Hydrodynamic Flow Past A Semi- Infinite Rough Plate Embedded In Porous Medium

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Abstract:- The boundary layer flow of a fluid past a semi-infinite rough plate embedded in porous medium is investigated. The transformed non-linear ordinary differentional equation describing the problem is solved numerically using the shooting technique. The effects of permeability parameter (M), inertia coefficient parameter (N) and the roughness parameter (R_w) on the velocity are shown on graphs. Numerical data for the skin fraction have been tabulated for various values of M, N and R_w.

Key-Words:- MHD, Flow, Porous, numerical solution, semi-infinite

1- Introduction

The analysis of hydrodynamic flow through porous medium is of interest in a wide range of technical problems. The importance of inertia effects were discussed by many authors [1-4].

In the a forementioned studies, the plates were asumed to be smooth. In most practical application connected with the flat plate, the wall can not be considered smooth. Consequently the flow past a rough plate is of as much practical interest [6].

To the end of our knowledge there have been no studies concerning the laminar boundary layer flow past a rough surfaces through a non-Darcian porous medium.

It is now proposed to study the effects of permeability parameter, inertia coefficient parameter and the roughness parameter on the flow past a semi-infinite plate embedded in non-Darcian porous medium.

2- Basic Equations

The boundary layer steady flow of an incompressible fluid through a porous medium bounded by a semi- infinite rough flat plate is studied. The x-axis is taken along the plate and y-axis normal to it We assume that:

(a) the magnitude of the free- stream velocity is maintained and the flow is steady and two-dimensional;

(b) the boundary-layer approximations hold;

(c) the physical properties are constant; and

(d) the velocity at the plate is proportional to the gradient of velocity.

Under these assumptions the governing boundary layer equation for fluid flow through a porous medium by using a generalized

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+\frac{\in\nu_e}{K}(u_\infty-u)+c\in^2\left(u_\infty^{\ 2}-u^2\right) \qquad (1)$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (2)$$

Where u,v are the velocity components in the x,y direction, respectively, $\in$ and K are the porosity and the permeability of the porous medium, ν the kinemetic viscosity, ν_e the effective kinematic viscosity.

3- Boundary conditions

As for the surface roughness of the plate, we shall assume the slip velocity u_s to be proportional to the gradient of the velocity u at the wall. The condition to be considered is

$$u_s = R\frac{\partial u}{\partial y} \qquad (3)$$

at the plat $(y=0)$

The proportionality factor R is approximately the height of the roughness element, and is taken to be of the same order of magnitude as theboundary layer thickness δ. In other words from the model assumed in equation (3), we can write

$$\frac{u_s}{u_\infty}\cong\frac{R}{\delta}$$

In our calculations we shall take $R\cong O\left(\frac{\delta}{10}\right)$ which seems to be practically reasonable even for extremely smooth surfaces.

The boundary conditions for our problem are then given by:

$$y=0; \qquad u=R\frac{\partial u}{\partial y}, \qquad v=0$$
$$y\to\infty: \ u=u_\infty \qquad (4)$$

A stream function of the form $\psi=\sqrt{2u_\infty\nu x}\,f(\eta)$ where $f(\eta)$ denotes the dimensionless stream function and $\eta=\sqrt{\frac{u_\infty}{2\nu x}}\,y$ is introduced to satisfy the continuity equation (2). The expressions for u and v are

$$u=u_\infty\ f'(\eta),\ v=\sqrt{\frac{\nu u_\infty}{2x}}\left[-f(\eta)+\eta\ f'(\eta)\right]$$

.

Where prime denotes differentiation with respect to η, and for the roughness parameter $R_w=\frac{R(x)}{u_\infty}\sqrt{\frac{u_\infty}{2\nu x}}$ is constant, (hence $R(x)\sim x^{-1/2}$ for similarity solution).

Substitution in equation (1) and simplification leads to the following non-linear ordinary differential equation:

$$f''' + f f'' + M\left(1 - f'\right) + N\left(1 - f'^{2}\right) = 0, \qquad (5)$$

Where $M = \frac{2 \in x}{K u_{\infty}}$ (*permebility parameter*),

$N = 2 c \in x$ (*Inertia coefficient parameter*).

The boundary condition now become.

$$y = 0 : f' = R_w f'', \quad f = 0 \qquad (6)$$

$$y \to \infty : f' = 1$$

4- Numerical Solution And Discussion

Equation (5) with the boundary conditions (6) are solved numerically using the shooting method, with the fourth order Rung-kutta.

For engineering application, the local wall shear stress is of importance. The shear stress.

$$\tau_w = \mu u_{\infty} \sqrt{\frac{u_{\infty}}{2 \nu x}}\, f''(0) \qquad (7)$$

At the plate the coefficient of skin friction C_f is given by

$$C_f = \frac{\tau_w}{\frac{1}{2} \rho u_{\infty}^{2}} = \sqrt{2}\, R_{ex}^{-1/2} f''(0) \qquad (8)$$

where $R_{ex} = \frac{\rho u_{\infty} x}{\mu}$

Figure 1 displays the effect of the permeability parameter M on the distribution of velocity within the boundary layer. The results indicate that the velocity increases as M increases.

Figure 2 shows the results for the velocity distributions in the boundary layer. As the coefficient of inertia parameter N increases, we observe that the velocity increases.

Figure 3 represents the distributions of the velocity within the boundary layer. It is found that the velocity increases with increasing the roughness parameter R_w.

From the numerical results obtained in Table 1, it is observed that:

(a) C_f decreases with the increasing values of R_w while N and M are kept constants.

(b) C_f increases with the increasing values of M while N and R_w are kept constants.

(c) C_f increases with the increasing values of N while M and R_w are kept constants.

Table 1: Summary of our numerical results for $f^{''}(0)$

M	N	R_w =0	R_w =0.05	R_w =0.1	R_w =0.15
0.5	0.1	0.90984	0.880226	0.851642	0.824059
1	0.1	1.14892	1.09587	1.04652	1.00066
2	0.1	1.52077	1.42187	1.33401	25561
0.5	0.3	1.044	1.00471	0.966987	0.930956
1	0.3	1.25873	1.19609	1.13804	1.08434
2	0.3	1.6058	1.49754	1.40151	1.31604
0.5	0.5	1.16342	1.11446	1.06776	1.02349
1	0.5	1.35991	1.28773	1.22107	1.15966
2	0.5	1.6866	1.56899	1.46486	1.3724

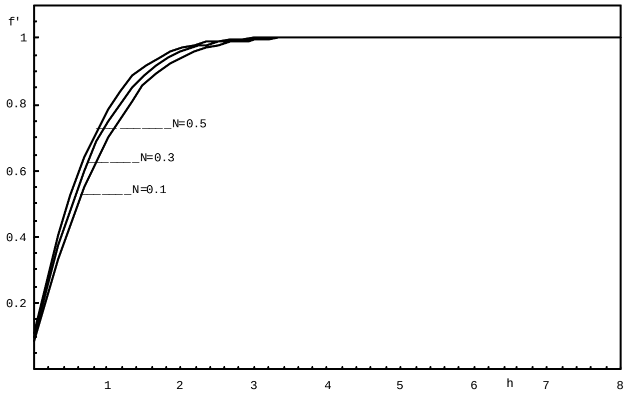

Figure2. velocity profiles for various values of *N* at *M*=0.5 and R_w=0.1

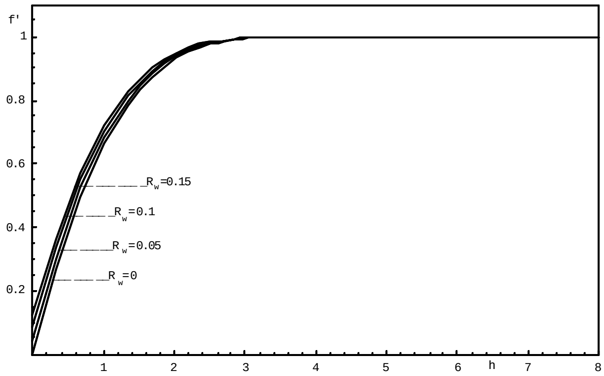

Figure3. velocity profiles for various values of R_w at *N* =0.1 and *M* =0.5

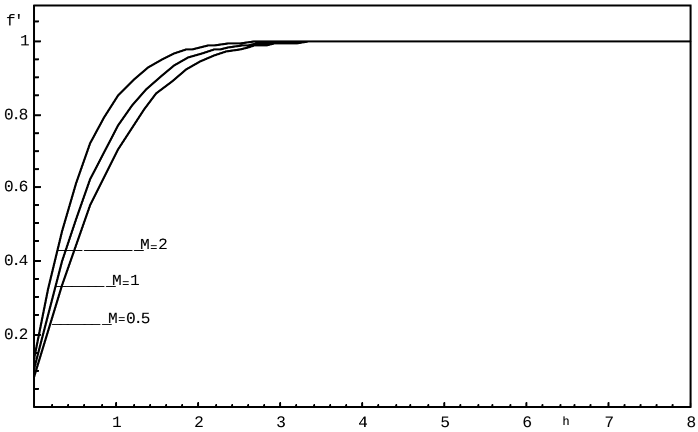

Figure1. Velocity profiles for various values of *M* at *N* = 0.1 and R_w =0.1

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