Performance Analysis of Binary and Quaternary Modulations on a κ–μ Frequency-Nonselective Fading Channel

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Abstract – This paper presents the performance analysis of binary and quaternary modulations on a κ–μ nonselective fading channel. The κ–μ distribution is used to model a frequency-nonselective fading channel considering similar time delays for the scattered waves arriving at the receiver. The modulations analyzed are: BPSK, BFSK Coherent, BFSK Noncoherent, DPSK, QPSK, OQPSK and π/4–DQPSK. This analysis shows that BPSK, QPSK and OQPSK present a better performance than the other modulations with respect to the average bit error probability. It will be shown that for one value of the Nakagami parameter, *m*, it is possible to obtain different average bit error probability performance. Thus, it permits better fit of the experimental data to the theoretical curves, by choosing the appropriates values of the κ–μ parameters.

Key Words – κ–μ distribution nonselective fading, average error probability, digital modulation.
Introduction

1 Introduction

In the analysis of radio propagation for cellular mobile communications, many effects are responsible to affect the received signal strength like terrestrial path loss, diffraction and multipath. One of the most important effects, multipath fading, is modeled on a statistical basis using mathematical formulas such as Nakagami-m, Rice and Rayleigh distributions [1]. Recently, a new statistical model, the κ–μ distribution, has been published [2]. This new distribution has a better performance than the commonly used distributions when it is applied to fit experimental data. It results on a more accurate modeling of the mobile radio propagation channel. Besides the increasing in the modeling accuracy, the $\kappa-\mu$ is a general fading distribution the includes as especial cases Nakagami-m, Rice, One-Sided Gaussian and Rayleigh distribution [2]. The lognormal distribution can also be very well approximated by the $\kappa-\mu$ distribution.

In order to evaluate the usefulness of the $\kappa-\mu$ distribution for the radio channel modeling this work presents expressions, not in a closed-form, for the average error probability of different modulations schemes such as BPSK, coherent and noncoherent BFSK, DPSK, QPSK, OQPSK and $\pi/4$ DQPSK. The equations obtained allow us to plot average error probability (P_e) versus SNR (signal-to-noise ratio) curves for each modulation. These curves can be analyzed to find out which modulation scheme has the best performance under a $\kappa-\mu$ fading channel. This paper is organized as follows. In Section 1 we present a summary of the $\kappa-\mu$ distribution [2].

In Section 2, the expressions for the average error probability (P_e) are derived curves for one. In Section 3 we perform an analysis to find which modulation has a better performance. Finally, in Section 4 the conclusions of this work are shown.

2 The κ–μ Distribution

The $\kappa - \mu$ distribution model is obtained based on the assumption of a non-homogeneous environment composed of clusters of multipath waves propagating in an homogeneous environment. For each cluster is assumed the existence of a particular dominant component with an arbitrary power, and a sum of scattered waves with random phases and similar time delays. As derived in [2], the $\kappa - \mu$ distribution is given by:

$$p(r;\Omega,\mu,\kappa) = \frac{2\mu(1+\kappa)^{\frac{\mu+1}{2}}}{\sqrt{\Omega}\kappa^{\frac{\mu-1}{2}}\, exp(\ \mu\kappa\ )} \left(\frac{r}{\sqrt{\Omega}}\right)^{\mu} exp\left(-\mu(1+\kappa)\left(\frac{r}{\sqrt{\Omega}}\right)^{2}\right)\times I_{\mu-1}\left(2\mu\sqrt{\kappa(1+\kappa)}\left(\frac{r}{\sqrt{\Omega}}\right)\right) \quad (1)$$

where $\Omega = E(r^2)$ is the average power, $I_v(\bullet)$ is the modified Bessel function of first kind and arbitrary order v (v real), κ is the ratio between the total power of the dominant components and the total power of the scattered waves, and the variable μ is given by [2]

$$\mu = \frac{E^2(r^2)}{Var(r^2)} \times \frac{1+2\kappa}{(1+\kappa)^2} \quad (2)$$

where

$$\frac{\mu(1+\kappa)^2}{1+2\kappa} \geq \frac{1}{2} \quad (3)$$

Note that from the κ–μ distribution we can derive such as other distributions through simple transformations, like the Rice distribution is obtained by setting $\mu = 1$ in Equation 1. The Rice statistics generates the Rayleigh distribution when $\kappa \to 0$ and the Nakagami-*m* distribution is obtained from Rice setting $\kappa = m - 1 + \sqrt{m(m-1)}$, where m is the Nakagami parameter.

3 Average Error probability for the κ–μ Distribution

In this section, we derive the error bit rate performance of binary (PSK, FSK Coherent, FSK Noncoherent, DPSK) and quaternary (QPSK, OQPSK, $\pi/4$-DQPSK) modulations when signals are transmitted over a frequency-nonselective fading channel modeled by a $\kappa - \mu$ distribution. Let us assume that the channel fading is sufficiently slow that the phase shift can be estimated from the received signal without error. In that case, we can achieve ideal coherent detection of the received signals. Thus, the received signal can be processed passing it through a matched filter in case of BPSK, QPSK, OQPS, BFSK Coherent and $\pi/4$-DQPSK [3]. One method employed to determine the performance of a mobile radio communication system is to evaluate the probabilities of error on a time-invariant channel and from these determine referred error probability. For instance, the error rate expression of binary FSK as a function of the received SNR (γ) on a time-invariant channel is given by [3]

$$P_{BFSK}(\gamma) = Q(\sqrt{\gamma}\,) \quad (4)$$

where $\gamma = \alpha^2 E_b/N_0$ is the signal-to-noise ratio for a particular value of α, and $Q(x)$ can be expressed in terms of complementary error function as follows:

$$Q(x) = \frac{1}{2} erfc\left(\frac{x}{\sqrt{2}}\right) \quad (5)$$

The $P_{BFSK}(\gamma)$ probability it is seen as being a conditional probability, where the condition is that α is fixed. To obtain the error probability when α is a random variable, we must average $P_{BFSK}(\gamma)$, over the probability density function of γ, such as

$$P_e = \int_0^{\infty} P_{BFSK}(\gamma)p(\gamma)d\gamma \quad (6)$$

where $p(\gamma)$ is the probability density function of γ when α is a random variable.

In our analysis the use of $p(\gamma)$ is necessary since it represents the average error probability as function of the average signal-to-noise ratio received. Through a transformation of random variable from Equation (1) we have

$$p(\gamma;\bar{\gamma},\mu,\kappa)=\frac{\mu}{exp(\mu\kappa)}\left(\frac{\gamma}{\kappa}\right)^{\frac{\mu-1}{2}}\left(\frac{1+\kappa}{\bar{\gamma}}\right)^{\frac{\mu+1}{2}}\times$$
$$exp\left(-\mu(1+\kappa)\frac{\gamma}{\bar{\gamma}}\right)I_{\mu-1}\left(2\mu\sqrt{\kappa(1+\kappa)\frac{\gamma}{\bar{\gamma}}}\right) \tag{7}$$

where $\bar{\gamma}=\Omega E_b/N_0$ is the average signal-to-noise ratio of γ when γ is $\kappa-\mu$ distributed.

For the $\kappa-\mu$ distribution is necessary to evaluate the integral

$$P_e=\int_0^{\infty}P_e(\gamma)p(\gamma;\bar{\gamma},\mu,\kappa)d\gamma \tag{8}$$

where P_e is the average bit or symbol error probability, $p(\gamma;\bar{\gamma},\mu,\kappa)$ is defined by the Equation 7 and $P_e(\gamma)$ is defined in Table 1 for the different types of modulation.

Table 1 - Average error probability for different types of modulation in the time invariant channel corrupted by an AWGN noise.

Modulation	**$P_e(\gamma)$**
BPSK, QPSK, OQPSK	$Q(\sqrt{2\gamma})$
BFSK Coherent	$Q(\sqrt{\gamma})$
BFSK No Coherent	$\exp(-\gamma/2)/2$
DPSK	$\exp(-\gamma)/2$
$\pi/4$-DQPSK	$Q(\sqrt{4\gamma}\,\text{sen}(\pi/4\sqrt{2}))$

Substituting the expressions of Table 1, as well as the Equation (7) in Equation (8), we find the Equations (9) to (13). These integrals were evaluated numerically since we could not find an analytical closed-form. The average error probabilities curves are presented in the Figures 1 to 10. The graphics were depicted for the parameters κ, μ and m obtained from [2], which are listed on Table 2 and Table 3.

In each graphic the curves are in accordance with Equations (9) to (13). The curves for the case where $\kappa\to 0$ matches with the curves of Nakagami-m when $\mu=m$. The curve where μ=1 coincides with the curve of Rice. It is important to note that the value of the parameter μ must obey the following relation [2]

$$0\le\mu\le m \tag{14}$$

Table 2– Values of $\kappa-\mu$ and m=1.5 from [2].

	Curves - Figures 1, 2, 3, 7, 8						
	A	**B**	**C**	**D**	**E**	**F**	**G**
κ	0.01	0.69	1.37	2.41	4.45	10.48	28.49
μ	1.50	1.25	1.00	0.75	0.50	0.25	0.100

Table 3– Values of $\kappa-\mu$ and $m=0.75$ from [2].

	Curves - Figures 4, 5, 6, 9, 10						
	H	**I**	**J**	**L**	**M**	**N**	**O**
κ	0.01	0.69	1.37	2.41	4.45	10.48	28.49
μ	0.75	0.625	0.500	0.375	0.250	0.125	0.050

4 Results

By comparing the results of Figures 1 to 10, we can note that BPSK, QPSK and OQPSK present a better average bit error probability performance than the

$$P_e(BPSK)=\int_0^{\infty}\frac{1}{\sqrt{2\pi}}\int_{\sqrt{2\gamma}}^{\infty}exp\left(\frac{-x^2}{2}\right)dx\,\frac{\mu}{exp(\mu\kappa)}\left(\frac{\gamma}{\kappa}\right)^{\frac{\mu-1}{2}}\left(\frac{1+\kappa}{\bar{\gamma}}\right)^{\frac{\mu+1}{2}}exp\left(-\mu(1+\kappa)\frac{\gamma}{\bar{\gamma}}\right)I_{\mu-1}\left(2\mu\sqrt{\kappa(1+\kappa)\frac{\gamma}{\bar{\gamma}}}\right)d\gamma \tag{9}$$

$$P_e(BFSK-C)=\int_0^{\infty}\frac{1}{\sqrt{2\pi}}\int_{\sqrt{\gamma}}^{\infty}exp\left(\frac{-x^2}{2}\right)dx\,\frac{\mu}{exp(\mu\kappa)}\left(\frac{\gamma}{\kappa}\right)^{\frac{\mu-1}{2}}\left(\frac{1+\kappa}{\bar{\gamma}}\right)^{\frac{\mu+1}{2}}exp\left(-\mu(1+\kappa)\frac{\gamma}{\bar{\gamma}}\right)I_{\mu-1}\left(2\mu\sqrt{\kappa(1+\kappa)\frac{\gamma}{\bar{\gamma}}}\right)d\gamma \tag{10}$$

$$P_e(BFSK-NC)=\int_0^{\infty}\frac{1}{2}\frac{\mu}{exp(\mu\kappa)}\left(\frac{\gamma}{\kappa}\right)^{\frac{\mu-1}{2}}\left(\frac{1+\kappa}{\bar{\gamma}}\right)^{\frac{\mu+1}{2}}exp\left(\left(-\mu(1+\kappa)+2\bar{\gamma}\right)\frac{\gamma}{2\bar{\gamma}}\right)I_{\mu-1}\left(2\mu\sqrt{\kappa(1+\kappa)\frac{\gamma}{\bar{\gamma}}}\right)d\gamma \tag{11}$$

$$P_e(DPSK)=\int_0^{\infty}\frac{1}{2}\frac{\mu}{exp(\mu\kappa)}\left(\frac{\gamma}{\kappa}\right)^{\frac{\mu-1}{2}}\left(\frac{1+\kappa}{\bar{\gamma}}\right)^{\frac{\mu+1}{2}}exp\left(\left(-\mu(1+\kappa)+\bar{\gamma}\right)\frac{\gamma}{\bar{\gamma}}\right)I_{\mu-1}\left(2\mu\sqrt{\kappa(1+\kappa)\frac{\gamma}{\bar{\gamma}}}\right)d\gamma \tag{12}$$

$$P_e(\pi/4)=\int_0^{\infty}\frac{1}{\sqrt{2\pi}}\int_{\sqrt{4\gamma}\,sin(\pi/4\sqrt{2})}^{\infty}exp\left(\frac{-x^2}{2}\right)dx\,\frac{\mu}{exp(\mu\kappa)}\left(\frac{\gamma}{\kappa}\right)^{\frac{\mu-1}{2}}\left(\frac{1+\kappa}{\bar{\gamma}}\right)^{\frac{\mu+1}{2}}exp\left(-\mu(1+\kappa)\frac{\gamma}{\bar{\gamma}}\right)I_{\mu-1}\left(2\mu\sqrt{\kappa(1+\kappa)\frac{\gamma}{\bar{\gamma}}}\right)d\gamma \tag{13}$$

other modulation studied. Note that, the QPSK modulation, which is adopted in IS-95 standard (CdmaOne), presents better performance than $\pi/4$–DQPSK modulation used in IS-136 (US TDMA). We can also see that, for any given error probability, the average SNR required for BFSK-C signals is 3 dB greater than that for BPSK for any values of κ and μ.

5 Conclusion

This paper presented the performance analysis of binary and quaternary modulations on a $\kappa-\mu$ nonselective fading channel. The $\kappa-\mu$ distribution is used to model a frequency nonselective fading channel. The modulations analyzed in this paper are: BPSK, BFSK Coherent, BFSK noncoherent, DPSK, QPSK, OQPSK and $\pi/4$-DQPSK. The analysis shows that BPSK, QPSK and OQPSK modulations present a better average bit error probability performance when compared to the other modulations schemes studied. An important conclusion, obtained from simulations, is that for one value of the Nakagami parameter, m, it is possible to obtain different average bit error probability performance. Thus, it permits better fit of the experimental data to the theorical curves, by choosing the appropriates values of the $\kappa-\mu$ parameters. For future works in the performance analysis of modulations schemes in a $\kappa-\mu$ channel, we will investigate the influence of using diversity techniques.

References

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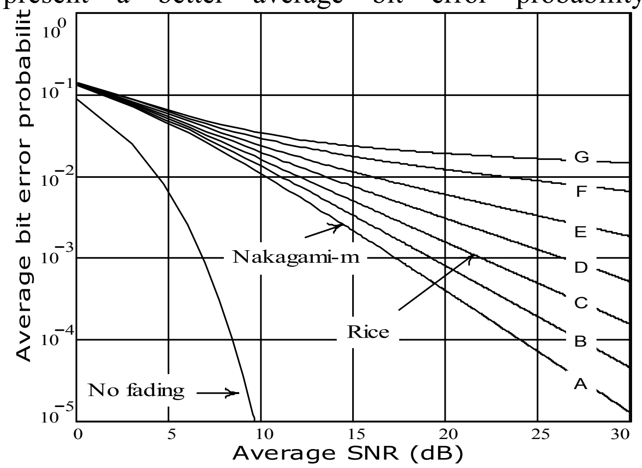

Fig. 1 - Average error probability for BPSK, QPSK, OQPSK and $m = 1.5$ with $\kappa-\mu$ fading.

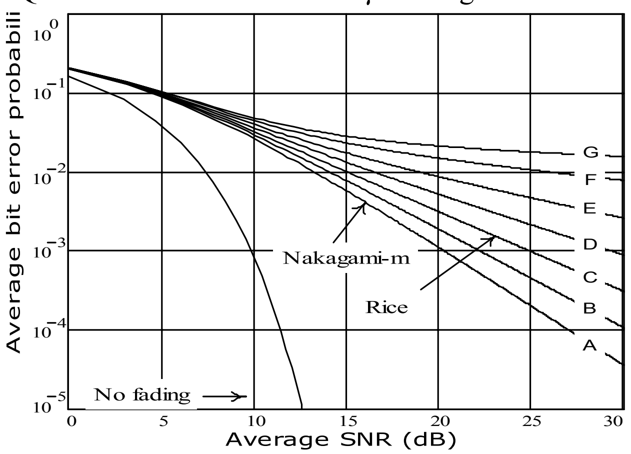

Fig. 2 - Average error probability for BFSK Coherent and $m = 1.5$ with $\kappa-\mu$ fading.

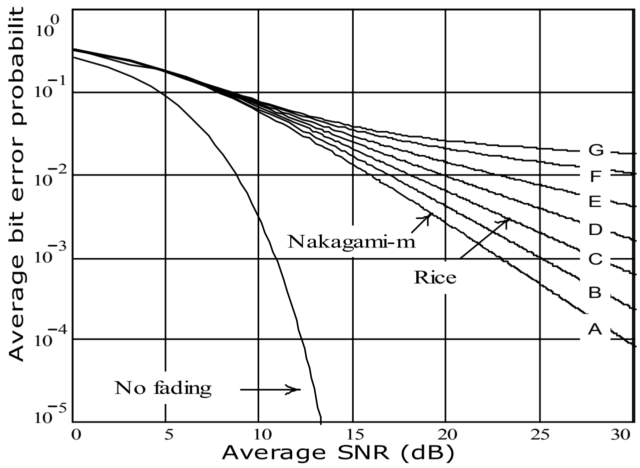

Fig. 3 - Average error probability for BFSK nonCoherent and $m = 1.5$ with $\kappa-\mu$ fading.

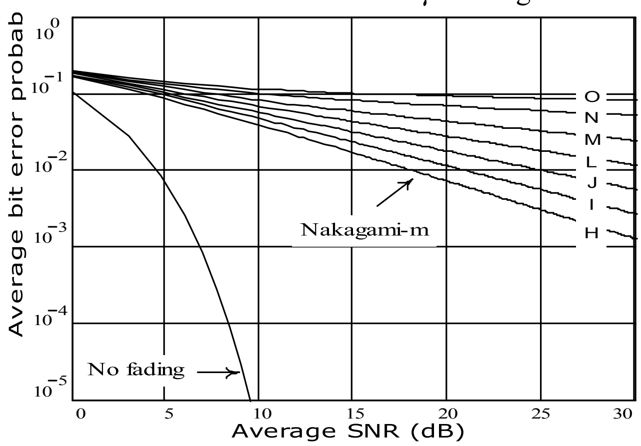

Fig. 4 - Average error probability for BPSK, QPSK, OQPSK and $m = 0.75$ with $\kappa-\mu$ fading.

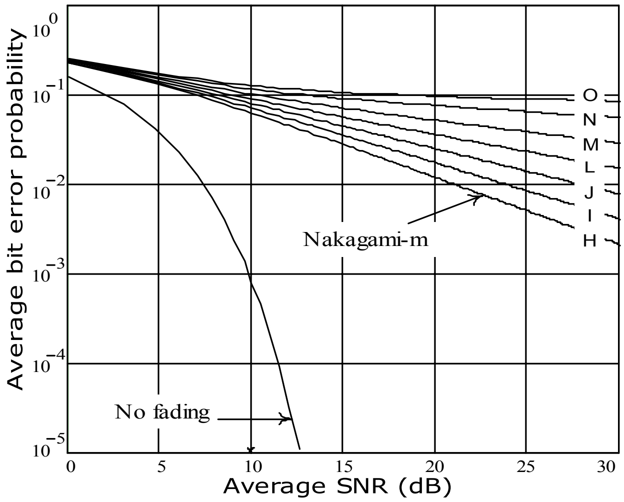

Fig. 5 - Average error probability for BFSK Coherent and $m = 0.75$ with $\kappa - \mu$ fading.

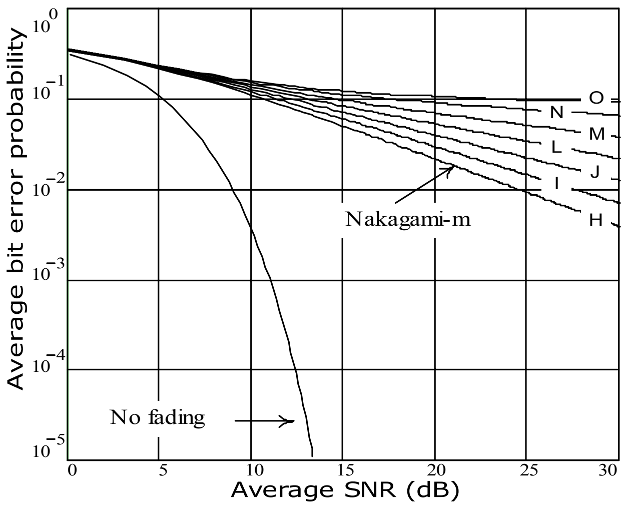

Fig. 6 - Average error probability for BFSK noncoherent and $m = 0.75$ with $\kappa - \mu$ fading.

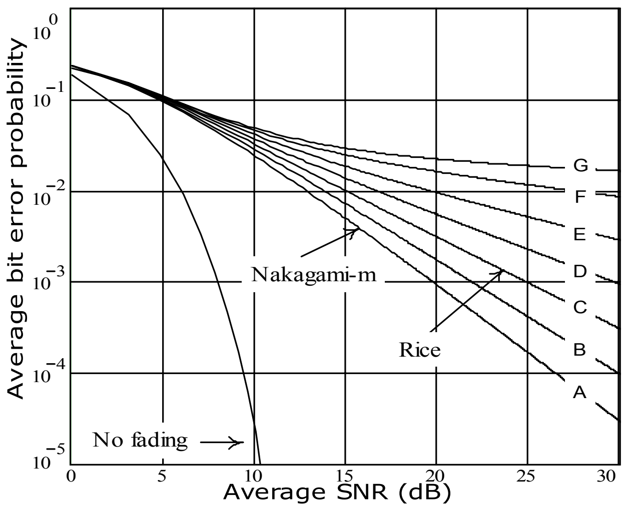

Fig. 7 - Average error probability for DPSK and $m = 1.5$ with $\kappa - \mu$ fading.

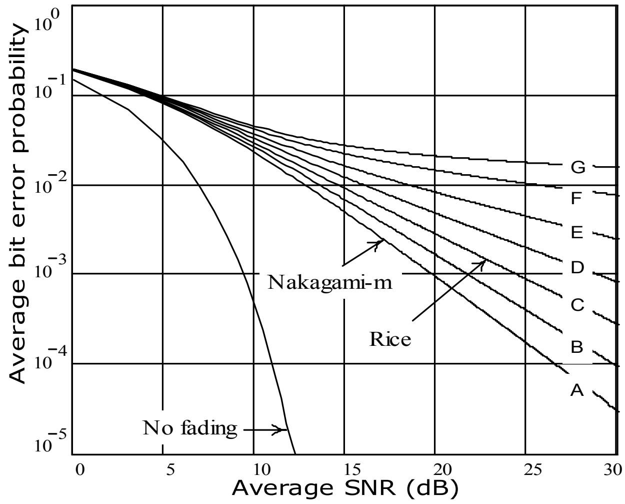

Fig. 8 - Average error probability for $\pi/4$-DQPSK com $m = 1.5$ with $\kappa - \mu$ fading.

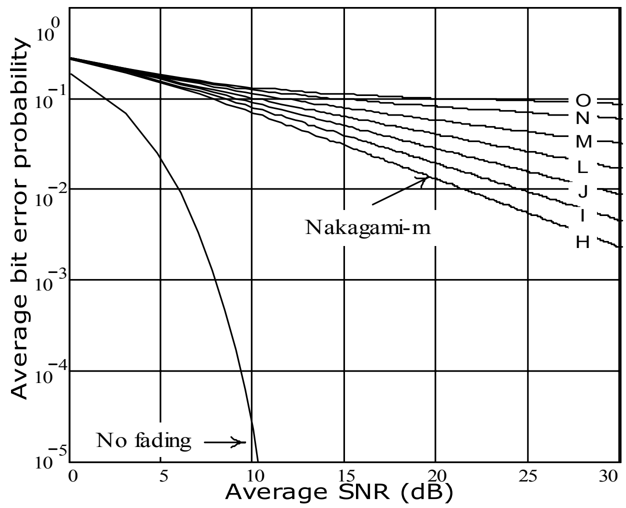

Fig. 9 - Average error probability for DPSK and $m = 0.75$ with $\kappa - \mu$ fading.

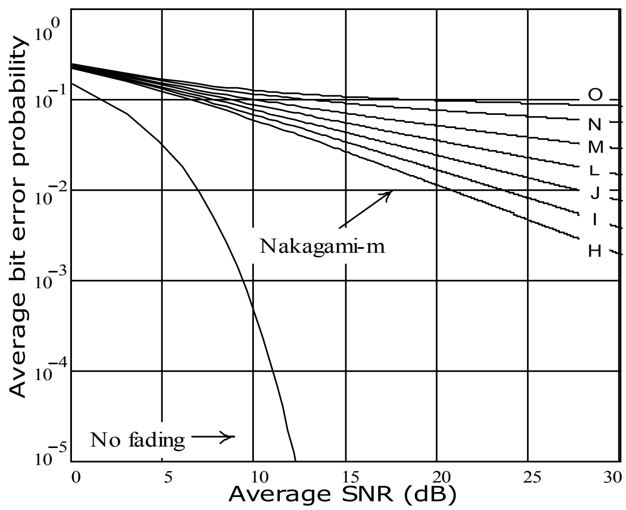

Fig. 10 - Average error probability for $\pi/4$-DQPSK and $m = 0.75$ with $\kappa - \mu$ fading.