A New Multivariable Robust Model Reference Adaptive Controller

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Abstract: This paper proposes a new multivariable model reference adaptive controller (MIMO RMRAC), which consists of two parts: the first part involves the characterization of the integral structure of the modeled part of the plant, and the associated parameterization of the controller structure; and the second part involves the development of a robust adaptive law based on a modified least-squares algorithm for adjusting the controller parameters so that the closed-loop plant is globally stable despite the presence of unmodeled dynamics and bounded disturbances.

Key-Words: Model reference adaptive control Multivariable control Robust adaptive control

1 Introduction

Adaptive continuous-time controllers for single-input single-output (SISO) plants have been developed in the literature. In last years, it has grown the amount of algorithms with robustness characteristics in the sense of stability as well as performance. In Ioannou and Tsakalis [1], a robust model reference adaptive controller (RMRAC) is presented, which has advantages over other algorithms when applied to adverse real situations [2]. This scheme requires some assumptions over the characteristics of the plant, as, for example, knowing the sign of the high frequency gain. Lozano and others [3] developed a controller that relaxes this assumption, applying a projection technique which avoids division by zero in the controller. Another advantage presented in [3] is the use of a modified least-squares algorithm, which presents faster parametric convergence characteristics than the gradient algorithm used in [1]. Other developments for SISO adaptive control are presented in [2]. In [4] it was developed an RMRAC-RP controller, where RP is a repetitive control technique mixed to the RMRAC controller.

Parallel to the SISO case, adaptive control techniques have been developed for multiple-input multiple-output (MIMO) plants. The coupling and parameterization problem is focused in literature [5]-[6], arriving to algorithms for the estimation of the interaction matrix [7]-[8].

If the multivariable plant to be controlled is weakly coupled, it is possible to develop a decentralized adaptive control, as described in [9] and [10]. In [11], it was developed a decentralized RMRAC for a three-phase uninterruptible power supply, and in [12], the same algorithm was applied to develop a three-phase AC power source. In both works it was obtained encouraging practical results. Unfortunately this technique is not effective when applied to strongly coupled multivariable plants. In this case, centralized multivariable control, which deals directly with coupling, has to be applied.

Tao and Ioannou present in [13] a MIMO RMRAC, which uses parameterization with base on the modified left interactor (MLI) matrix or the modified right interactor (MRI) matrix. This work solves the matrix commutability problem that arises in the development of a MIMO controller. The scheme developed uses a gradient adaptation algorithm for on-line update of the controller parameters, so that the closed-loop plant is globally stable despite the presence of unmodeled dynamics and bounded disturbances.

This paper presents a MIMO RMRAC, which uses, differently from [13], a modified least-squares algorithm to provide faster parameter convergence characteristics. The adaptor uses s-modification and normalization techniques. Inspired in [1] and [3], stability proofs are developed making the adequate considerations to the MIMO case. Hence, it is shown that the proposed algorithm presents robustness characteristics regarding additive and multiplicative stable plant perturbations. For limited small plant perturbations it is shown that the tracking error is small in the mean, and, in the absence of plant perturbations, tracking error tends asymptotically to zero. The controller is applicable to MIMO plants with the same number of inputs and outputs. Some of the assumptions of this scheme are the prior knowledge the MLI matrix of the modeled part of the plant and the knowledge of a lower bound of the norm of the high frequency gain matrix, which is assumed to be positive definite. Without satisfying these assumptions, the controller may become unstable due to the inversion of a singular matrix.

This paper is organized as follows. In Section 2 we present the integral structure and parameterization of a multivariable system. Plant description and the control objective are presented in Section 3. The controller structure is given in Section 4. Section 5 is devoted to the parameter adaptation algorithm and its properties. In Section 6 the robustness properties are analyzed.

2 System Integral Structure

An important concept for designing MIMO model reference control schemes is the plant integral structure, which may be characterized by the plant interactor matrix. A full description of right and left interactor matrix can be found in [13]. In this paper it will be presented only the concepts involving the modified left interactor (MLI) matrix. All developments will be made supposing the use of the MLI matrix and filtering. The use of the modified right interactor (MRI) matrix and pre-compensation is equivalent, and will not be treated in this paper.
The following lemma [13] serves to define the multivariable counterpart to high frequency gain and relative degree of MIMO plants.

Lemma 2.1. [13] For any $N \times N$ strictly proper rational full rank transfer matrix $G_0(s)$ there exists a lower triangular polynomial matrix $x_\ell^m(s)$, defined as the modified left interactor (MLI) matrix of $G_0(s)$, of the form

$$x_\ell^m(s) = \begin{bmatrix} d_{\ell 1}^m(s) & 0 & \cdots & \cdots & 0 \\ h_{\ell 21}^m(s) & d_{\ell 2}^m(s) & 0 & \cdots & 0 \\ & \cdots & \cdots & \cdots & \\ h_{\ell N1}^m(s) & \cdots & \cdots & h_{\ell N(N-1)}^m(s) & d_{\ell N}^m(s) \end{bmatrix} \quad (1)$$

where $h_{\ell ij}^m(s)$, $j = 1, \ldots, N-1$, $i = 2, \ldots, N$ are polynomials, and $d_{\ell i}^m(s) = s^{\ell_i} + a_1^i s^{\ell_i - 1} + \ldots + a_{\ell_i}^i$, $i = 1, \ldots, N$, are Hurwitz arbitrary polynomials so that $\lim_{s\to\infty} x_\ell^m(s) G_0(s) = K_{\ell p}^m$ is finite and non-singular.

Proof. The proof is presented in [13].

Remark 2.1. [13] Since the polynomials $d_{\ell i}^m(s)$ in (1) are Hurwitz, the MLI matrix $x_\ell^m(s)$ has stable inverse, and so it can be used for the design of MRC schemes.

The following lemma employs the notion of the MLI matrix to give a parameterization for the plant with which it will be possible do design MRC schemes.

Lemma 2.2. [13] The MIMO LTI plant

$$y = G_0(s) u \quad (2)$$

can be represented as

$$y = f_\ell(s)(x_\ell^m(s))^{-1} y_f \quad y_f = G_0^\ell(s) u \quad (3)$$

where $G_0^\ell(s)$ is a $N \times N$ transfer matrix whose MLI matrix is $f_\ell(s) I$, and $f_\ell(s)$ is an arbitrary Hurwitz polynomial of degree d_ℓ, and $d_\ell \geq$ the maximum degree of the elements of $x_\ell^m(s)$. Furthermore, the high-frequency gain matrix of $G_0^\ell(s)$ is equal to $K_{\ell p}^m$.

Proof. [13] Equation (2) can be written as $y = f_\ell(s)(x_\ell^m(s))^{-1}(f_\ell(s))^{-1} x_\ell^m(s) G_0(s) u$. Defining $G_0^\ell(s) = (f_\ell(s))^{-1} x_\ell^m(s) G_0(s)$ and $y_f = G_0^\ell(s) u$, and noting that $f_\ell(s) G_0^\ell(s) = x_\ell^m(s) G_0(s)$, the proof is complete.

3 Plant Description and Control Objective

Consider the $N \times N$ MIMO LTI plant described by

$$y = G(s) u,\ G(s) = \{G_0(s)[I + m\Delta_m(s)] + m\Delta_a(s)\} \quad (4)$$

where $y \in \mathcal{R}^N$, $u \in \mathcal{R}^N$. $G_0(s)$ is the modeled part of the plant, and $\Delta_m(s)$, $\Delta_a(s)$ are the respectively multiplicative and additive non-modeled parts of the plant. Consider that $x_\ell^m(s)$ is the modified left interactor (MLI) matrix of $G_0(s)$, and consider the following system

$$y_f = G_\ell(s) u,\ G_\ell(s) = \{G_0^\ell(s)[I + m\Delta_m^\ell(s)] + m\Delta_a^\ell(s)\} \quad (5)$$

where $y_f = (f_\ell(s))^{-1} x_\ell^m(s) y$, $y = f_\ell(s)(x_\ell^m(s))^{-1} y_f$, $G_0^\ell(s) = f_\ell^{-1}(s) x_\ell^m(s) G_0(s)$, $\Delta_m^\ell(s) = \Delta_m(s)$ and $\Delta_a^\ell(s) = f_\ell^{-1}(s) x_\ell^m(s) \Delta_a(s)$.
The constant $\bar{n}_0$ is a known higher bound for the observability index of $G_0(s)$, and $\bar{n}_\ell$ is a known higher bound for the observability index of $G_0^\ell(s)$.

We can now state the control objective as follows:
Given the reference model

$$y_m = W_m(s) r \quad \text{(or equivalently } y_{mf} = W_m^\ell(s) r\text{)} \quad (6)$$

where $W_m(s)$ (or $W_m^\ell(s) = f_\ell^{-1}(s) x_\ell^m(s) W_m(s)$) is an $N \times N$ strictly proper stable minimum phase transfer matrix to be selected, and $r \in \mathcal{R}^N$ is a known uniformly bounded and piecewise continuous input reference signal, find in (4) (or (5)) the control input $u \in \mathcal{R}^N$ so that the output $y \in \mathcal{R}^N$ (or y_f) follows $y_m \in \mathcal{R}^N$ (or $y_{mf} = f_\ell^{-1}(s) x_\ell^m(s) y_m$) in (6) as close as possible, and all signals in the closed-loop plant are uniformly bounded for any bounded initial conditions.

In order to satisfy the control objective, it is necessary that the plant and the reference model satisfy the following assumptions:

A1. $G(s)$ is strictly proper and full rank.

A2. $G_0(s)$ is strictly proper, non-singular, it has stable zeros, and its MLI $x_\ell^m(s)$ is known.

A3. $\Delta_m(s)$ and $\Delta_a(s)$ are rational transfer matrices, and $\Delta_a(s)$ is strictly proper.

A4. Let $f_\ell(s)$ be a monic Hurwitz polynomial with degree d_ℓ, which is the maximum degree of $x_\ell^m(s)$, and let $f_\ell(s)$ have all its roots in $\mathrm{Re}[s] < -p_0$, and define

$$D_a = \lim_{s\to\infty} \Delta_a(s)\, s \;,\quad D_m = \lim_{s\to\infty} \frac{1}{f_\ell(s)} \Delta_m(s)\, s \;, \tag{7}$$

then there exists constants k_a, $k_m > 0$ so that

$$\|D_a\| < k_a \;,\quad \|D_m\| < k_m \tag{8}$$

$$\left\| \big(\Delta_a(s-p_0) - D_a\big)(s+p) \right\|_\infty < k_a \tag{9}$$

$$\left\| \left(\frac{1}{f_\ell(s-p_0)} \Delta_m(s-p_0) - D_m \right)(s+p) \right\|_\infty < k_m \tag{10}$$

for some $p > 0$, where $\|X(s)\|_\infty \triangleq \sup_{w\in\mathcal{R}} \|X(jw)\|$.

A5. An upper bound $\bar{n}_\ell$ for the observability index n_ℓ of $f_\ell^{-1}(s) x_\ell^m(s) G_0(s)$ is known.

A6. A matrix K_ℓ is known so that $K_\ell K_{\ell p}^m$ is positive definite, where $K_{\ell p}^m$ is the high frequency gain matrix of $G_0(s)$ associated to the MLI matrix $x_\ell^m(s)$.

A7. A lower bound r for $\| K_{\ell p}^m \|$ is known.

A8. An upper bound M_0 for $\| q^* \|$ is known, so that $\| q^* \| + d_3 \le M_0$ for some $d_3 > 0$, where q^* is the desired parameter matrix of the controller.

A9. The reference model $W_m(s)$ has all its poles and zeros stable, and it is chosen so that $f_\ell(s) W_m(s)$ is proper. Without loss of generality, we can choose $W_m(s) = (x_\ell^m(s))^{-1}$.

4 Controller Structure

The control input is computed from:

$$\begin{gathered} q^T w + x_\ell^m(s) W_m(s) r = 0 \\ q_1^T w_1 + q_2^T w_{2f} + q_3^T y_f + q_4 u + x_\ell^m(s) W_m(s) r = 0 \end{gathered} \tag{11}$$

where $q = [q_1^T, q_2^T, q_3^T, q_4]^T$ and $w = [w_1^T, w_{2f}^T, y_f^T, u^T]^T$. The order of the filters is $M = (\bar{n}_\ell - 1)N$, and they can be represented by

$$w_1 = \Lambda^{-1}(s)\, A(s)\, u \qquad w_{2f} = \Lambda^{-1}(s)\, A(s)\, y_f \tag{12}$$

where $\Lambda(s)$ is an arbitrary Hurwitz polynomial with degree $(\bar{n}_\ell - 1)$, and $A(s) = [Is^{\bar{n}_\ell - 2}, Is^{\bar{n}_\ell - 3}, \cdots, Is, I]^T$ is a polynomial $(M\times N)$ matrix, and $q_1 = [q_{11}, \cdots, q_{1(\bar{n}_\ell - 1)}]^T$, $q_2 = [q_{21}, \cdots, q_{2(\bar{n}_\ell - 1)}]^T$; $q_3, q_{ij} \in \mathcal{R}^{N\times N}$. We have also that q_1, $q_2 \in R^{N\times M}$; $w_1, w_{2f} \in R^{M\times 1}$; $q \in \mathcal{R}^{N\times p}$ and $w \in \mathcal{R}^{p\times 1}$, where $p = 2M + 2N = 2\bar{n}_\ell N$. If we write

$$f = q - q^* \tag{13}$$

where $q^* = [q_1^{*T}, q_2^{*T}, q_3^{*T}, q_4^{*}]^T$ have the same dimensions as q, then (11) can be written as

$$f^T w + x_\ell^m(s)\, W_m(s)\, r = \left[-q_4^* - F_1(s) - F_2(s)\, G_\ell(s) \right] u \tag{14}$$

where $F_1(s)$ and $F_2(s)$ are $(N\times N)$ matrices:

$$F_1(s) \triangleq q_1^{*T} \frac{A(s)}{\Lambda(s)}, \quad F_2(s) \triangleq q_2^{*T} \frac{A(s)}{\Lambda(s)} + q_3^{*T}. \tag{15}$$

Lemma 4.1. Combining (4)-(6) and (11)-(15), the filtered tracking error can be expressed as

$$e_f \triangleq y_f - y_{mf} = f_\ell^{-1}(s)\, f^T w + mh \tag{16}$$

with

$$h = \Delta_\ell(s) u \tag{17}$$

where $\Delta_\ell(s)$ is a strictly proper transfer matrix.

Proof. Considering (14) and (15), in view of the controllability of the modeled part of the plant, there exists a vector q^*, with $q_4^{*T} = -K_{\ell p}^m$, such that

$$\left[-q_4^* - q_1^{*T} \frac{A(s)}{\Lambda(s)} - q_2^{*T} \frac{A(s)}{\Lambda(s)} G_0^\ell(s) - q_3^{*T} G_0^\ell(s) \right] = f_\ell(s)\, G_0^\ell(s) \tag{18}$$

Using (14) and (18), (5) can be rewritten as

$$y_f = f_\ell^{-1}(s) \left[f^T w + x_\ell^m(s) W_m(s) r \right] + m\Delta_\ell(s) u \tag{19}$$

where (20)

$$\Delta(s) = f_\ell^{-1}(s)[-q_4^* - F_1(s)]\Delta_m^\ell(s) + m[I + f_\ell^{-1}(s) F_2(s)]\Delta_a^\ell(s).$$

Thus, $\Delta_\ell(s)$ is a strictly proper transfer matrix. Equations (16) and (17) are obtained from (6) and (19).

Finally, define the filtered augmented error

$$\boldsymbol{e}_f \triangleq e_f + \left[q^T f_\ell^{-1}(s) w - f_\ell^{-1}(s) q^T w \right] = f^T z + mh \tag{21}$$

with $\boldsymbol{e}_f \in R^{N\times 1}$, and where

$$z = f_\ell^{-1}(s) w\,. \tag{22}$$

5 Parameter Adaptation Algorithm

Consider the following modified least-squares algorithm

$$\dot{f} = \dot{q} = -s P q - \frac{P z e_f^T}{m^2} \quad (23)$$

$$\dot{P} = -\frac{P z z^T P}{m^2} + \left(l P - \frac{P^2}{R^2} \right) \overline{m}^2 \quad (24)$$

where $P = P^T$ is a $(M \times M)$ matrix so that

$$0 < P(0) < l R^2 I, \qquad m^2 \le k_m \overline{m}^2 \quad (25)$$

and

$$\dot{m} = -d_0 m + d_1 (\| u \| + \| y_f \| + 1), \quad m(0) \ge d_1 / d_0 \quad (26)$$

where l, $\overline{m}$, R^2, d_0 and d_1 are positive constants and d_0 satisfies

$$d_0 + d_2 \le \min[p_0, q_0] \quad (27)$$

where $q_0 > 0$ is such that the poles of $W_m(s - q_0)$ and $\Lambda(s - q_0)$ are stable and d_2 is a positive constant. $p_0 > 0$ is defined in assumption A4, and s in (23) is given by

$$s = \begin{cases} 0 & \text{if} \quad \| q \| < M_0 \\ s_0 \left(\frac{\| q \|}{M_0} - 1 \right) & \text{if} \quad M_0 \le \| q \| \le 2M_0 \\ s_0 & \text{if} \quad \| q \| > 2M_0 \end{cases} \quad (28)$$

where $M_0 > \| q^* \|$ (Assumption A9), and $s_0 > 2\overline{m}^2 / R^2$ are project parameters.

The following lemma gives an important property to the normalizing signal $m(t)$ which is necessary in the stability analysis, and the proof is similar to that presented to the SISO case in [1].

Lemma 5.1. Consider the system

$$z = W(s) U \quad (29)$$

where $z, U \in R^{N \times 1}$ and $W(s)$ is an $(N \times N)$ stable and strictly proper transfer matrix, whose poles p_j satisfy

$$d_0 + d_2 \le \min_j \left| \mathrm{Re}(p_j) \right| \quad (30)$$

and $\| U(t) \| \le d_4 m(t)$ for some $d_4 > 0$ $(\| U(t) \| \le \| u(t) \| + \| y(t) \| + m(t))$ $\forall t \ge 0$. Then there exists a constant $c_1 > 0$ such that

$$\frac{\| z(t) \|}{m(t)} \le c_1 + e_t \quad (31)$$

where e_t is a term which depends on the initial conditions and decays exponentially to zero with a rate at least as fast as $e^{(-d_0 t)}$.

Now we can establish the following lemma, which generalizes to the MIMO case the lemma stated in [3].

Lemma 5.2. The parameter adaptation algorithm in (23)-(28) and (21) subject to the Assumptions A2, A4 and A9, has the following properties

1) $$I / l R^2 \le P^{-1} \le I \left(1/ l R^2 + g_3^2 / \overline{m}^2 \right) \quad (32)$$

where g_3 is the upper bound for $\| z \| / m$.

2) $$s \ \mathrm{tr}\left(f^T q \right) \ge 0 . \quad (33)$$

3) $$V = (1/2) \mathrm{tr}\left(f^T P^{-1} f \right) \le \overline{V} \triangleq \begin{cases} 2 \max\left(\frac{2 k_m g_5^2}{l}, \frac{9 M_0^2}{l R^2} \right) & \text{for} \quad \overline{m} > 0 \\ V(0) & \text{for} \quad \overline{m} = 0 \end{cases} \quad (34)$$

where g_5 is the upper bound for $\| h \| / m$.

4) $$\| f \| \le k_f \triangleq 2 l R^2 \overline{V} . \quad (35)$$

5) $$\frac{1}{T} \int_{t_0}^{t_0 + T} \left(\frac{\| f^T z \|^2}{m^2} + s \ \mathrm{tr}\left(f^T q \right) \right) dt \le \frac{g_1}{T} + m^2 g_2 , \quad \forall\, t_0 \ge 0, \ T > 0 . \quad (36)$$

6) $$\frac{1}{T} \int_{t_0}^{t_0 + T} \frac{(p_j^T z)^2}{m^2} dt \le \frac{g'_1}{T} + \overline{m}^2 g'_2 , \quad \forall\, t_0 \ge 0, \ T > 0, \ j = 1, \ldots, M \quad (37)$$

where g_1, g_2, g'_1 e g'_2 are positive constants and p_j is the *j*-th line of *P*.

Proof. The proofs of 1), 4), 5) and 6) are similar to that presented in [3] and will be omitted.

2) Using matrix theory, we can define $\mathrm{tr}(A^T A) = \| A \|^2$ for every matrix *A*. Also, using the property $\mathrm{tr}(f^T q) = \mathrm{tr}(q^T f)$ we can arrive to

$$2 s \ \mathrm{tr}\left(f^T q \right) \ge s \left(\| f \|^2 - \| q^* \|^2 \right) \quad (38)$$

Similarly, using (28) and Assumption A8,

$$2 s \ \mathrm{tr}\left(f^T q \right) \ge s \left(\| q \|^2 - \| q^* \|^2 \right) \ge 0 . \quad (39)$$

3) Define the positive definite function

$$V = (1/2) \mathrm{tr}(f^T P^{-1} f) \quad (40)$$

Using (21), (24), the time derivative of *V* along (23) is

$$\dot{V} = -s \ \mathrm{tr}\left(f^T q \right) - \frac{1}{4} \frac{\| f^T z \|^2}{m^2} - \frac{1}{4} \left\| \frac{f^T z}{m} + \frac{2 m h}{m} \right\|^2 + \frac{m^2 \| h \|^2}{m^2} - \overline{m}^2 l V + \frac{\overline{m}^2 \| f \|^2}{2 R^2} \quad (41)$$

$$\dot{V} \le -s \ \mathrm{tr}\left(f^T q \right) - \overline{m}^2 l V + m^2 \frac{\| h \|^2}{m^2} + \frac{\overline{m}^2 \| f \|^2}{2 R^2} \quad (42)$$

From result of (28) and (38) we have for $\| f \| \ge 3 M_0$, $s \ \mathrm{tr}(f^T q) - \overline{m}^2 \| f \|^2 / 2 R^2 \ge 0$. Therefore, from (25) and (42) it follows that $\dot{V} \le 0$ for $V \ge \overline{V}$ with $\overline{V}$ as in (34). Thus, *V* is bounded by $\overline{V}$.

6 Stability Analysis

Consider the following non-minimal state-space representation for (16), which has order $N_c = \bar{n}_\ell + 2(\bar{n}_\ell - 1)N$. It can be obtained using similar considerations to that made in [3] for the SISO case.

$$\dot{e}_{cf} = A_c e_{cf} + B_c (f^T w) + m\, B_{c1} h_1 + m\, B_{c2} h_2 \quad (43)$$

$$e_f = C_c e_{cf} + m h \quad (44)$$

where A_c is a stable matrix, $h_1 = \Delta_a^\ell(s)u$ and $h_2 = \bar{\Delta}_m^\ell(s)u$, $\bar{\Delta}_m^\ell(s)$ is a proper matrix, and the poles of $\bar{\Delta}_m^\ell(s - p_0)$ are stable.

To analyze (43) and (44), consider the positive definite function

$$W = k_1 e_{cf}^T \bar{P}\, e_{cf} + \frac{m^2}{2} \quad (45)$$

where $k_1 > 0$ is an arbitrary constant and $\bar{P} = \bar{P}^T > 0$ satisfies

$$\bar{P} A_c + A_c^T \bar{P} = -I\,. \quad (46)$$

Lemma 6.1. The time derivative of W in (45) satisfies

$$\dot{W} \le -b\, W_0 + b_3 \frac{\| f^T w \|}{m} W + b_4 \quad (47)$$

for each $m \in [0, m_0]$, where b, b_3, b_4 e m_0 are positive constants.

The proof of this lemma is similar to the proof of Lemma 5.1 in [3] and will be omitted.

Define W_0 as

$$\dot{W}_0 = -b\, W_0 + b_3 \frac{\| f^T w \|}{m} W_0 + b_4 \quad (48)$$

with $W_0(t_0) = W(t_0)$. The homogeneous part of (48) is

$$\dot{\bar{W}}_0 = -b\, \bar{W}_0 + b_3 \frac{\| f^T w \|}{m} \bar{W}_0 \quad (49)$$

and

$$\bar{W}_0 = \bar{W}_0(t_0) \exp\left[-b(t - t_0) + b_2 \int_{t_0}^{t} \frac{\| f^T w \|}{m} dt \right],\ \forall t \ge t_0\,. \quad (50)$$

From Theorem A.1, stated in the Appendix, we have

$$\int_{t_0}^{t_0+T} \frac{\| f^T w \|}{m} dt \le \left(h_0 \sqrt{e_0} + h_1 \bar{m} + h_2 \bar{m}^2 + h_3 \frac{\bar{m}^2}{e_0^{\,2}} \right) T + \frac{h_4}{e_0^{\,2}} \quad (51)$$

where h_0 to h_4 are positive constants and $e_0 \in (0,1]$ is an arbitrary constant. Introducing (51) in (50), it follows that $\bar{W}_0 = 0$ is an exponentially stable equilibrium point if

$$b \ge b_3 \left(h_0 \sqrt{e_0} + h_1 \bar{m} + h_2 \bar{m}^2 + h_3 \frac{\bar{m}^2}{e_0^{\,2}} \right). \quad (52)$$

Let us chose e_0 such that

$0 < e_0 \le \min\left((b / 5 b_3 h_2)^2, 1 \right)$ and take

$$\bar{m} \le \bar{m}^* = \min\left(\left(\frac{b}{5 b_3 h_1} \right), \left(\frac{b}{5 b_3 h_2} \right)^{1/2}, \left(\frac{b}{5 b_3 h_3} \right)^{1/2} e_0\,, m_0 \right). \quad (53)$$

If $m \le k_m \bar{m} \le k_m \bar{m}^*$, then

$$\bar{W}_0(t) \le \bar{W}_0(t_0) \exp\left[\frac{h_4}{e_0^{\,2}} \right] \exp\left[-\frac{b}{5}(t - t_0) \right]\ \forall t \ge t_0\,. \quad (54)$$

Hence, $\bar{W}_0 = 0$ is exponentially stable and, therefore, $W_0(t)$ is limited. Using the comparison theorem, boundedness of $W_0(t)$ implies boundedness of $W(t)$, and therefore m and e_{cf} are bounded. Boundedness of m implies that all the signals in the adaptive loop are bounded. We can now establish the main result.

Theorem 6.1. Consider the multivariable plant in (4) and its left representation in (5) using the MLI matrix in (1). Subject to the Assumptions A1-A9, the multivariable adaptive control structure in (6), (11)-(17), (21), (22) together with the parameter adaptation algorithm in (23)-(28), with $|q_4(0)| \ge r$, then $\bar{m}^* > 0$ in (53) can be computed so that for each $m \in [0, k_m \bar{m}^*)$ all the signals in the closed-loop system are bounded for any initial conditions. Furthermore, the tracking error belongs to the residual set

$$D\, e_f = \left\{ e_f :\ \lim_{T\to\infty} \sup_{T>0} \left(\frac{1}{T} \int_{t_0}^{t_0+T} \left\| e_f(t) \right\| dt \right) \le \right. \quad (55)$$
$$\left. \le \left(\bar{e} + g_1 \bar{m} + g_2 \bar{m}^2 \right) \right\}.$$

Proof. Boundedness of all the signals has already been proved. In order to prove (55), consider the following minimal state representation for (16):

$$\dot{e}_{0f} = A_m e_{0f} + B_m (f^T w) \quad (56)$$

$$e_f = C_m e_{0f} + m h \quad (57)$$

where A_m, B_m e C_m have respective dimensions $(N_m \times N_m)$, $(N_m \times N)$ and $(N \times N_m)$, with $N_m = d_\ell N$. Therefore,

$$\| e_f \| \le b_5 \| e_{0f} \| \exp[-q_0 (t - t_0)] +$$
$$+ b_6 \int_{t_0}^{t} \left(\left\| f^T w \right\| \exp[-q_0 (t - t)] \right) dt + m\, b_7 \quad (58)$$

where q_0, b_5, b_6 e b_7 are positive constants. Hence,

$$\frac{1}{T} \int_{t_0}^{t_0+T} \| e_f \| dt \le \frac{1}{T} \frac{b_5}{q_0} \| e_{0f}(t_0) \| + I_p + m\, b_7 \quad (59)$$

where

$$I_p = \frac{1}{T} b_6 \int_{t_0}^{t_0+T} \left\{ \int_{t_0}^{t} \left(\left\| f^T(t)\, w(t) \right\| \exp[-q_0 (t - t)] \right) dt \right\} dt$$
$$= \frac{1}{T} b_6 \int_{t_0}^{t_0+T} \left\{ \int_{t_0}^{t_0+T} \left(\left\| f^T(t)\, w(t) \right\| \exp[-q_0 (t - t)] \right) dt \right\} dt$$

$$I_p \le b_6 \frac{1}{T}\int_{t_0}^{t_0+T}\left(\left\|f^T(t)\,w(t)\right\|\left(\frac{1}{q_0}\right)\right)dt\,. \qquad (60)$$

Substituting (51) and (60) in (59) we obtain (55) with $\bar{e} = \left(b_6 h_0 \sqrt{e_0}\right)/q_0$, $g_1 = (b_6 h_1/q_0 + k_m\, b_7)$ and $g_2 = (b_6 h_3)/(e_0^{\,2} q_0)$.

Corollary 6.1: In the absence of modeling error (i.e., when $m = 0$) and when we choose $\bar{m} = 0$, the adaptive control algorithm considered in Theorem 6.1 guarantees boundedness of all the signals as well as convergence of the tracking error e_f to zero.

Proof. The proof is similar to that presented in [3] and will be omitted.

7 Conclusion

This paper presented a new multivariable robust model reference adaptive controller, which uses a modified left interactor matrix and filtering to deal with coupling, and applies a direct model reference controller whose parameters are updated by a modified least-squares estimator. It was shown that for small multiplicative and additive plant perturbations the tracking error is small in the mean and all the signals in the closed-loop are bounded.

A topic for future research is the use of a projection procedure to avoid singularities in the controller even without the knowledge of the high frequency gain matrix.

A possible practical application for this algorithm is the control of the electrical currents in a three-phase induction motor, which is a challenge at certain frequencies of operation due to coupling and modeling errors.

Appendix

Theorem A.1. If

$$\frac{1}{T}\int_t^{t+T}\left(\frac{\| f^T z \|^2}{m^2} + s\ \mathrm{tr}\left(f^T q\right)\right)dt \ \le\ \frac{g_1}{T} + m^2 g_2 \qquad (61)$$

and

$$\frac{1}{T}\int_t^{t+T}\frac{(p_j^{\,T} z)^2}{m^2}dt \le \frac{g'_1}{T} + \bar{m}^2 g'_2\,,\ j = 1,\ldots,M \qquad (62)$$

where g_1, g_2, g'_1 e g'_2 are positive constants, $T > 0$, $t \ge 0$, then

$$\int_t^{t+T}\frac{\|f^T w\|}{m}dt \le \left(h_0\sqrt{e_0} + h_1\bar{m} + h_2\bar{m}^2 + h_3\frac{\bar{m}^2}{e_0^{\,2}}\right)T + \frac{h_4}{e_0^{\,2}} \qquad (63)$$

where h_0 to h_4 are positive constants and $e_0 \in (0,1]$ is an arbitrary constant.

The proof is inspired in [3] and is omitted.

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