Fuzzy Linear Programming for the Optimization of Land Use Scenarios

ARKADIUSZ SALSKI [a] and CHRISTIAN NOELL [b]
[a] Institute of Computer Science and Applied Mathematics
University of Kiel, Olshausenstr.40, D-24098 Kiel, GERMANY

[b] Department of Economics and Natural Ressources
Royal Agricultural University, Rolighedsvej 23, DK-1958 Frederiksberg C, DENMARK

Abstract: - In real-life situations we have to deal not only with stochastic uncertainty. Uncertainty can also be caused by imprecise data or vagueness of the semantic meaning of events or statements. The Fuzzy Set Theory provides a modeling technique to handle this class of uncertainty problems. One of these techniques is fuzzy linear programming (LP), a special type of model for decision making. In this paper the "fuzzy" version of a standard LP-problem is presented as the result of "fuzzification" of the primary goal function and constraints (secondary goal functions). Then the approach is applied to the optimization of land use scenarios for a particular farm. The primary goal function "profit for the farmer" and the constraints "share of the grassland" and "leisure time of the farmer family" are defined as fuzzy sets based on linear membership functions. The primary goal function together with the two constraints (secondary goals) are employed as optimization criteria in the fuzzy farm planning model. The goal of this integrative approach is to find the optimal organization of the farm firm, resulting in the highest fulfillment degree or highest degree of satisfaction respectivelly. In economic terms the " maximum satisfaction" derived from the weighted economic, ecological and social criteria can be interpreted as a measure for utility.

Key-Words: - linear programming, fuzzy approach, land use scenarios, linear models, fuzzy optimization

"The strength of the fuzzy set approach is
that it starts from the premise
that nature may be inherently vague or imprecise
and does not try pretend that the real world……
is more exact, or more perfect than it really is."
(Burrough, 1989)

1 Fuzzy logic and operations research

The Fuzzy Set Theory has been applied to quite a number of operations research problems, e.g. logistics, production control, scheduling or some optimization problems in decision making models [6,2]. Real-life situations in these areas are often not crisp and deterministic and they cannot be described precisely. L. Zadeh, the founder of the Fuzzy Set Theory, wrote in [4]: "As the complexity of a system increases, our ability to make precise and yet significant statements about its behavior diminishes until a threshold is reached beyond which precision and significance (or relevance) become almost mutually exclusive characteristics."

Real-life situations are often vague and uncertain; that means not only stochastic uncertainty (appropriately handled by statistics), but also uncertainty which results from the vagueness concerning the description of the semantic meaning of the events or statements themselves, which is called fuzziness [6]. The Fuzzy Set Theory provides powerful modeling techniques that can cope with fuzziness in many operations research problems. The "fuzzification" of standard problems in operation research and decision making often leads to a better approximation of real-life situations by fuzzy models.

One of these problems is the modeling of problem situations closely related to human evaluations and decisions. The search for optimal solutions to such situations requires often the consideration of several criteria which can be in conflict with each other. That can often lead to a very limited (or empty) set of solutions. The "fuzzification" of this multi criteria problem enables us to make better use of imprecise and vage information, fuzzy sets can be used to deal with the imprecision of data and fuzzy logic to handle inexact reasoning in knowledge-based models.

2 Fuzzy linear programming

H.-J. Zimmermann [6] considers linear programming (LP) models as “a special kind of decision model: the decision space is defined by the constraints; the “goal” (utility function) is defined by the objective function; and the type of decision is decision making under certainty”. Such a classical model of linear programming can be defined as follows:

$$\text{Maximize} \qquad F(x) = c^T x = \sum_{j=1}^{n} c_j x_j \qquad (1)$$

with linear constraints

$$(Ax)_i = \sum_{j=1}^{n} a_{ij} x_j \le b_i, \qquad \forall i = 1,\dots,m \qquad (2)$$

$$x_j \ge 0, \qquad \forall j = 1,\dots,n \qquad (3)$$

where: $x, c \in R^n$, $b \in R^m$, $A \in R^{m\times n}$

and F is a goal function.

In a classical linear programming model all coefficients of *A*, *b* and *c* are crisp numbers and all constraints in (2) and (3) are crisp. There is a number of ways for the “fuzzification” of this model. The first one is the “fuzzification” of the goal function, which means that we only want to reach some acceptable levels of this function instead of its maximum (or minimum). The second one is the “fuzzification” of constraints, which means the constraints might be vague, e.g. either we can define the constraint coefficients as fuzzy numbers or the signs $\le$ and $\ge$ might not be meant in a crisp but in a fuzzy sense. For this last way of the “fuzzification” of constraints and for the “fuzzification” of the goal function Zimmermann proposed [6,5] a formulation of the fuzzy linear programming problem (based on Bellman’s and Zadeh ‘s approach, [1]) as follows:

Find $x \in R^n$ such that

$$F(x) = c^T x = \sum_{j=1}^{n} c_j x_j \underset{\sim}{\ge} b_0 \qquad (4)$$

$$(Ax)_i = \sum_{j=1}^{n} a_{ij} x_j \underset{\sim}{\le} b_i, \qquad \forall i = 1,\dots,m \qquad (5)$$

$$x_j \ge 0, \qquad \forall j = 1,\dots,n \qquad (6)$$

where $\underset{\sim}{\ge}, \underset{\sim}{\le}$ means vague constrains

(the “fuzzification” of $\ge$ and $\le$).

We can formulate the constraints (4) and (5) as fuzzy sets with the linear form of the membership functions [3,6]:

$$m_0 : R^n \to [0,1], \qquad (7)$$

$$m_0(x) = \begin{cases} 1, & falls \quad c^T x > b_0 \\ 1 - \dfrac{b_0 - c^T x}{d_0}, & falls \quad b_0 - d_0 \le c^T x \le b_0 \\ 0 & falls \quad c^T x < b_0 - d_0 \end{cases}$$

where $b_0, d_0 \in R$,

for “fuzzy” goal, and

$$m_i : R^n \to [0,1], \; i = 1,\dots,m \qquad (8)$$

$$m_i(x) = \begin{cases} 1, & falls \quad (Ax)_i < b_i \\ 1 - \dfrac{(Ax)_i - b_i}{d_i}, & falls \quad b_i \le (Ax)_i \le b_i + d_i \\ 0 & falls \quad (Ax)_i > b_i + d_i \end{cases}$$

where $b, d \in R^m$,

for the constraints (5).

We can reach the fuzzy decision set *D* (based on Bellman’s and Zadeh’s approach) from:

$$m_D(x) = \min\left\{m_0(x), \min_{i=1,\dots,m} m_i(x)\right\} \qquad (9)$$

for all $x \in R^n$. Now we can find the optimal solution $x_{opt} \in R^n$ with the highest membership value:

$$m_D(x_{opt}) = \sup_{x \in R^n} m_D(x). \qquad (10)$$

Because of the linearity of the membership functions m_0 and m_i, *i*= 1,…,*m*, we can formulate the equivalent standard (crisp) linear programming problem with a new variable l :

Maximize $\quad l \qquad (11)$

such that

$m_0(x) \ge l$

$m_i(x) \ge l, \qquad \forall i = 1,\dots,m$

$x_j \ge 0, \qquad \forall j = 1,\dots,n$

$l \in [0,1]$

where

$l = m_D(x) = \min\{m_0(x), m_1(x), \dots, m_m(x)\}$

for all $x \in R^n$.

The variable l can be interpreted here as the common fulfillment degree for all fuzzy constraints of the model. From (7), (8) and (11) we have:

$$l \le m_0(x) \Leftrightarrow c^T x = \sum_{j=1}^{n} c_j x_j \ge b_0 - (1 - l) d_0 \qquad (12)$$

	Winter wheat	Winter barley	Winter rye	Summer barley	Straw gathering	Winter rape	Sunflowern	Sugar beet	Grassland / pasture	Geese	Liming	Single room	Double room	Holidaylatoolid	Wage fam. ab. SP	Wage fam.lab. RC	Wage fam. lab. GH	Wage fam. lab. RH	Wage fam. lab. RS	Wage fam. lab.RW	Wage SP	Wage RC	Wage GH	Wage RH	Wage RS	Wage RW	Operators	Capacities
Acreage	1	1	1	1		1	1	1	1																		<=	65
Grassland									1																		<=	1
Max. sugar beet acreage								1																			<=	3.3
Max. Wheat acreage	1																										<=	21.7
Max. summer barley ac.				1																							<=	10.8
Max. rape + sunflower + sugar beet acreage						1	1	1																			<=	21.7
Liming area											1																=	22
Straw gathering area	-1	-1	-1	-1	1																						<=	0
Amount of straw					-4.5					2																	<=	0
Feedenergy summer									-2337	2337																	<=	0
Feedenergy winter									-1137	1137																	<=	0
Single room												1															<=	2
Double room													1														<=	5
Holiday flat														1													<=	1
Fam. labor spring (SP)															1												<=	251
Fam. Labor root crop / hay harvest (RC)																1											<=	368
Fam. labor grain harvest (GH)																	1										<=	401
Fam. labor root crop harvest (RH)																		1									<=	410
Rest of fam. labor sum.																			1								<=	276
Rest of fam. labor win.																				1							<=	803
Total family labor															1	1	1	1	1	1							<=	2007.2
Labor hours spring	1.2	1.2	1	2.1		0.3	2.3	5.1				4.8	7.2		-1						-1						=	0
Labor hours root crops/ hay harvest	1.1	1.1	0.4	0.4		0.7	1.9	1.3	12	4		7.2	10.8	9.8		-1						-1					=	0
Labor hours grain harvest	2	2	2	2	2	2.8		0.5		17		12	18	39.8			-1						-1				=	0
Labor h. root crop harvest	5.7	5.7	5.7			1.9	1.2	3.1	12.3	15	0.4	9.6	14.4	19.6				-1						-1			=	0
Rest of labor h. summer				3.4			1.8	7.5		56		4.8	7.2	29.4					-1						-1		=	0
Rest of labor h. winter	0.3	0.3	0.3							130		9.6	14.4							-1						-1	=	0
Raw profit	967	826	813	983	-137	982	708	2225	-169	4186	-90	3454	7025	5942	0	0	0	0	0	0	-15	-15	-15	-15	-15	-15		

Table1: The linear land use model for a Northern German farm.

$$l \le m_i(x) \Leftrightarrow (Ax)_i = \sum_{j=1}^{n} a_{ij} x_j \le b_i + (1 - l) d_i \quad (13)$$

Finally we have:

Maximize l (14)

such that

$$c^T x = \sum_{j=1}^{n} c_j x_j \ge b_0 - (1 - l) d_0$$

$$(Ax)_i = \sum_{j=1}^{n} a_{ij} x_j \le b_i + (1 - l) d_i, \ \forall i = 1,\dots,m$$

$$x_j \ge 0, \qquad \forall j = 1,\dots,n$$

$$l \in [0,1].$$

Now we can find the optimal decision x_{opt} by the solution of this equivalent LP problem using the standard methods of the linear programming.

3 Fuzzy linear model of the land use of a particular farm

As an empirical example the standard linear farm modeling approach as it is regularly applied in agricultural economics was chosen, namely the linear land use model for a Northern German farm [3]. For the purpose of simplicity we consider a highly aggregated conventional LP model – a more realistic farm model would consist of several hundred variables and equations, but would rather decrease than increase the depth of insight we gain from our example. Table 1 shows typical constraints (row numbers 2 to 28) with the technical coefficients a_{ij} (values in columns 2 to 27), b_i (values in the last column) and c_j (last row), corresponding to the coefficients in (1), (2) and (3).

All constraints are formulated as sharply defined upper boundaries in this model, e.g. the first constraint (row No 2): total crop land used for farming should be lower or equal to 65 ha. Such sharply defined boundaries often truly reflect absolute limits of the availability of essential resources like land, labor or capital. Also the structural flexibility of parts of the model (e.g. the substitution of family labor with hired labor or the crop rotation) is accompanied by sharp boundaries of the corresponding restrictions. In the same way input-output balances (e.g. Nitrogen demand of crops and fertilisation) and technical relations are necessarily formulated as sharply bounded restrictions or equations. Of course this may lead to a very limited (or even empty) set of solutions under certain circumstances, but the reason is then (modeling errors left aside) rather to be found in real economic or technical infeasibilities than in ineffective modeling.

Aside from this "natural crispness" of some restrictions sharply defined boundaries are often unrealistic and make decision models less flexible and appropriate. A highly justified criticism to the use of pure economic goal functions in land use planning is for example that secondary goals of the decision maker (here: the farmer and his family) like environmental protection (here indicated by the share of grassland of total crop land) or social wellness (here indicated by the preference for leisure time) can only be included in the decision process in a very limited way if they are represented as conventional constraints.

In these cases the "fuzzification" of this model can be very useful. As shown above the "fuzzification" means here the formulation of the equivalent model with not sharply defined secondary goals (constraints) and the primary goal ($m_i(x)$ and $m_0(x)$ in (7) and (8)).

The primary goal function is "profit for the farmer" (last row). But now we want also take into account ecological and social criteria as additional optimization criteria. For this purpose we "fuzzified" two constraints, that is we defined them as fuzzy sets. The first one was the share of grassland. The highest and lowest possible values of the share of grassland were defined by the farmer (Figure 1). He agreed to increase grassland to more than 1 ha but not bigger than 10 ha (d_i = 9 in this case, see (14)). The second constraint we "fuzzified" was the leisure time of the farmer family. The highest and lowest possible values (hours per year) were also subjectively defined by the farmer.

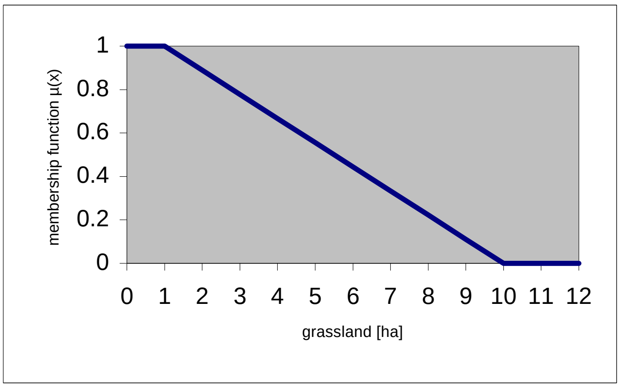

Fig. 1. The fuzzy constraint "share of grassland".

Furthermore we turned the primary goal function of the conventional model into a fuzzy constraint. The new primary goal function contains only one parameter: l, the measure for overall satisfaction with the realization of the different goals. Because all single goals are linked to each other by their link to l, we can now let the economic, ecological and social preferences of the decision maker determine the optimal farm structure instead of using only profit or any other single goal. The main advantage of the fuzzy formulation is that the solution space is increased because explicit substitutions among the goals are possible.

We calculated the highest and lowest possible values of the yield (125590 DM/year and 115058 DM/year, respectively) by means of the standard LP method (simplex method) taking into account the highest and lowest possible values of the grassland share and the leisure time of the farmer family (Figure 2).

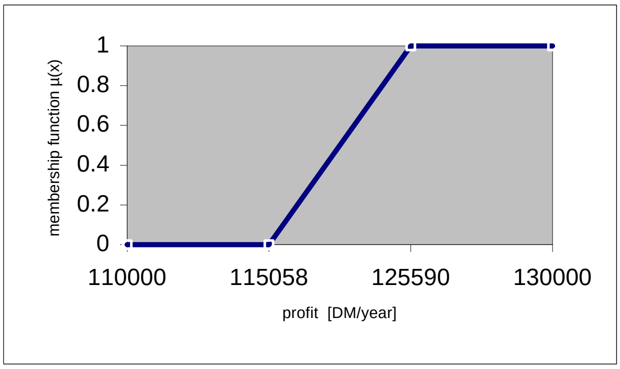

Fig. 2. The fuzzy constraint “profit for the farmer”.

Together with the other constraints of the model the fuzzy constraints define the space in which we search for the optimal solution. Using the equivalent standard model (14) we found the optimal value of the common constraint’s fulfillment l equal to 0.69. The optimal solution x_{opt} of our LP problem is corresponding to the optimum of l. The corresponding value of the grassland size was 3.78 ha (Figure 3), the leisure time of the farmer family 2163 hours per year and the value of the profit for the farmer 122 325 DM per year (Figure 4).

It is quite obvious that the level of l is driven by (1) the range of single goal fulfillment between the membership values 0 and 1 as well as (2) the preference for maximizing or minimizing the individual goal. The other constraints form the technical framework for this multicriteria decision making setting. With a few modifications this setting can be used to investigate different types of decision makers and their influence on land use patterns and social behavior.

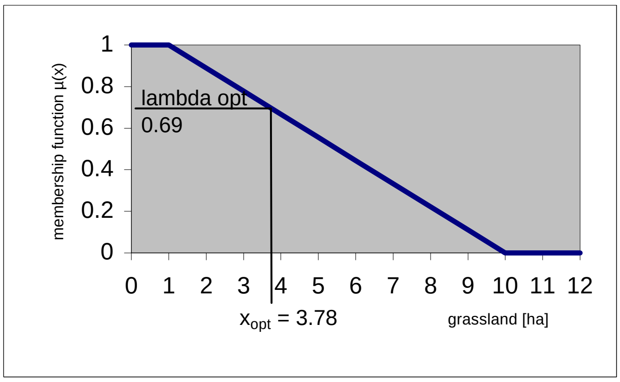

Fig. 3. The optimal solution (x_{opt}) for “share of the grassland”.

If we assume that every farmer would be profit maximizing at least to a certain extent and that he could furthermore be maximizing or minimizing his ecological or social goals, then we can define four basic types of farmers:

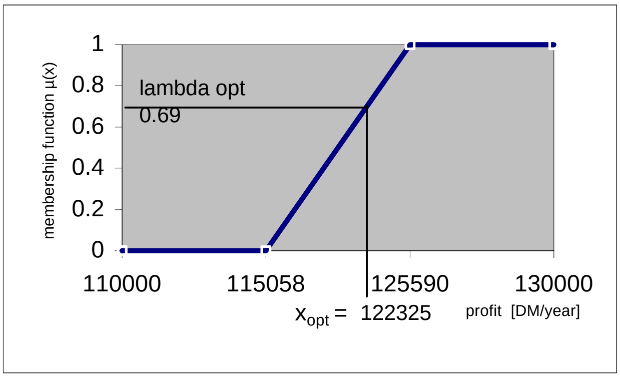

Fig. 4. The optimal solution (x_{opt}) for “the profit for the farmer”.

First, the “hardliner”, that is the environmentally unfriendly workoholic. Secondly, the “ignorant”, that is the lazy ignorant of the natural environment. Third, the “idealist”, that is the hard working friend of the environment. Fourth, the “bon vivant”, that is the easy going friend of the environment. With the adjustment of the above mentioned main drivers of the level of l the fuzzy programming approach offers nearly unlimited possibilities of behavioral modifications.

4 Conclusions

In real-life situations the sharply defined boundaries as the model constraints are often unrealistic. The definition of the model constraints as not sharply defined (fuzzy) boundaries extends the space in which we search for the optimal solution. To find this solution we can use the standard LP tools. This is possible if we formulate the conventional LP model equivalent to our fuzzy model.

Fuzzy LP models can be particularly useful as an integrative approach to linear models with different optimization criteria, e.g. ecological, economical and social criteria. It serves two main purposes, both reduce conflicts among single goals or restrictions and to model certain preference structures which have behavioral implications and as such influence individual land use preferences. It is intuitively easy to understand that this context can be extended to higher scales of land use or use of other natural resources. Furthermore other and more detailed indicators for single goals could be applied.

References

[1] Bellmann, R.E. and Zadeh, L.A., Decision making in a fuzzy environment, *Management Science*, Vol. 17, No. 4, 1970, pp. 141-164.

[2] Dubois, D. and Prade, H., *Fuzzy sets and systems: theory and applications*, Academic Press, 1980.

[3] Quang Hai Tran, *Entwicklung einer LP-Prozedur für Fuzzy-Optimierungsprobleme*, M.Th., University of Kiel, 1999.

[4] Zadeh, L.A., *The concept of a linguistic variable and its application to approximate reasoning*, Memorandum ERL-M Berkeley, 1973.

[5] Zimmermann H-J., *Description and optimization on fuzzy systems*, International Journal of General Systems, Vol. 2, 1976, pp. 209-215.

[6] Zimmermann H-J., *Fuzzy set theory and its applications*, Kluwer Academic Publishers, 1996.