Analog Implementation of MOS-Translinear Morlet Wavelets

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Abstract.- The design of a low-voltage current-mode Morlet Wavelet, using MOS transistors in weak inversion, is presented. The proposed design is based on two translinear building blocks: a normalized gaussian-function generator and a four-quadrant analog-multiplier. Simulation results using BSIM3.v3 model for a 0.6μm AMS process parameters, in a CADENCE environment, are presented.

Keywords: Wavelets, weak inversion, MOS transistors, translinear circuits.

1. Introduction

Wavelets have been a very popular research topic in scientific and engineering fields. Since wavelets theory is a resourceful tool along a strong mathematical background, it can be applied in a wide variety of engineering applications [1]. Wavelets transforms were early applied to engineering by Grossman and Morlet [1, 2], and their results are related to the *continuous wavelets transform* (CWT) [2]. The CWT is based on a complex method of interpolation and intermediate-spaces, which provides a tool to describe spaces of functions and their approaches [2].
Nowadays, analog wavelets have been reported as an alternative to highly-complex digital implementations [3]. Moreover, the fast development of sub micron technologies, and the increment demand of portable electronic systems has been leading to a reduction in supply voltages, encouraging analog implementations. For that reason, several design techniques for low-voltage low-power analog circuits has been proposed in recent literature [4 - 8].

In this work, the design of a current-mode Morlet CWT, using MOS-translinear circuits, is presented. In section 2, a review of the CWT is depicted. A description of Morlet Wavelets, which are based on gaussian-functions, is presented in section 3. A four-quadrant translinear-multiplier is presented in section 4. The complete realization and the simulation-observed results are presented in section 5 and 6. Finally, the conclusions are summarized in section 7.

2. Continuous Wavelet Transform

The CWT is defined as the correlation between a signal to be processed, and a family of modulated functions, which are composed by arbitrary windowed functions, and generated by a mother wavelet. Lets consider a continuous time function $\psi(t)$ with the following properties [1,2]:

1. The function integrates to zero:

$$\int_{-\infty}^{\infty}\psi(t)dt = 0 \tag{2.1}$$

2. It is square integrable or has finite energy.

$$\int_{-\infty}^{\infty}\left|\psi(t)\right|^{2}dt < \infty \tag{2.2}$$

If the function satisfies both properties, $\psi(t)$ can be considered as a *mother wavelet*. Property 2 implies that most of the energy in $\psi(t)$ is confined to a time-finite interval, while property 1 is suggestive of an oscillatory function. If $f(t)$ is any square integrable function, the CWT of $f(t)$ respect to $\psi(t)$ can be defined as [3]:

$$W(a,b) \equiv \int_{-\infty}^{\infty} f(t)\frac{\mathbf{1}}{\sqrt{|a|}}\psi^{*}\left(\frac{t-b}{a}\right)dt \tag{2.3}$$

Where $\boldsymbol{a}$ and $\boldsymbol{b}$ are real numbers, and * denotes complex conjugation. Consequently, the wavelet transform can be written in a more compact form by defining $\psi_{a,b}(t)$ as:

$$\psi_{a,b}(t) \equiv \frac{1}{\sqrt{|a|}}\psi\left(\frac{t-b}{a}\right) \tag{2.4}$$

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combining (2.3) and (2.4), we obtain:

$$W(a,b)=\int_{-\infty}^{\infty} f(t)\psi^{*}_{a,b}(t) \tag{2.5}$$

The normalizing factor of $1/\sqrt{|a|}$ ensures that the energy becomes the same for all ***a*** and ***b***, that is:

$$\int_{-\infty}^{\infty}\left|\Psi_{a,b}(t)\right|^{2}dt=\int_{-\infty}^{\infty}\left|\Psi(t)\right|^{2}dt \tag{2.6}$$

For any given value of a, the function $\psi_{a,b}(t)$ represents a shift of b along the time axis of $\psi_{a,0}(t)$. Hence, the variable b represents either a time-shift or a translation, such that:

$$\psi_{a,0}(t)=\frac{1}{\sqrt{|a|}}\psi\left(\frac{t}{a}\right) \tag{2.7}$$

From (2.7), it follows that $\psi_{a,0}(t)$ is a time and amplitude scaled version of $\psi(t)$, where ***a*** is the scale or *dilatation variable*. The CWT is generated using dilatations and translations of the single function $\psi(t)$. The mother wavelet is a modulated function composed by two components in quadrature, it is characterized by equation [3]:

$$\Psi^{*}_{a,b}(t)=\left[\cos\omega_o\left(\frac{t-b}{a}\right)+j\sin\omega_o\left(\frac{t-b}{a}\right)\right]\Psi_{a,b}(t) \tag{2.8}$$

Although a windowed function must satisfy (1) and (2) to be considered as a mother wavelet, it is possible to have wavelets that are not supported compactly. For example, a *Morlet Wavelet*, which is constructed by modulating a sinusoidal function by a gaussian function [4, 10] is not a finite-time function. However, most of the energy in this wavelet is confined to a finite interval. In this work, Morlet wavelets are considered for short time and no stationary signal analysis [2].

3. Morlet Wavelets

From the concept of Mother Wavelet, introduced in the previous section, the required oscillatory condition leads us to consider sinusoidal building blocks. On the other hand, we also need a quick decaying condition as a tapering or windowing operation. Both conditions must simultaneously be satisfied for the wavelet function [3], and the basic Morlet wavelet can be expressed as follows:

$$\psi(t)=e^{j\omega_o t}e^{-\frac{t^2}{2}}=(\cos\omega_o t+j\sin\omega_o t)e^{-\frac{t^2}{2}} \tag{3.1}$$

The Fourier transform for the real part of (3.1) is given by:

$$\Psi_{\mathbf{Real}}(\omega)=\sqrt{\frac{\pi}{2}}\left\{e^{-\left(\frac{\omega-\omega_o}{2}\right)}+e^{-\left(\frac{\omega+\omega_o}{2}\right)}\right\} \tag{3.2}$$

It can be noticed that property (1) is not satisfied in equation (3.2). Nevertheless, if ω_0 is large enough, e.g. $\omega_0>5$, the Fourier transform expressed in (10) valued at the origin, can be practically considered as zero and the Morlet function given by (3.2) can be used as a mother wavelet [1]. The equation that governs a gaussian function [2] is given as:

$$h_{s,\tau}(t)=\frac{1}{\sqrt{S}}e^{-\frac{1}{2}\left(\frac{t-\tau}{S}\right)^2} \tag{3.3}$$

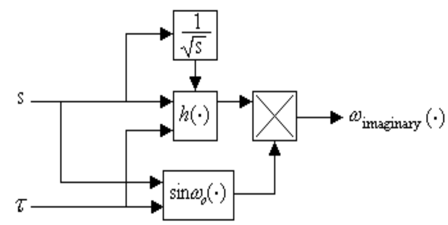

Figure 1. Functional blocks for the implementation of the imaginary component of a Morlet wavelet.

Since the term $1/\sqrt{S}$ denotes a scaling factor to preserve the energy in different scales, equation (3.3) represents a normalized gaussian-function, where τ represents the translation in time, and *S* relates different scale positions for the analysis of the signal. From (3.3), the Morlet wavelet is given by:

$$\psi_{a,b}(t)=h_{s,\tau}(t) \tag{3.4}$$

From (2.8) and (3.3), it can be noticed that a gaussian function, sinusoidal generators, as well as energy normalization, are needed for the implementation of Morlet wavelets. The functional block diagram of the imaginary component of the Morlet wavelet is sketched in figure 1. A circuit that generates a normalized gaussian function is shown in figure 2 [3, 9]. The circuit is composed of four blocks, namely: the quadrature circuit, the active resistor, the gaussian function generator circuit, and the normalizing gain element.

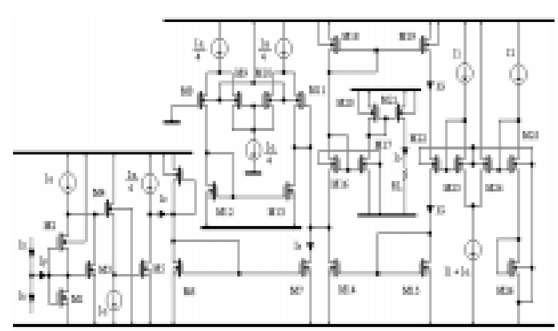

Figure 2. Normalized Gaussian-Function.

4. A Four Quadrant Analog Multiplier

In figure 3, the basic architecture of the multiplier cell is illustrated. It is composed of two translinear loops, to perform true current-mode analog multiplication. Let's consider the first translinear loop, which is formed by transistors M1-M2-M3-M4. Using the translinear theory, the following equation arises:

$$\frac{I_{d1}}{I_{d2}} = \frac{I_{d3}}{I_{d4}} = \frac{I_{x1}}{I_{x2}} \quad (4.1)$$

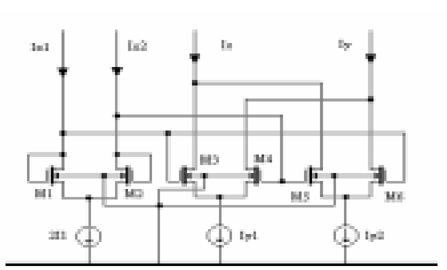

Figure 3. Four-quadrant current-mode analog multiplier.

Similarly, the resulting equation of the second loop, formed by transistors M1-M2-M5-M6, becomes:

$$\frac{I_{d1}}{I_{d2}} = \frac{I_{d6}}{I_{d5}} = \frac{I_{x1}}{I_{x2}} \quad (4.2)$$

The differential output current (I_x-I_y) is equal to:

$$I_x - I_y = I_{d3} + I_{d5} - I_{d4} - I_{d6} \quad (4.3)$$

By setting:

$$I_{y1} = I_{d3} + I_{d4} \quad (4.4a)$$

$$I_{y2} = I_{d5} + I_{d6} \quad (4.4b)$$

Replacing (4.4a) and (4.4b) into (4.3), it results:

$$I_0 = I_{y1} + I_{y2} - 2(I_{d4} + I_{d6}) \quad (4.6)$$

Solving for I_{d4} and I_{d6} in terms of I_{x1}, I_{x2}, I_{y1}, I_{y2} , and then (4.6) becomes:

$$I_0 = \frac{(I_{y1} - I_{y2})(I_{x1} - I_{x2})}{I_{x1} + I_{x2}} \quad (4.7)$$

For a four quadrants multiplier, one may define the following conditions.

$$I_{x1} = I_1 + i_b \text{ , (DC + AC)} \quad (4.8.a)$$

$$I_{x2} = I_1 - i_b \text{, (DC + AC)} \quad (4.8.b)$$

$$I_{y1} = I_1 + i_a \text{, (DC + AC)} \quad (4.8.c)$$

$$I_{y2} = I_1 \text{, (DC)} \quad (4.8.d)$$

By substituting (4.8a)-(4.8d) into (4.7), it results the equation:

$$I_0 = \frac{i_a * i_b}{I_1} \quad (4.9)$$

where the following conditions

$$I_1 + i_b > 0 \text{ and } \boldsymbol{I}_1 + \boldsymbol{i_a} > 0 \quad (4.10)$$

should be satisfied. That topology inherently minimizes the body effect due to the two translinear loops [7, 8]. The drain-current of the NMOS transistors can be modeled [5 -8], such as:

$$I_D = \frac{W}{L} I_{DO} e^{\frac{(n-1)V_{BS}}{nVt}} e^{\left(\frac{V_{GS}-V_{TH}}{nVt}\right)} \left(1 - e^{\frac{-V_{DS}}{Vt}} + \frac{V_{DS}}{V_0}\right) \quad (4.11)$$

where V_{GS} is the gate-source voltage, V_{DS} is the drain-source voltage, V_{BS} is the bulk-source voltage (body effect), V_{TH} is the threshold voltage, V_t is the thermal voltage, V_0 is the voltage Early and n is the slope factor. I_{D0} is a current related to the transconductance parameter *K'* and is given by [3]:

$$I_{D0} \cong \frac{2K'(nVt)}{e^2} \quad (4.12)$$

5. Morlet Wavelet Generation.

The block diagram circuit realization is given in figure 4. It involves the output current multiplication of the train of normalized gaussian functions with a sinusoidal current signal. Output currents i_a and i_b are the sinusoidal current source and the train of normalized gaussian functions, respectively. Modulated gaussian windows, corresponding to Morlet wavelets, are obtained at the multiplier output, such that:

$$i_{om} = \frac{i_a * i_b}{I_{1M}} = \frac{\sqrt{I_{1FG}^3}\, I_A}{I_{1M}} \left[\frac{1}{\sqrt{I\alpha}} e^{-\left(\frac{2*(Ix-Iu)}{I\alpha}\right)^2} \sin \omega_0(t) \right] \quad (5.1)$$

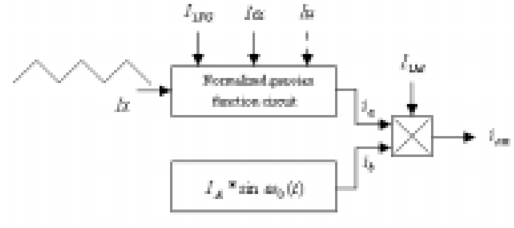

Figure 4. Block diagram for the imaginary-component of the Morlet wavelet.

6. Simulated Results.

Simulation results of the normalized gaussian function are shown in figure 5, where it can be observed the variation of the maximum value and the standard deviation when I_y is varying. In table 1, the ranges of the control currents are listed. The DC transfer function of the current multiplier is shown in figure 6. The input current i_b goes from -100nA to 100nA in steps of 25nA, while i_a is a continuous-signal varying from -100nA to 100nA.

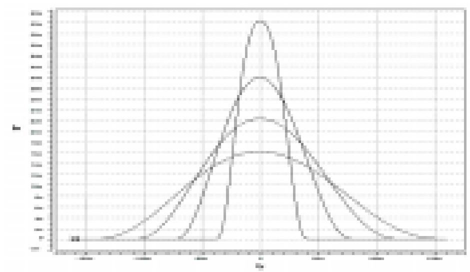

Figure 5. Output-current of the normalized gaussian function.

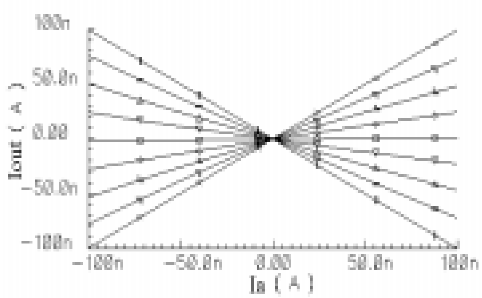

Figure 6. DC-transfer characteristics.

The imaginary component of the Morlet wavelet, which is found as the output current of the analog multiplier, is presented in figure 7. The final design of the Morlet wavelet is totally programmable. All simulation results were obtained using a BSIM3.v3 model for 0.6μm AMS process parameters, in a CADENCE environment. A single power supply of 1.5V it was used, with power consumption of 2.2 μW.

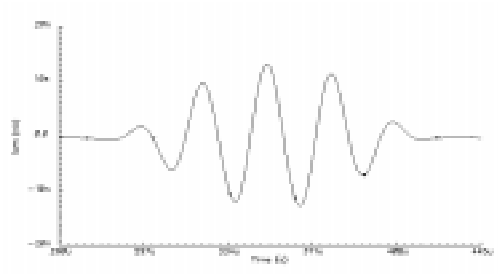

Figure 7. Imaginary-component of the Morlet wavelet.

7. Conclusions

The implementation of a fully-programmable current-mode low-voltage Morlet wavelet, using MOS transistors working in weak inversion, has been presented. A translinear gaussian function generator circuit, and a four-quadrant analog multiplier, which topology inherently minimizes the body effect has been presented, and used for the design of Morlet wavelets. Finally, some nonlinear effects were observed in the simulations, mainly due to the body effect, which can be minimized using a double well technology.

8. References

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Table 1.

Bias conditions for the normalized gaussian-function

V_{DD}	**1.5 V**
Power Consumption	**1.534 μW**
Iα	**40n -- 160n**
Iy	**-160n -- 160n**
I1	**40n**