	0	0	2	0	1
24	0	0	0	0	0	0	2	0	0

It is clear, from the comparison of Tables 2 and 4, that dispatch policies calculated by purely active dispatch methodologies are significantly different from those obtained by general active/reactive models, such as the proposed HOPF. Such differences become more clear when the dispatches calculated for heavy loaded profiles are evaluated.

For such time intervals coupling between active and reactive aspects are weaker.

5 Conclusion

A new formulation to the Hydrothermal Optimal Power Flow (HOPF) problem is proposed in this work. In such model active and reactive power aspects of the transmission system are fully represented. The proposed HOPF is a trade-off between two conflicting objectives: i) the minimization of non-linear losses in the transmission system ii) the optimization of hydraulic and thermal generation resources. The hydraulic generation system is represented in detail through the generation loss function (involving losses related to turbine-generator efficiency and effective water head.).

The solution methodology proposed to solve the HOPF is based on Lagrangian Relaxation technique. This technique decomposes the problem into a set of independent OPF problems, which are solved in a concatenated manner. The results highlights the discrepancies between purely active and general active/reactive HOPF dispatches. It is clear from the results that the inclusion of reactive analysis in dispatch studies are very important, specially for heavy load profiles. Results also point-out the efficiency of the proposed solution methodology which is robust enough to handle HOPF, a large scale optimization problem.

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