

# A Three-Phase Uninterruptible Power Supply using Multivariable Repetitive Robust Model Reference Adaptive Control

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*Abstract:* This paper presents a multivariable repetitive robust model reference adaptive controller (MIMO RMRAC-RP) applied to a three-phase uninterruptible power supply (UPS). Using  $\alpha\beta 0$  transformation and balanced three-phase load, the plant can be considered weakly coupled, and then a multivariable law may have its parameters reduced in number, reducing the computational effort to adapt them. For this MIMO RMRAC-RP with reduced number of parameters, the augmented error equation is written, and the adaptation algorithm convergence mathematical proves are developed, as well as the proof of the overall closed-loop system stability, despite the presence of unmodeled dynamics and bounded disturbances. Experimental results confirm good performance for linear and non-linear balanced three-phase loads.

*Key-Words:* Multivariable control  
Repetitive control

Model reference adaptive control  
Uninterruptible power supply

Robust adaptive control

## 1 Introduction

A usual way to deal with coupling in three-phase inverter systems is the use of  $\alpha\beta 0$  transformation or synchronous transformation in dq coordinates. In a real setup implementation, errors in the measured variables may lead to coupling in the final system. If the existing coupling is not significant, it may be considered unmodeled dynamics, and a single-input single-output (SISO) robust adaptive control technique may be applied to assure performance. Otherwise, if coupling becomes significant, there will be no guarantee to obtain system good performance. Carati and others [1] introduce the use of two SISO robust model reference adaptive controllers (RMRAC) applied to a three-phase uninterruptible power supply (UPS), using synchronous transformation.

In this work we propose a repetitive multivariable robust model reference adaptive controller (MIMO RMRAC-RP) to control a three-phase UPS. Different from the decentralized case [2], where weak coupling is considered unmodeled dynamics in the multivariable plant, the developed multivariable control law is able to deal with coupling. Moreover, assuming that the three-phase UPS is weakly coupled, the proposed direct multivariable controller has its parameters reduced in number, reducing the consequent computational effort to adapt them. This reduction of parameters is inspired in a discrete-time indirect MIMO MRAC developed by

Moctezuma and Lozano [3], where an ARMA model together with the knowledge of the Interactor matrix lead to a reduced number of plant parameters to adapt. Different from this controller, the proposed one is a direct continuous-time robust controller.

In [4] it was developed a MIMO RMRAC which, inspired in [5], [6] and [7], uses a modified least-squares parameters adaptation algorithm. The proposed scheme reduces the number of parameters to be adapted assuming that the plant is weakly coupled. In addition, the developed controller has a portion of repetitive controller to avoid repetitive errors in the sinusoidal waveforms of the output.

The augmented error equation is written for the proposed MIMO RMRAC-RP scheme, and it is shown that for small additive and multiplicative stable plant perturbations, the tracking error is small in the mean and all the signals in the closed loop are bounded.

Simulation and experimental results are obtained showing good performance of the proposed scheme with linear and non-linear balanced three-phase loads.

This paper is organized as follows. In Section 2 we describe a three-phase UPS, and in Section 3 we show the structure of the overall closed loop system. Section 4 contains the plant description and the control objective. In Section 5 we show the controller structure. Section 6 is devoted to the parameter adaptation algorithm and its properties. In Section 7 the robustness properties are analyzed. Section 8 shows experimental results.

## 2 Three-Phase Uninterruptible Power Supply (UPS)

Fig. 1 shows the plant of a three-phase UPS, composed by a three-phase full-bridge inverter, an LC delta ( $\Delta$ )-connected filter and a three-phase load. System state-space model is given by

$$\begin{bmatrix} \dot{i}_R \\ \dot{i}_S \end{bmatrix} = \frac{1}{3L_f} \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} V_{RS} - V_{UV} \\ V_{ST} - V_{VW} \end{bmatrix} - \frac{R_f}{L_f} \begin{bmatrix} i_R \\ i_S \end{bmatrix}, \quad (1)$$

$$i_T = -(i_R + i_S),$$

$$\begin{bmatrix} \dot{V}_{RS} \\ \dot{V}_{ST} \end{bmatrix} = \frac{1}{3C_f} \begin{bmatrix} 1 & -1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} -i_R - i_{RL} \\ -i_S - i_{SL} \end{bmatrix}, \quad (2)$$

$$V_{TR} = -(V_{RS} + V_{ST}).$$

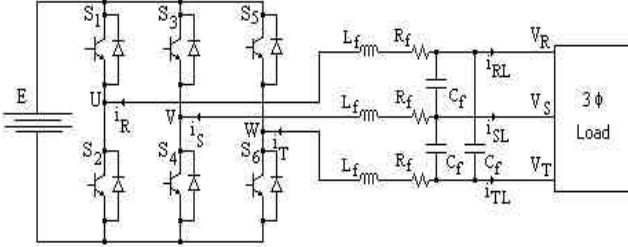


Fig. 1. Three-phase PWM inverter system.

Inverter line voltages,  $V_{UV}$ ,  $V_{VW}$  and  $V_{WU}$ , are pulses with amplitude  $E$ , 0 and  $-E$ , due the operation of  $S_{1-6}$  switches, which open and close at each sampling period  $t_s$ . The PWM inverter voltages are used to control the output voltages,  $V_{RS}$ ,  $V_{ST}$  and  $V_{TR}$ .

In (2), the load currents  $i_{RL}$  and  $i_{SL}$  depend on the used load. In case of three wye (Y)-connected resistors  $R$ , the currents are given by

$$i_{RL} = \frac{2V_{RS}}{3R} + \frac{V_{ST}}{3R}, \quad i_{SL} = -\frac{V_{RS}}{3R} + \frac{V_{ST}}{3R}. \quad (3)$$

## 3 Closed Loop System Structure

Fig. 2 shows that the three-phase reference model generates phase outputs  $y_{mr}$ ,  $y_{ms}$  and  $y_{mt}$ , which are converted in two orthogonal independent reference model variables  $V_{ma}$  and  $V_{mb}$  through the use of  $\alpha\beta 0$  transformation represented by  $T$  matrix, given in (4). Fig. 3 shows that the MIMO RMRAC-RP controller uses the transformed reference variables  $r_a$  and  $r_b$ , from  $r_r$ ,  $r_s$ ,  $r_t$ , and the transformed output voltage variables  $V_a$  and  $V_b$ , from the plant output phase voltages  $V_r$ ,  $V_s$  and  $V_t$ . The controller generates the control variables  $u_a$  and  $u_b$ , which are transformed

into the three-phase control variables  $u_U$ ,  $u_V$  and  $u_W$ . These variables are used to command the PWM inverter, so that it generates the three-phase voltages  $V_U$ ,  $V_V$  and  $V_W$ , which applied to the plant generate the output three-phase voltages  $V_r$ ,  $V_s$  and  $V_t$ .

$$T = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix}, \quad T^{-1} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & 0 \\ -1/2 & \sqrt{3}/2 \\ -1/2 & -\sqrt{3}/2 \end{bmatrix}. \quad (4)$$

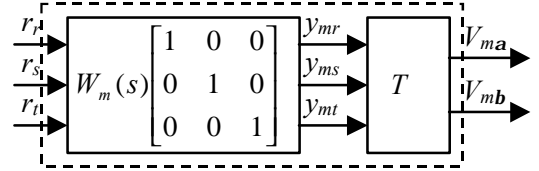


Fig. 2. Reference model for a three-phase UPS.

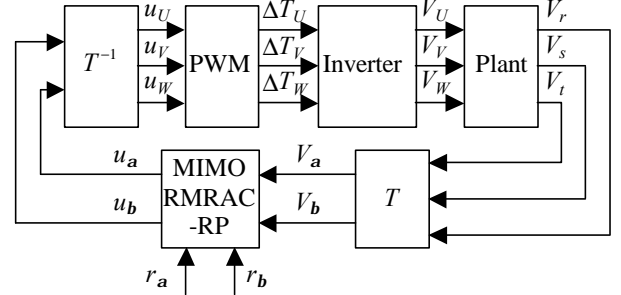


Fig. 3. Block diagram of a three-phase UPS with a MIMO RMRAC-RP controller.

## 4 Plant Description and Control Objective

In the following sections we will describe a multivariable control system, which is inspired in that of [4]. The assumption that the plant is weakly coupled makes the parameterization problem simpler to solve. Consider the  $N \times N$  MIMO LTI plant described by

$$y = G(s)u, \quad G(s) = \{G_0(s)[I + m\Delta_m(s)] + m\Delta_a(s)\} \quad (5)$$

where  $y \in \mathcal{R}^N$ ,  $u \in \mathcal{R}^N$ .  $G_0(s)$  is the modeled part of the plant, and  $\Delta_m(s)$ ,  $\Delta_a(s)$  are the respectively multiplicative and additive unmodeled parts of the plant.

In the UPS application, the plant has  $N=2$  and  $G_0(s)$  comes from the  $\alpha\beta 0$  transformation of the system described by (1), (2) and (3). Input and output vectors can be expressed as  $y = [y_a, y_b]^T$  and  $u = [u_a, u_b]^T$ .

As stated in [4], the integral structure of a multivariable system may be characterized by the plant modified left interactor (MLI) matrix  $\mathbf{x}_m(s)$ , which is a polynomial matrix that satisfies  $\lim_{s \rightarrow \infty} \mathbf{x}_m(s)G_0(s) = K_p$ , where  $K_p$  is a finite and non-singular matrix. Considering a weakly coupled multivariable system, we can define the interactor matrix as

$$\mathbf{x}_m(s) = d_m(s)I \quad (6)$$

where  $I$  is the identity matrix and  $d_m(s)$  is a monic Hurwitz polynomial matrix of degree  $d$ .

We can now state the control objective as follows:

Given the reference model

$$y_m = W_m(s)r = w_m(s)Ir \quad (7)$$

where  $W_m(s)$  is an  $N \times N$  matrix and  $w_m(s)$  is a strictly proper stable minimum phase transfer function to be selected, and  $r \in \mathcal{R}^N$  is a known uniformly bounded and piecewise continuous input reference signal, find in (5) the control input  $u \in \mathcal{R}^N$

so that the output  $y \in \mathcal{R}^N$  follows  $y_m \in \mathcal{R}^N$  in (7) as close as possible, and all signals in the closed-loop plant are uniformly bounded for any bounded initial conditions.

In order to satisfy the control objective, it is necessary that the plant and the reference model satisfy the following assumptions:

- A1.**  $G(s)$  is strictly proper and full rank.
- A2.**  $G_0(s)$  is strictly proper and non-singular, it has stable zeros, and it has a known MLI matrix of the form  $\mathbf{x}_m(s) = d_m(s)I$ .
- A3.**  $\Delta_m(s)$  and  $\Delta_a(s)$  are rational transfer matrices, and  $\Delta_a(s)$  is strictly proper.
- A4.** Let  $d_m(s)$  have all its roots in  $\text{Re}[s] < -p_0$ , and define

$$D_a = \lim_{s \rightarrow \infty} \Delta_a(s)s, \quad D_m = \lim_{s \rightarrow \infty} \frac{1}{d_m(s)} \Delta_m(s)s, \quad (8)$$

then there exists constants  $k_a, k_m > 0$  so that

$$\|D_a\| < k_a, \quad \|D_m\| < k_m \quad (9)$$

$$\|(\Delta_a(s - p_0) - D_a)(s + p)\|_\infty < k_a \quad (10)$$

$$\left\| \left( \frac{1}{d_m(s - p_0)} \Delta_m(s - p_0) - D_m \right) (s + p) \right\|_\infty < k_m \quad (11)$$

for some  $p > 0$ , where  $\|X(s)\|_\infty \triangleq \sup_{\mathbf{w} \in \mathcal{R}} \|X(j\mathbf{w})\|$ .

- A5.** An upper bound  $\bar{n}_0$  for the observability index  $\mathbf{n}_0$  of  $G_0(s)$  is known.
- A6.** The high frequency gain matrix  $K_p$  of  $G_0(s)$  associated to the MLI matrix  $\mathbf{x}_m(s)$  is a known finite positive definite matrix.

**A7.** An upper bound  $M_0$  for  $\|\mathbf{q}^*\|$  is known, so that  $\|\mathbf{q}^*\| + \mathbf{d}_3 \leq M_0$  for some  $\mathbf{d}_3 > 0$ , where  $\mathbf{q}^*$  is the desired parameter matrix of the controller.

**A8.** The reference model  $W_m(s)$  in (7) has all its poles and zeros stable, and it is chosen so that  $d_m(s)W_m(s)$  is proper. Without loss of generality, we can choose  $W_m(s) = (\mathbf{x}_m(s))^{-1}$ .

## 5 Controller Structure

The control input is computed from

$$\begin{aligned} u &= \mathbf{q}^T \mathbf{w} + u_{RP} + C_0 r \\ u &= \mathbf{q}_1^T \mathbf{w}_1 + \mathbf{q}_2^T \mathbf{w}_2 + \mathbf{q}_3^T y + u_{RP} + C_0 r \end{aligned} \quad (12)$$

where  $\mathbf{q} = [\mathbf{q}_1^T, \mathbf{q}_2^T, \mathbf{q}_3^T]^T$ ,  $\mathbf{w} = [\mathbf{w}_1^T, \mathbf{w}_2^T, y^T]^T$ . For simplicity and without loss of generality, we can consider  $C_0 = I$ . The variable  $u_{RP}$  is the repetitive part of the controller, and it is defined as

$$u_{RP} = c_{RP} \frac{\exp(-sT_s)}{1 - \exp(-sT_s)} (y - y_m), \quad (13)$$

where  $T_s$  is the period of the sinusoidal waveform and  $c_{RP}$  is a small constant, avoiding a great influence of  $u_{RP}$  in the overall control  $u$  in (12).

Provided the system is weakly coupled, the order of the filters is  $M = dN$ , and they can be represented by

$$\mathbf{w}_1 = \Lambda^{-1}(s)A(s)u \quad \mathbf{w}_2 = \Lambda^{-1}(s)A(s)y \quad (14)$$

where  $\Lambda(s)$  is an arbitrary Hurwitz polynomial with degree  $d$ , and  $A(s) = [Is^{d-2}, Is^{d-3}, \dots, Is, I]^T$  is a  $(M \times N)$  polynomial matrix, and  $\mathbf{q}_1 = [\mathbf{q}_{11}, \dots, \mathbf{q}_{1d}]^T$ ,  $\mathbf{q}_2 = [\mathbf{q}_{21}, \dots, \mathbf{q}_{2d}]^T$ ;  $\mathbf{q}_3, \mathbf{q}_{ij} \in \mathcal{R}^{N \times N}$ . We have also that  $\mathbf{q}_1, \mathbf{q}_2 \in \mathcal{R}^{N \times M}$ ,  $\mathbf{w}_1, \mathbf{w}_2 \in \mathcal{R}^{M \times 1}$ ,  $\mathbf{q} \in \mathcal{R}^{N \times p}$  and  $\mathbf{w} \in \mathcal{R}^{p \times 1}$ , where  $p = (2d + 1)N$ .

In the UPS controller implementation in (12), we can make diagonal the matrices  $\mathbf{q}_2 = \mathbf{q}_{21}^T$  and  $\mathbf{q}_3$ , reducing the number of parameters to adapt. It makes  $u_a$  independent of  $y_b$  and  $u_b$  independent of  $y_a$ , as shown in [3]. This diagonalization is only possible when the MLI matrix is diagonal.

If we write

$$\mathbf{f} = \mathbf{q} - \mathbf{q}^* \quad (15)$$

where  $\mathbf{q}^* = [\mathbf{q}_1^{*T}, \mathbf{q}_2^{*T}, \mathbf{q}_3^{*T}]^T$  have the same dimensions as  $\mathbf{q}$ , then (12) can be written as

$$\mathbf{f}^T \mathbf{w} + u_{RP} + r = [I - F_1(s) - F_2(s)G(s)] u \quad (16)$$

where  $F_1(s)$  and  $F_2(s)$  are  $(N \times N)$  matrices:

$$F_1(s) \triangleq \mathbf{q}_1^{*T} \frac{A(s)}{\Lambda(s)}, \quad F_2(s) \triangleq \mathbf{q}_2^{*T} \frac{A(s)}{\Lambda(s)} + \mathbf{q}_3^{*T}. \quad (17)$$

**Lemma 5.1.** Combining (5)-(7) and (12)-(17), the tracking error can be expressed as

$$e \triangleq y - y_m = W_m(s) (\mathbf{f}^T \mathbf{w} + u_{RP}) + \mathbf{m}\mathbf{h} \quad (18)$$

with

$$\mathbf{h} = \Delta(s)u \quad (19)$$

where  $\Delta(s)$  is a strictly proper transfer matrix.

**Proof.** Considering (16) and (17), in view of the controllability of the modeled part of the plant, there exists a vector  $\mathbf{q}^*$  such that

$$\left[ I - \mathbf{q}_1^{*T} \frac{A(s)}{\Lambda(s)} - \mathbf{q}_2^{*T} \frac{A(s)}{\Lambda(s)} G_0(s) - \mathbf{q}_3^{*T} G_0(s) \right] = W_m^{-1}(s) G_0(s) \quad (20)$$

Using (16) and (20), (5) can be rewritten as

$$y = W_m(s) [\mathbf{f}^T \mathbf{w} + u_{RP} + r] + \mathbf{m}\mathbf{h} \quad (21)$$

where

$$\Delta(s) = W_m(s) [I - F_1(s)] \Delta_m(s) + \mathbf{m} [I + W_m(s) F_2(s)] \Delta_a(s). \quad (22)$$

Thus,  $\Delta(s)$  is a strictly proper transfer matrix. Equations (18) and (19) are obtained from (7) and (21).

Finally, we can define the augmented error as

$$\mathbf{e} \triangleq e + \left[ \mathbf{q}^T W_m(s) \mathbf{w} - W_m(s) \mathbf{q}^T \mathbf{w} \right] - W_m(s) u_{RP} \quad (23)$$

$$\mathbf{e} = \mathbf{f}^T \mathbf{z} + \mathbf{m}\mathbf{h}$$

with  $\mathbf{e} \in R^{N \times 1}$ , and where

$$\mathbf{z} = W_m(s) \mathbf{w}. \quad (24)$$

Note that (23) was determined assuming that the interactor matrix in (6) and the model reference matrix in (7) are scalar transfer functions multiplied by the identity matrix. This makes them commutative when multiplied by other matrix.

## 6 Parameter Adaptation Algorithm

Consider the following modified least-squares algorithm

$$\dot{\hat{\mathbf{q}}} = \dot{\mathbf{q}} = -\mathbf{S}P\mathbf{q} - \frac{P\mathbf{z}\mathbf{e}^T}{m^2} \quad (25)$$

$$\dot{P} = -\frac{P\mathbf{z}\mathbf{z}^T P}{m^2} + \left( \mathbf{I} P - \frac{P^2}{R^2} \right) \bar{\mathbf{m}}^2 \quad (26)$$

where  $P = P^T$  is a  $(M \times M)$  matrix so that

$$0 < P(0) < \mathbf{I} R^2 \mathbf{I}, \quad \mathbf{m}^2 \leq k_m \bar{\mathbf{m}}^2 \quad (27)$$

and

$$\dot{m} = -\mathbf{d}_0 m + \mathbf{d}_1 (\|u\| + \|y_f\| + 1), \quad m(0) \geq \mathbf{d}_1 / \mathbf{d}_0 \quad (28)$$

where  $\mathbf{l}$ ,  $\bar{\mathbf{m}}$ ,  $R^2$ ,  $\mathbf{d}_0$  and  $\mathbf{d}_1$  are positive constants and  $\mathbf{d}_0$  satisfies

$$\mathbf{d}_0 + \mathbf{d}_2 \leq \min[p_0, q_0] \quad (29)$$

where  $q_0 > 0$  is such that the poles of  $W_m(s - q_0)$  and  $\Lambda(s - q_0)$  are stable and  $\mathbf{d}_2$  is a positive constant.  $p_0 > 0$  is defined in assumption A4, and  $\mathbf{S}$  in (25) is given by

$$\mathbf{S} = \begin{cases} 0 & \text{if } \|\mathbf{q}\| < M_0 \\ \mathbf{s}_0 \left( \frac{\|\mathbf{q}\|}{M_0} - 1 \right) & \text{if } M_0 \leq \|\mathbf{q}\| \leq 2M_0 \\ \mathbf{s}_0 & \text{if } \|\mathbf{q}\| > 2M_0 \end{cases} \quad (30)$$

where  $M_0 > \|\mathbf{q}^*\|$  (Assumption A8), and  $\mathbf{s}_0 > 2\bar{\mathbf{m}}^2 / R^2$  are project parameters.

Note that in the UPS implementation, continuous-time equations are discretized, and the control variables  $u_a$  and  $u_b$  can be computed independently. In this case, the parameters of  $u_a$  and  $u_b$  can be separated, and two independent parameter estimators (25)-(30) can be used, reducing computational effort.

The following lemma [4] gives an important property to the normalizing signal  $m(t)$  which is necessary in the stability analysis, and the proof is similar to that presented to the SISO case in [5].

**Lemma 6.1.** [4] Consider the system

$$\dot{z} = W(s)U \quad (31)$$

where  $z, U \in R^{N \times 1}$  and  $W(s)$  is an  $(N \times N)$  stable and strictly proper transfer matrix, whose poles  $p_j$  satisfy

$$\mathbf{d}_0 + \mathbf{d}_2 \leq \min_j |\operatorname{Re}(p_j)| \quad (32)$$

and  $\|U(t)\| \leq \mathbf{d}_4 m(t)$  for some  $\mathbf{d}_4 > 0$  ( $\|U(t)\| \leq \|u(t)\| + \|y(t)\| + m(t)$ )  $\forall t \geq 0$ . Then there exists a constant  $c_1 > 0$  such that

$$\frac{\|z(t)\|}{m(t)} \leq c_1 + \mathbf{e}_t \quad (33)$$

where  $\mathbf{e}_t$  is a term which depends on the initial conditions and decays exponentially to zero with a rate at least as fast as  $e^{(-\mathbf{d}_0 t)}$ .

Now we can establish the following lemma [4], which generalizes to the MIMO case that lemma stated in [6].

**Lemma 6.2.** [4] The parameter adaptation algorithm in (25)-(30) and (23) subject to the Assumptions A2, A4 and A8, has the following properties

$$1) \quad I/IR^2 \leq P^{-1} \leq I(1/IR^2 + g_3^2/\bar{m}^2) \quad (34)$$

where  $g_3$  is the upper bound for  $\|z\|/m$ .

$$2) \quad \mathbf{s} \operatorname{tr}(\mathbf{f}^T \mathbf{q}) \geq 0. \quad (35)$$

$$3) \quad V = (1/2) \operatorname{tr}(\mathbf{f}^T P^{-1} \mathbf{f}) \\ \leq \bar{V} \triangleq \begin{cases} 2 \max\left(\frac{2k_m g_5^2}{I}, \frac{9M_0^2}{IR^2}\right) & \text{for } \bar{m} > 0 \\ V(0) & \text{for } \bar{m} = 0 \end{cases} \quad (36)$$

where  $g_5$  is the upper bound for  $\|h\|/m$ .

$$4) \quad \|\mathbf{f}\| \leq k_f \triangleq 2IR^2 \bar{V}. \quad (37)$$

$$5) \quad \frac{1}{T} \int_{t_0}^{t_0+T} \left( \frac{\|\mathbf{f}^T \mathbf{z}\|^2}{m^2} + \mathbf{s} \operatorname{tr}(\mathbf{f}^T \mathbf{q}) \right) dt \leq \frac{g_1}{T} + \bar{m}^2 g_2, \quad (38) \\ \forall t_0 \geq 0, T > 0.$$

$$6) \quad \frac{1}{T} \int_{t_0}^{t_0+T} \frac{(p_j^T \mathbf{z})^2}{m^2} dt \leq \frac{g'_1}{T} + \bar{m}^2 g'_2, \quad (39) \\ \forall t_0 \geq 0, T > 0, j = 1, \dots, M$$

where  $g_1, g_2, g'_1 \in g'_2$  are positive constants and  $p_j$  is the  $j$ -th line of  $P$ .

**Proof.** The proof is presented in [4], and will be omitted.

## 7 Stability Analysis

Following the same steps as presented in [4], we can establish the main result, which is similar to that obtained in Theorem 6.1 in [4], but now applied to a MIMO RMRAC-RP.

**Theorem 7.1.** Consider the multivariable plant in (6). Subject to the Assumptions A1-A8, the multivariable adaptive control structure in (8), (13)-(19), (23)-(24) together with the parameter adaptation algorithm in (25)-(30), then  $\bar{m}^* > 0$  can be computed so that for each  $m \in [0, k_m \bar{m}^*)$  all the signals in the closed-loop system are bounded for any initial conditions. Furthermore, the tracking error belongs to the residual set

$$D e = \left\{ e : \limsup_{T \rightarrow \infty} \left( \frac{1}{T} \int_{t_0}^{t_0+T} \|e(t)\| dt \right) \leq \right. \\ \left. \leq (\bar{e} + g_1 \bar{m} + g_2 \bar{m}^2) \right\}. \quad (40)$$

**Proof.** Similar to that presented in [4], it will be omitted.

The following corollary is similar to Corollary 6.1 in [4].

**Corollary 7.1:** In the absence of modeling error (i.e., when  $m=0$ ) and when we choose  $\bar{m}=0$ , the adaptive control algorithm considered in Theorem 7.1 guarantees boundedness of all the signals as well as convergence of the tracking error  $e$  to zero.

**Proof.** The proof is similar to that presented in [3] and will be omitted.

## 8 Experimental Results

In order to show and verify the performance of the proposed MIMO RMRAC-RP scheme, experimental results are presented. Table 1 presents the parameters of the three-phase PWM inverter, the LC filter and the load used in experimental results.

The implemented prototype is a system as shown in Fig. 1, whose parameters are shown in Table 1. The continuous-time multivariable controller was discretized at sampling period  $t_s$ , as shown in Table 1. The model reference matrix in (7) is

$$W_m(s) = w_m(s)I \quad (53)$$

and  $w_m(s)$  is a RLC transfer function where  $R, L$  and  $C$  have the respective values of  $20 \Omega, 10 \text{ mH}$  and  $60 \mu\text{F}$ .

Table 1. Parameters of the Three-Phase UPS.

|                             |                                    |
|-----------------------------|------------------------------------|
| LC filter inductance        | $L_f = 5.4 \text{ mH}$             |
| LC filter capacitance       | $C_f = 75 \mu\text{F}$             |
| Inductors resistance        | $R_f = 0.1 \Omega$                 |
| Load                        | $R = 24 \Omega$                    |
| Reference voltage           | $V_{\text{phase}} = 110 \text{ V}$ |
| Sine wave voltage frequency | $f = 60 \text{ Hz}$                |
| Rectifier output capacitor  | $C_{\text{rec}} = 330 \mu\text{F}$ |
| Inverter DC source voltage  | $E = 300 \text{ V}$                |
| Sampling period             | $t_s = 1/1800 \text{ s}$           |

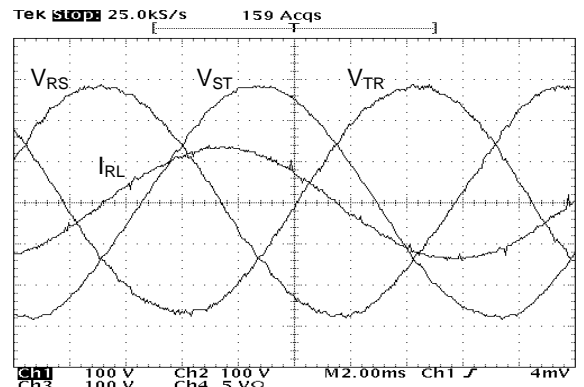


Fig. 4. Output line voltages (100 V/div) and phase current (5 A/div) with three-phase linear load.

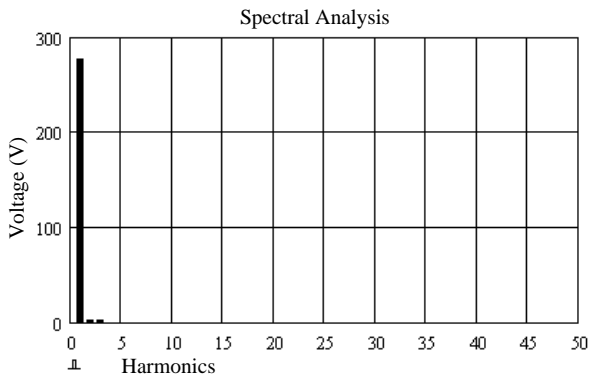


Fig. 5. Spectral analysis of the output line voltages with three-phase linear load.

Fig. 4 shows the system response to linear load, showing three line voltages and one phase current. Fig. 5 shows the spectral analysis of the line voltages. In this case, the percent total harmonic distortion (%THD) obtained in each line voltage is 2.088%.

Fig. 6 shows system response to the non-linear load in parallel connection with the linear load. Again, three line voltages and one phase voltage are shown. Fig. 7 shows the spectral analysis of the line voltages. The percent total harmonic distortion (%THD) obtained in each line voltage is 3.342%.

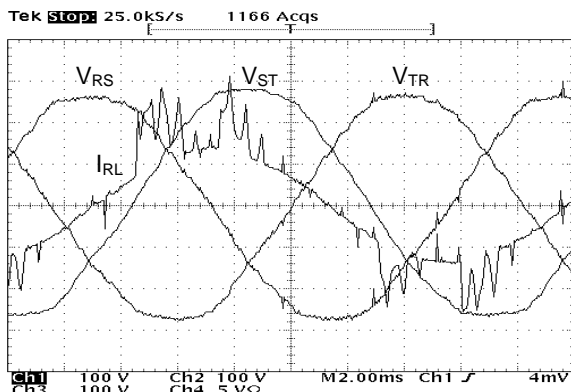


Fig. 6. Output line voltages (100 V/div) and phase current (5 A/div) with non-linear three-phase load in parallel connection with three-phase linear load.

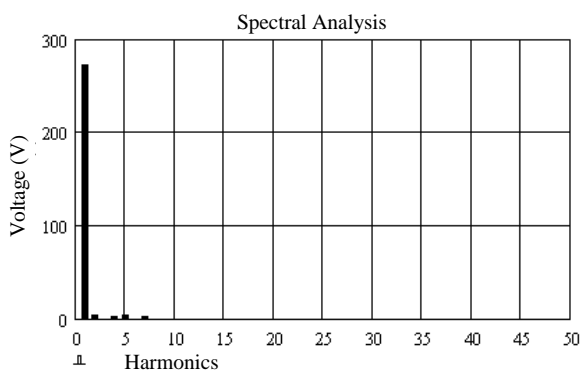


Fig. 10. Spectral analysis of the output line voltages with non-linear three-phase load in parallel connection with three-phase linear load.

We can consider that the total harmonic distortions (%THD) we obtained in the experiments are reasonable values for the real setup used, because all sort of disturbances may deteriorate system response. For example, a PWM inverter generates noise; load and filters may not be perfectly balanced; etc.

In order to improve performance, we can increase sample frequency, but being careful to avoid exceeding the limits of the PWM switches.

## 9 Conclusion

This paper presented a repetitive multivariable robust model reference adaptive controller (MIMO RMRAC-RP) applied to a three-phase uninterruptible power supply (UPS). Using  $\alpha\beta 0$  transformation and balanced three-phase load, it was assumed that the plant is weakly coupled, and great reduction in the number of parameters was obtained. For small plant perturbations, it was shown that the tracking error is small in the mean and all the signals in the closed loop are bounded.

Experimental results were obtained, showing that the scheme presents good performance for both linear and non-linear balanced three-phase loads.

## References:

- [1] Carati, E. G., Gründling, H. A. & Richter, C. M. A Robust High Performance Three-Phase Uninterruptible Power Supply. In: *Proc. 2000 IEEE Int. Conf. Control Appl.* Anchorage, USA, pp.896-901, 2000.
- [2] Wen, C. & Soh, Y. C. Decentralized Model Reference Adaptive Control Without Restriction on Subsystem Relative Degrees. *IEEE Trans. Automat. Contr.*, Vol.44, N.7, pp.1464-1469, 1999.
- [3] Moctezuma, R. G. & Lozano, R. Singularity-Free Multivariable Model Reference Adaptive Control. *IEEE Trans. Automat. Contr.*, Vol.39, N.9, pp.1856-1860, 1994.
- [4] Richter, C. M. et al. A New Multivariable Robust Model Reference Adaptive Controller. *WSEAS Int. Conf. on Systems Science*, Rio de Janeiro, 2002.
- [5] Ioannou, P. A. & Tsakalis, K. A Robust Direct Adaptive Controller. *IEEE Trans. Automat. Contr.*, V.AC-31, N.11, pp.1033-1043, 1986.
- [6] Lozano, R., Collado, J. & Mondié, S. Model Reference Robust Adaptive Control Without *A Priori* Knowledge of the High Frequency Gain. *IEEE Trans. Automat. Contr.*, Vol.35, N.1, pp.71-78, 1990.
- [7] Tao, G. & Ioannou, P. A. Robust Model Reference Adaptive Control for Multivariable Plants. *Int. J. Adaptive Control and Signal Processing*, Vol.2, pp.217-248, 1988.