Application of the wavelets in 2D scattering problems

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Abstract: - Wavelets technique is applied for solving of 2D Dirichlet, Neumann and mix boundary problems. The developed method utilizes Haar or linear B-spline functions and Fredholm integral equations (-or a system of the Fredholm integral equations) of fist kind with smooth or singular kernels. The problems of accuracy, choosing auxiliary surfaces end stable results are discussed.

Key-Words: - Haar and Battle-Lemarie wavelets, Fredholm integral equations (or a system equations) of fist kind with smooth or singular kernel.

1 Introduction

One of the most attractive ideas were appeared in last years had been connected with utilizing of wavelets as basis functions in method of the moments [1].

This paper is concerned the extending wavelets technique for solving an auxiliary currents integral equations [2], system of auxiliary currents integral equations, currents integral equations [3] and system of currents integral equations, which could be presented as a Fredholm integral equations of the first kind with smooth and singular kernals. In this case the unknown function is an auxiliary current on some surface Σ located within (or within and outside) the scattering surface S and it could be presented in form of set with wavelets function. The results of utilizing wavelets technique are illustrated by solving of a Dirichlet, Neumann and mix 2D boundary scattering problems by cylinders and bands.

2 2D Scattering Problem

In the beginning let consider a scattering of E and H polarized wave $u_0(\vec{r})$ by perfect conducting cylinder with cross section contour as $\rho(\varphi)$ function in cylindrical system of coordinates (z,r,φ). Diffracted field $u^1(\vec{r})$ is a solution of the Helmholts equation

$$\Delta u^1 + k^2 u^1 = 0 \qquad (1)$$

without of S and satisfies the Dirichlet boundary condition (E polarized incident wave) on S

$$u(\vec{r})|_S \equiv [u_0(\vec{r}) + u^1(\vec{r})]|_S = 0. \qquad (2)$$

or the Neunamm boundary condition (H polarized incident wave)

$$\frac{\partial}{\partial n_S}[u_0(\vec{r}) + u^1(\vec{r})]|_S = 0 \qquad (3)$$

and Sommerfeld radiation condition

$$\frac{\partial u^1(\vec{r})}{\partial r} + iku^1(\vec{r}) = o(r^{-1/2}), |r| \to \infty \qquad (4)$$

in case of perfect conducting cylinders .

For dielectric cylinder we have a nix boundary conditions and we have to replace the (2) and (3) by

$$u^i(\vec{r})|_S = [u_0(\vec{r}) + u^1(\vec{r})]|_S, \qquad (5a)$$

$$\frac{\partial}{\partial n_S}[u_0(\vec{r}) + u^1(\vec{r})]|_S = \chi \frac{\partial}{\partial n_S}[u^i(\vec{r})] \qquad (5b)$$

where k is a wave number of the free space, $u^i(\vec{r})$ - is a field inside of the dielectric body satisfies (1) with $k = k_i$, $\chi = 1$ for E polarized incident field and $\chi = 1/\varepsilon_i$ for H polarized incident field, ε_i is a relative dielectric penetrability of dielectric.

In case of multiple scattering by system of M perfect conducting bands we have to execute a boundary condition (2) or (3) on each band.

In accordance with the method of an auxiliary current (MAC) [2] the boundary problems (2),(3),(5) mention above could be reduced to the Fredholm integral equations (-or system) of the first kind with smooth kernel

$$u_0(\vec{r}_S) = \int_\Sigma \mu(\vec{r}_\Sigma) H_0^{(2)}(k|\vec{r}_S - \vec{r}_\Sigma|) d\sigma, \qquad (6)$$

$$\frac{\partial}{\partial n_S} u_0(\vec{r}_S) = \int_\Sigma \mu_1(\vec{r}_\Sigma) \frac{\partial}{\partial n_S} H_0^{(2)}(k|\vec{r}_S - \vec{r}_\Sigma|) d\sigma, \qquad (7)$$

$$u_0(\vec{r}_S) + \int_\Sigma \mu(\vec{r}_\Sigma) H_0^{(2)}(k|\vec{r}_S - \vec{r}_\Sigma|) d\sigma = \\ = \int_\Sigma \mu(\vec{r}_{\Sigma 1}) H_0^{(2)}(k_i|\vec{r}_S - \vec{r}_{\Sigma 1}|) d\sigma, \qquad (8)$$

$$\frac{\partial}{\partial n_S} u_0(\vec{r}_S) + \int_\Sigma \mu_1(\vec{r}_\Sigma) \frac{\partial}{\partial n_S} H_0^{(2)}(k|\vec{r}_S - \vec{r}_\Sigma|) d\sigma = \\ = \chi \int_{\Sigma 1} \mu_1(\vec{r}_{\Sigma 1}) \frac{\partial}{\partial n_S} H_0^{(2)}(k_i|\vec{r}_S - \vec{r}_{\Sigma 1}|) d\sigma,$$

In case of the multiple scattering by a system of M perfect conducting bands we have a system of M currents integral equations of the first kind with singular kernels. For Dirichlet problem (2) it could be written as follow

$$u_0(\vec{r}_i) = \sum_{j=1}^{M} \int_S \mu_j(\vec{t}_j) H_0^{(2)}(k \,|\, \vec{r}_i - \vec{t}_j \,|\, d\tau_j\,, \qquad (9)$$

$\vec{r}_i \in S_i ; \vec{t}_j \in S \equiv S_j$; $i = 1,...,M$.

In (6)-(9) $\mu(\vec{r}_\Sigma), \mu_1(\vec{r}_\Sigma)$ - allocated on Σ and $\Sigma 1$ an auxiliary currents; $\Sigma, \Sigma 1$ - auxiliary closed contours within S and outside of S respectively; $\vec{r}_\Sigma$ - is the radius-vector of the integration's points on auxiliary contour Σ, $\vec{r}_S$ - is the radius-vector of the points on contour S; $d\sigma$ - is the element's length of shaft-bow on Σ or $\Sigma 1$; $|\, \vec{r}_S - \vec{r}_\Sigma \,| = [r^2(\alpha) + \rho^2(\theta) = 2r(\alpha)\rho(\theta)\cos(\alpha - \theta)]^{1/2}$ is a distance between points with $\vec{r}_\Sigma$ and $\vec{r}_S$; $r(\alpha)$ - is the equation of the contour S and $\rho(\theta)$ - is the equation of the contour Σ inside S in cylindrical coordinate system, $\rho_1(\theta)$ - is the equation of the contour $\Sigma 1$ outside of S in cylindrical coordinate system; $|\, \vec{r}_S - \vec{r}_{\Sigma 1} \,| = [r^2(\alpha) + \rho_1^2(\theta) = 2r(\alpha)\rho_1(\theta)\cos(\alpha - \theta)]^{1/2}$ is a distance between points with $\vec{r}_S$ and $\vec{r}_{\Sigma 1}$; $\mu_j(\vec{t}_j)$ - current on S_j; S_i is the surface of the i-th band; $\vec{t}$ is the radius-vector of the integration's points on S_j; j=1,…,M; M is the total number of bands.

One of the main problems of the method (ACIE) is the constructing of the auxiliary contours and detecting its location. We constructed these contours as a result of the analytical transformation of the original contour S:

$\zeta = \rho(\varphi)\exp\{i\varphi\}$; $\varphi = \varphi' + i\varphi''$; $r_\Sigma = |\,\zeta\,|$; $\theta = \arg\zeta$.

This sort of transformation is possible as long as the analytical transformation remain a one-to-one mapping. The points at which this one-to-one correspondence is violated and singular points of the analytical extension of the diffracted field to the region inside of the original contour constitute a set of so-called principal singularities of the diffracted field [8]. As it was shown in [3,7] for integral equations mention above to have a solution, it is sufficient for auxiliary contour to enclose these singularities. For such types of original contours as ellipse (-or multifoil): $\rho(\varphi) = a / \sqrt{1 - \varepsilon^2 \cos(\varphi)}$; $\varepsilon = a / b$; (- or $\rho(\varphi) = a + b\cos(q\varphi)$) these singularities could be calculated in analytical form [3,7,8]. For arbitrary analytical original contour these singularities could be calculated by developed numerical procedure.

Another arising problems of MAC method is the applying of most effective basis function when integral equations are solved. The solution of this problem was achieved by utilizing wavelets technique. So using the wavelet technique one can obtain the presentation for $\mu(\theta)$ in form of set with Haar functions [1]

$$\mu(\theta) = c_0\phi_0(\theta) + \sum_{j=0}^{\infty} \sum_{k=0}^{2^j-1} d_{jk}\psi_{jk}(\theta)\,, \qquad (10)$$

where

$$\phi_0(\theta) = \begin{cases} 1, & 0 \le \theta < 2\pi, \\ 0, & \theta \notin [0, 2\pi[, \end{cases}$$

$$\psi_{jk}(\theta) = 2^{j/2}\psi_H(2^j\theta - 2k\pi),$$

$$\psi_H(t) = \begin{cases} 1, & 0 \le t < \pi, \\ -1, & \pi \le t < 2\pi, \\ 0, & t \notin [0, 2\pi[. \end{cases}$$

. Another basis on $0 \le \theta < 2\pi$ which we had also applied were the linear B-spline wavelets [1]

$$\psi_{jk}(\theta) = 2^{j/2}\psi_0(\theta); \qquad (11)$$

$$\psi_0(\theta) = \phi_0(2\theta - \pi) - \frac{1}{2}\phi_0(2\theta) - \frac{1}{2}\phi_0(2\theta - 2\pi);$$

$$\phi_0(\theta) = \begin{cases} 1 - \dfrac{1}{\pi}\,|\,\theta - \pi\,|, & 0 \le \theta \le 2\pi \\ 0, & \theta \notin [0, 2\pi[\end{cases}$$

Placing (9) or (10) into (6) (or into (7),(8)) one could get

$$c_0 L\phi_0 + \sum_{j=0}^{p} \sum_{k-0}^{2^j-1} d_{jk} L\psi_{jk} \approx \psi(\alpha), \qquad (12)$$

were L is a integral operator , $\psi(\alpha) = u_0(\vec{r}_S)$. Making a procedure of description with basis functions $\xi_m(\tau), m = 1,..., M1$. we have a system of linear equations which can be written in the matrix form as

$$[A_{mn}][d_n] = [b_m] \qquad (13)$$

where

$$A_{mn} = \langle \xi_m, L\Psi_{ik} \rangle, m = 1,..., M1; b_m = \langle \xi_m, \Psi \rangle. \quad j = 0,..., p; k = 0,..., 2^j - 1; n = p + 2^j \qquad (14)$$

Let as take ξ_m in form of the delta functions $\xi_m = \delta(\alpha - \alpha_m)$,where α_m are the points on contour S from the interval [0,2π] .In result we obtain a scheme for the method of point matching . When we take the Haar functions as a basis functions ξ_m we get the Galerkin method and we have to make two dimensional integrations.

The $\mu(\theta)$ function having been found from (6)-(8), the scattering pattern $g(\varphi)$ could be calculated as follows

$$g(\varphi)=\int_0^{2\pi}\exp[ik\rho_0(\theta)\cos(\theta-\varphi)]\mu(\theta)d\theta \quad . \quad (16)$$

The accuracy of the solving problems we estimate as the residual Δ of the boundary conditions. It was detected that residual Δ depends as on the number N and type of the basis functions as on the degree δ of the closeness auxiliary contour at singularities. For example, in case of dielectric multifiol with q=5; ka=10; kb=2; ε=4; (N is a total number of Haar wavelets functions) we had: max(Δ)=0.013 if N=128,$\delta=10^{-3}$ and max(Δ)=$5{,}8*10^{-4}$ if N= =128,. $\delta=10^{-4}$; max(Δ)=$5*10^{-9}$ if N=256,$\delta=10^{-4}$.

It was detected that the Haar wavelets gives much better accuracy then linear B-spline wavelets when the total number of the basis functions is the same. It was shown that the stable results in case of the point-matching method could be obtained when descriptive points are choosing not only in the middle of the intervals but in the points closed at the ends of the intervals. Described above method was applied for calculation a scattering pattern $g(\varphi)$ in high frequency region $(kD>>1)$. As example, we had calculated $g(\varphi)$ for E polarized plane incident wave $u_0=\exp\{-ikr\cos(\varphi-\varphi_0)\}$: for dielectric cylinder with elliptic cross section with kb=30. ka=10,$\varepsilon=4$, N=256,$\delta=10^{-6}$, max(Δ)=$6{,}2*10^{-7}$, $\varphi_0=\pi/2$ (kD=120); for metal cylinder with multifoil cross section and q=4, ka=30, kb=15, N=256, $\delta=10^{-5}$,$\varphi_0=0$, max(Δ)=$1{,}85*10^{-7}$, (kD=90); for dielectric cylinder with multifoil cross section and q=24, ka=8,2 , kb=1,2, N=256, $\delta=10^{-5}$,$\varphi_0=0$, max(Δ)=0,018 .

Solution of the 2D scattering problems on the base of currents integral equations of the first kind with singular kernel(9 or like it [3]) we had made as by ordinary way (with extracting singularities) as by developed methods - "prolonged" boundary conditions in complex region and applying Haar or linear Battle-Lemarie wavelets as a basis functions. On this way we had considered the scattering problem by system M-th perfect conducting bands, corner reflector and plane (or cylindrical) incident wave. We had calculated $g(\varphi)$ for one band with ka=200 (a is the band's width) when N=128, max(Δ)<10^{-3} for E polarized plane incident wave with $\varphi_0=\pi/2$; $g(\varphi)$ for corner reflector with $ka=5\lambda$ and $\varphi_0=\pi/4$ and the scattering pattern $g(\varphi)$ (see Fig.1) for M=9 perfect conducting bands located in space as a two liners array with distance between liners H=a (a – width of the one band, b – distance between neighboring bands) .

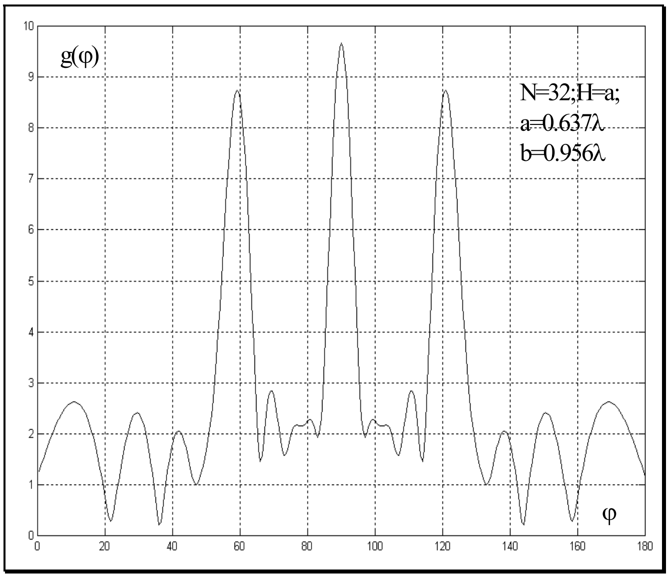

Fig.1 *Scattering pattern by the 9 bands*

3.Conclusion

The developed method allows making of the essential step into high frequency domain. It has advantages over the traditional techniques and largely extends the class of problems, which could be solved. The method can easily be extended to plane-layered media and vector fields.

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