# Isolated Regions in Ad Hoc Networks 

MARKUS BORSCHBACH and WOLFRAM-M. LIPPE<br>Dep. of Mathematics and Natural Science, Institute for Computer Science, University of Münster, Germany, Einsteinstr. 62, D-48149 Münster,


#### Abstract

The major prerequisites for successful wireless ad hoc networkingare an almost homogeneous distribution of a nontrivial number of nodes and the determination of an almost ideal selective connectivity of the nodes in the network. To give a basic characterization of network connectivity, an ad hoc network model based on planar graphs is introduced. According to this underlying mathematical network description, the features of homogeneous connectivity for ad hoc networks are defined. Due to a specific physical layer ratio of wireless capacity utilization, a condition of isolation gives the opportunity to maintain isolated areas in any given ad hoc network distribution. To support identified isolated regions is a main goal of a successfully operating hybrid transfer network.


Key-Words: Ad hoc network models, Modeling inhomogeneous distributions, Condition of isolation.

## 1 Introduction

The rapidly expanding and changing market for wireless communication technologies through the last ten years has encouraged even a great amount of research towards an ad hoc networking future. Most of this research was directed either to routing protocols, optimization of medium access or requirements for a real time quality of service and mobile IPnetworking. One underlying assumption of this research is an acceptable degree of node density almost homogeneously distributed [11][14][15]. Some authors have termed ad hoc nets with a large scale of nodes analytically intractable [8].
This paper is introducing reliable models of connectivity in ad hoc networks up to a large scale of network nodes. Section 2 reviews the basic ad hoc networking principles, and a mathematical model is specified, based on notations similar to terms in graph theory. This general ad hoc network definition covers technical details of the underlying physical antenna system. Following the general node and link model, a connected network model is introduced.
In section 3, the local degree of connectivity, a multiple path and the three kinds of a connected net model are defined. These definitions of a homogeneous ad hoc network shape are used to mark the condition and degree of isolation. Finally, modeling of inhomogeneous ad hoc networks, based on the number of isolated areas, is introduced. The model of inhomogeneity, based on the number of independent paths among two different isolated areas, the degree and the condition of isolation are presented in section 5. Based on one example of a simple ad hoc network node distribution, a characterization of inhomogeneity is presented in subsection 5.1. Typical larger distributions of network nodes termed very small village, cities or china town are analyzed in a similar manner and are briefly reviewed in subsection 5.2 .

## 2 Basic Networking Principles and Graph Notation

The basic principles of a self-organizing wireless network maybe set down in two rules: Each node communicates wirelessly with nodes in its immediate vicinity. A connection between two nodes is made by reservation of wireless bandwidth between a chain of nodes, forming a route, step-by-step,
circuit channels or packets switched [7] from the source node to the destination node. The basic routing is executed in a decentralized manner, node by node [17][16]. Due to the principle of accessing wireless bandwidth, neighbor to neighbor from the calling node to the target node, the network can be termed multi-hop and peer to peer [13]. The basic route construction scheme is sequential. According to a taxonomy of routing [7] it is mainly uni-cast, distributive, adaptive, progressive or backtracking.
A graph model $N=(V, L)$ of network connectivity (1) provides a formal model of an ad hoc net. The network model is based on the number of nodes $|V|$, with a formal description given by the node model $V$ (3).

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- Net-model \(N:=\)
    \(\left\{\left((\right.\right.\) node \(),\left\{\right.\) node \(_{i}\), node \(\left.\left._{j}\right\}\right)||V|=|\{(\) node \() \mid\) node \(\in V\} \mid\)
    \(\wedge\) node \(\in V \wedge\) node \(_{i}\), node \(_{j} \in L \wedge|L|=\mid\left\{\right.\) node \(_{i}\), node \(\left.\left._{j}\right\} \mid\right\}\)
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From the transmission range of the node model $d_{j}=$ distance $_{\text {wireless }}$, the number of possible links $|L|=\mid\left\{\right.$ node $_{i}$, node $\left._{j}\right\} \mid$ with a formal description based on the link model $L$ (2) can be determined. All possible links can be termed edges of an undirected graph. A link exists, if the positions of both nodes $\left(\right.$ pos $_{i}=$ position $\left(\right.$ node $\left._{i}\right)$ ) are placed within the reach of the other nodes'radio interface ( $d_{i} \geq\left|p o s_{i}-\operatorname{pos}_{j}\right| \leq d_{j}$ ). A set of links $L$ is defined by all combinations of two different nodes in a given set of nodes $V$ fulfilling this link condition.

- Link-model : $L:=\left\{\left\{\right.\right.$ node $_{i}$, node $\left._{j}\right\} \mid$

$$
\begin{equation*}
\text { node } \left.\in V \wedge \operatorname{pos}_{i} \neq \operatorname{pos}_{j} \wedge d_{i, j} \geq\left|\operatorname{pos}_{i}-\operatorname{pos}_{j}\right|\right\} \tag{2}
\end{equation*}
$$

- Node-model : $V:=\{($ node $) \mid$
pos $=\operatorname{position}($ node $) \wedge d=$ distance $_{\text {wireless }}($ node $\left.)\right\}$
A necessary prerequisite for a low density of nodes performing an ad hoc network is that the transmission range be adjusted to the maximum distance of the radio interface. A communication is impossible, if the neighbors of each node were out of reach of the given maximum distance $d$ of the radio interfaces. A prerequisite for communication is an existing path from the
source to the target. An ad hoc network path $W$ is defined (4) as a linear sequence, $(\exists n \in \mathcal{N} \wedge n \geq 2 \wedge$ node $\in$ $V \wedge$ node $_{1}, \ldots$, node $_{n} \subseteq V$ ) forming a subset of different nodes in a network $N$. For every node in the subset of nodes termed path $\left(\right.$ node $_{i} \cap$ node $\left._{j}=\emptyset, \forall i \forall j \in \mathcal{N}, j \leq n\right)$, except the source $\left(\right.$ node $_{1}=$ node $\left._{A}\right)$ and the target node $\left(\right.$ node $_{n}=$ node $\left._{E}\right)$, a link to the preceding node of the sequence exists $\left(\left\{\right.\right.$ node $_{i}$, node $\left._{i-1}\right\} \in W \mid \exists\left\{\right.$ node $_{i}$, node $\left._{i-1}\right\} \in L$ ) and a link to the following node of the sequence exists $\left(\left\{\right.\right.$ node $_{i}$, node $\left._{i+1}\right\} \in W \mid \exists\left\{\right.$ node $_{i}$, node $\left.\left._{i+1}\right\} \in L\right)$. It follows that, for the path sequence of nodes to the successor of the first node, a link exists $\left(\left\{\right.\right.$ node $_{1}$, node $\left._{2}\right\} \in W \mid \exists\left\{\right.$ node $_{1}$, node $\left.\left._{2}\right\} \in L\right)$ and to the predecessor of the last node, a link exists also $\left(\left\{\right.\right.$ node $_{n}$, node $\left._{n-1}\right\} \in W \mid \exists\left\{\right.$ node $_{n}$, node $\left.\left._{n-1}\right\} \in L\right)$.

The first necessary condition of the network is to have at least one possible connection between every pair of nodes via a route of other nodes, all direct neighbors being in reach of each others'radio interface. In the connected network model $N^{+}$(5), for all permutations of two different nodes $(V)$ a path $W$ exists $\left(\exists W\left(\right.\right.$ node $_{i}$, node $\left.\left._{j}\right) \subseteq V\right)$.

- Connected network model : $N^{+}$:=

$$
\begin{align*}
& \left\{N \mid \forall \text { node }_{i}, \text { node }_{j} \in V \wedge \text { node }_{i} \neq \text { node }_{j}:\right.  \tag{5}\\
& \left.\exists W\left(\text { node }_{i}, \text { node }_{j}\right) \subseteq V, i, j \in\{1, \ldots|V|\}\right\}
\end{align*}
$$

A connected network model is a common prerequisite for communication networks. For channel-switched ad hoc networks [3], due to the demand of real time traffic, a connected network graph is a condition of similar significance. In a packetswitched ad hoc network all parts of a path should exist at least ones within a given period of time.
Avoiding bottlenecks caused by insufficient routing intelligence is one of the major purposes of designed routing protocols. The idea of a routing algorithm for the introduced wireless ad hoc network is to maximize the acceptable load and to avoid local congestion through a homogeneous loading of the ad hoc network elements, with a minimum of messaging overheads. The different routing protocols [11][14][17] introduced in the last decades can be distinguished according to their routing overheads for topology changes, overheads for mobility registry support, and their added logical hierarchy types for supporting real time services and maximum frequency reuse (see Corson and Park in [14] for example).
The key condition for successful ad hoc networking is a sufficient connectivity. A detailed exploration of the required degrees of connectivity and of an acceptable isolation condition is given in the following sections.

## 3 Degree of Connectivity

Based on the model of a connected ad hoc network $N^{+}$(5) with at least one path between all pairs of different nodes, a degree of node connectivity $d e g_{\text {con }}\left(\right.$ node $\left._{h}\right)$ is defined to distinguish between the subsets of nodes with different connectivity. The degree of node connectivity of a certain node $_{h}$ in a network $N=(V, L)$ is specified (6) by the number
of existing different links $L$ being in coincidence with node $h_{h}$. A subset of nodes $V^{i}$ with a specified node connectivity $i$ is defined (7) by all nodes in a given network $N=(V, L)$ with a degree of node connectivity equal to $i$. In addition, a subset of nodes with a degree of connectivity higher ( $>i$ ) or equal $i$ is defined (8) by all nodes with the corresponding degree in the network.

$$
\begin{align*}
& \text { Local degree of node connectivity }: \operatorname{deg}_{\text {con }}\left(\text { node }_{h}\right):= \\
& \qquad \begin{aligned}
& \mid\left\{\left\{\text { node }_{i}, \text { node }_{j}\right\} \in L \mid \text { node }_{i}=\text { node }_{h} \wedge \text { node }_{j} \neq \text { node }_{h}\right\} \mid \\
& V^{i}:=\left\{\text { node } \in V \mid \operatorname{deg}_{\text {con }}(\text { node })=i\right\} \\
& V^{\geq i}:=\left\{\text { node } \in V \mid \operatorname{deg}_{\text {con }}(\text { node }) \geq i\right\}
\end{aligned} \tag{6}
\end{align*}
$$

Obviously, for all nodes of the connected network model $\left(N^{+}=(V, L),(5)\right)$ at least one link $\left(\forall\right.$ node $_{i} \in V \exists$ node $_{j} \in$ $V:\left\{\right.$ node $_{i}$, node $\left._{j}\right\} \in L \wedge$ node $_{i} \neq$ node $\left._{j}\right)$ exists.
The definition of a simple connected network model $N^{*}(9)$ includes a basic amount of connectivity changes and excludes the minimum topologies mentioned before. The overall number of nodes in a simple connected net model is much greater than three $(|N| \gg 3)$ and the mightiness of the subset of nodes with a local node connectivity of two or one $\left(\left|V^{1} \cup V^{2}\right| \leq\right.$ $\left|V^{\geq 3}\right|$ ) is smaller than or equal to the mightiness of the subset of nodes with a local node connectivity of three or greater (node $\in V^{\geq 3} \mid$ deg $_{\text {con }}($ node $) \geq 3$ ).

- Simple connected net model : $N^{*}:=\left\{N^{+} \mid\right.$

$$
\begin{equation*}
\left.|V| \gg 3 \wedge \forall \text { node } \in V:\left|V^{1} \cup V^{2}\right| \leq\left|V^{\geq 3}\right|\right\} \tag{9}
\end{equation*}
$$

The number of nodes with a low node connectivity includes nodes with a changing connectivity.

## 4 Intensity of Connectivity

Prior to the definition of a multiply connected network model, a multiple path has to be defined. A multiple path (4) is defined by the number of independent paths (4) with identical source nodes $\left(\right.$ node $\left._{A}\right)$ and target nodes (node $\left.e_{E}\right)$. Except the first and the last node, these paths have no further nodes in common.

- Multiple path : $W^{i}\left(\operatorname{node}_{A}, \operatorname{node}_{E}\right):=$
$\left\{W \subseteq N \mid \forall h, j \in \mathcal{N} \wedge i>1 \wedge h, j>0 \wedge \forall j \leq i \exists W_{j} \subset N\right.$
$\wedge \forall h \neq j: h \leq i \wedge W_{h} \cap W_{j}=\left(\right.$ node $_{A}$, node $\left.\left._{E}\right)\right\}$
A multiply connected net model $N^{* i}(11)$ is based on the structure of the simple connected net model (9) as a prerequisite. The changeable intensity $i \in \mathcal{N} \wedge i>1$ defines the number of multiple paths between all combination of different nodes with a local node connectivity degree greater or equal $i$ (node $\in V^{\geq}{ }^{i}$ ). The mightiness of all nodes with a local node connectivity of $\geq i$ (node $\in V^{\geq i}$ ) is greater than or equal to the mightiness of all nodes with a local node connectivity smaller than $i$ (node $\in V^{<i}$ ).
- Multiple connected net model : $N^{* i}:=\left\{N^{*} \mid \forall\right.$ node $\in V$ :

$$
\begin{align*}
& \left|V^{1} \cup V^{2}\right|<|V \geq i| \wedge \forall \text { node }_{h}, \text { node }_{j} \in V \geq i  \tag{11}\\
& \text { node } \left._{h} \neq \text { node }_{j}: \exists W^{i}\left(\text { node }_{h}, \text { node }_{j}\right) \in N^{*}\right\}
\end{align*}
$$

In addition, a strict connected net model $N^{\bowtie s}(12)$ is defined on the structure of the simple net model (9). In the strict model, all nodes in the network have a local node connectivity equal to or greater than the given predefined intensity $s$ of the strict network model (node $\in V^{\geq}$).

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- Strict connected net model : \(N \bowtie s:=\left\{N^{*} \mid\right.\)
\(\forall\) node \(\in N^{*}:\) node \(\in V \geq s \wedge \forall\) node \(_{i}\), node \(_{j} \in V\),
node \(_{i} \neq\) node \(_{j}: \exists W^{s}\left(\right.\) node \(_{i}\), node \(\left.\left._{j}\right) \subseteq N^{*}\right\}\)
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The model definitions of connectivity in this section and the definitions of node connectivity in the previous section form the foundation of the common minimum topology connectivity of the whole network or in certain areas. The definition of the connected net model, the simple net model and the multiple model define the essential connectivity based on the local node connectivity of the major subset of nodes and the intensity of alternative paths beyond these major subset of nodes. The mightiness of low connective nodes can be either lower than or equal to the majority of highly connective nodes. For an ad hoc net environment the mightiness of nodes with a lower connectivity corresponds to the number of nodes that are moving or recently have been moving. In a quickly changing environment of all nodes, the majority of nodes represents the foundation of a stable [6][18][5] minimum topology connectivity.
In general, the second necessary condition for successful ad hoc networking is to have a relatively homogeneous node distribution, avoiding extremes. It is easy to see, that with a given bandwidth for each node to relay, a bottleneck in capacity is coming up soon between two extreme points of node density. In the following section, the model of topology connectivity is supplemented with an model of inhomogeneity, showing the magnitude and conditions of isolation.

## 5 Model of Inhomogeneity

To distinguish between areas of different connectivity, a model of inhomogeneity is specified, based on the connected net model (5) defined in one of the previous sections. Within two different areas of a connected network topology, the intensity of connectivity is defined by the number of different independent paths (13). The intensity of connectivity is defined by the number $i$ of independent paths $W^{i}$ within two unequal ( $V_{f}^{*} \cap V_{g}^{*}=\emptyset$ ) areas $N_{f}^{*}, N_{g}^{*}$ of the underlying connected net model $N^{+}$. Each source node of a path is part of one of the two areas, and the target is part of the other area $\left(W^{i}\left(\right.\right.$ node $\left.\left._{A} \in N_{f}^{+}, \operatorname{node}_{E} \in N_{g}^{+}\right)\right)$. Except the first and the last node as elements of both areas, none of the paths have further nodes in common $\left(W_{h} \cap W_{j}=\left(\operatorname{node}_{A} \in N_{f}^{+}\right.\right.$, node $\left.\left._{E} \in N_{g}^{+}\right)\right)$.

- Independent paths between two areas: $W_{N_{f}^{+}, N_{g}^{+}}^{i}:=$

$$
\begin{align*}
& \left\{W^{i} \subseteq N^{+} \mid\right. \\
& N_{f}^{+}, N_{g}^{+} \subset N^{+} \wedge \cap_{s=f, g}\left(N_{s}^{+}\right)=\emptyset \wedge h, j \in \mathcal{N} \\
& i>1 \wedge h, j>0 \wedge \forall j \leq i \exists W_{j} \subset N \wedge \forall W_{h} \neq W_{j}:  \tag{13}\\
& \left.h \leq i \wedge W_{h} \cap W_{j}=\left(\text { node }_{A} \in N_{f}^{+}, \text {node }_{E} \in N_{g}^{+}\right)\right\}
\end{align*}
$$

A set $W^{i}$ of paths between two different areas determines all possible sets of different paths with a mightiness of paths equal to $i$. To characterize the degree of connectivity $\operatorname{deg}\left(W_{N_{f}^{+}, N_{g}^{+}}\right)$ between two different areas $\left(N_{f}^{*} \cap N_{g}^{*}=\emptyset\right)$, the maximum number of independent paths has to be defined (14).

- Maximum number of independent paths

$$
\begin{align*}
& \operatorname{deg}\left(W_{N_{f}^{+}, N_{g}^{+}}\right):=\mid\left\{W \subseteq N^{+} \mid W=W^{h} \wedge\right. \\
& |W|=h \wedge N_{f}^{+}, N_{g}^{+} \subset N^{+} \wedge \cap_{s=f, g}\left(N_{s}^{+}\right)=\emptyset  \tag{14}\\
& \left.\wedge h, i \in \mathcal{N} \wedge \exists W_{N_{f}^{+}, N_{g}^{+}}^{h} \forall W_{N_{f}^{+}, N_{g}^{+}}^{i}: h \geq i\right\} \mid
\end{align*}
$$

To classify inhomogeneous planar ad hoc net topology and connectivity, at least one or more different interrelated parts of a connected network $\left(N^{+},(5)\right)$ are assumed. The magnitude $s$ of the inhomogeneous network model (15) defines the number of isolated areas $\left(N_{j} \mid\left(V_{j} \subset N^{+} \wedge N_{j} \hat{\equiv} N^{(+\vee * \vee * i \vee \bowtie h)}\right)\right)$. The network connectivity of an isolated area is either equal to the connected model $N^{+}$(Def. 5) or to one of the models of higher connectivity, the simple model $N^{*}$ (Def. 9), the multiple model $N^{* i}$ (Def. 11), or the strict model $N^{\bowtie s}$ (Def. 12).

An area is termed isolated, if the following condition of isolation is satisfied. This condition of isolation made up of three sub conditions. All nodes of a areas'subset of nodes are interrelated, are part of exactly one area $\left(\forall N_{<>j}: V_{j}<>V_{<>j}\right)$, and all nodes which are elements of one area are not elements of another area or the rest of the network $\forall N_{j}: V_{j}<>V^{+} \backslash V_{j}$.

$$
\begin{align*}
& \text { Inhomogeneity model based on a connected net : } \\
& I^{+s}:=\left\{N^{+} \mid j \in \mathcal{N}, s>0, j>0, j \leq s \wedge\right. \\
& \forall j \exists N_{j}:\left(V_{j} \subset N^{+} \wedge N_{j} \hat{=} N^{(+\vee * \vee * i \vee \bowtie h)}\right. \\
& \wedge\left(\left|N_{j}\right| \leq\left|N^{+} \backslash N_{j}\right|:\left(\alpha \operatorname{deg}\left(W_{N_{j}, N \backslash N_{j}}\right) \ll\left|N_{j}\right|\right)\right. \\
& \left.\vee\left|N^{+} \backslash N_{j}\right|<\left|N_{j}\right|:\left(\alpha \operatorname{deg}\left(W_{N_{j}, N \backslash N_{j}}\right) \ll\left|N \backslash N_{j}\right|\right)\right) \\
& \wedge \forall N_{<>j}:\left(V_{j}<>V_{<>j} \wedge\left(\left|N_{j}\right|<\left|N_{<>j}\right|:\right.\right. \\
& \left(\alpha \operatorname{deg}\left(W_{N_{j}, N_{<>j}}\right) \ll\left|N_{j}\right|\right) \\
& \left.\left.\left.\vee\left|N_{<>j}\right|<\left|N_{j}\right|:\left(\alpha \operatorname{deg}\left(W_{N_{j}, N_{<>j}}\right) \ll\left|N_{<>j}\right|\right)\right)\right)\right\} \tag{15}
\end{align*}
$$

The second sub condition is satisfied, if the number of independent paths (14) is much smaller than a certain system specific standard value. This value of isolation is defined by the ratio of the wireless capacity usage of a node $\alpha$ to the maximum number of independent paths between the isolated area and the rest of the network.
The wireless capacity usage ratio $\alpha$ determines the minimum amount of capacity a node is able to relay from a predecessor node to a successor node.
The necessary number of independent paths between the isolated area and the rest of the network, depends on the number of nodes in the isolated area, if the mightiness of nodes in the isolated area is lower than or equal to the mightiness of the rest of the network. The second isolation condition is satisfied, if the product of the number of independent paths existing between the area and the rest of the network and the wireless capacity usage is much lower than the number of nodes in the isolated area $\left(\left|N_{j}\right|<\left|N^{+} \backslash N_{j}\right|:\left(\alpha \operatorname{deg}\left(W_{N_{j}, N \backslash N_{j}}\right) \ll\left|N_{j}\right|\right)\right)$.
The necessary number of independent paths between the isolated area and the rest of the network, depends on the number of nodes in the rest of the network, if the mightiness of nodes in the rest of the network is lower than or equal to the mightiness of nodes in the isolated area. In this case, the second isolation condition is satisfied, if the product of the number of independent paths existing between the area and the rest of the network and the wireless capacity usage is much lower than the number of nodes in the rest of the network.
If the mightiness of the rest of the network is lower than in the isolated area, the necessary number of independent paths between the isolated area and the rest of the network depends on the product of the overall number of nodes in the rest of the network and the wireless capacity usage ratio $\left(\left|N^{+} \backslash N_{j}\right| \leq\left|N_{j}\right|\right.$ : $\left(\alpha \operatorname{deg}\left(W_{N_{j}, N \backslash N_{j}}\right) \ll\left|N \backslash N_{j}\right|\right)$.
The third sub condition of isolation is satisfied, if the second condition holds for every set of two different isolated areas.

### 5.1 Characterization of Inhomogeneity

To illustrate how to determine inhomogeneity of distribution within a given ad hoc network, a simple example can be found
in fig. 1. The general connectivity of the given network topology can be expressed by the simple connected network model (5) . Obviously, areas with more than the basic connectivity of the net do exist. In the upper part, an isolated area with the connectivity degree three $\left(N_{1}^{* 3}\right)$ of the multiple network model (11) can be found. For every pair of different nodes inside the isolated area, three independent paths exists.
In the lower part of the given distribution (fig. 1), a second iso-


Figure 1: Characterization of inhomogeneity
lated area with more than the basic connectivity compared to the basic connectivity of the net exists. For this area a network connectivity degree of two $\left(N_{1}^{* 2}\right.$ ) within the multiple network model can be maintained.
As a prerequisite to maintain both isolated areas, the capacity usage ratio $\alpha$ of the network nodes should not satisfy the isolation condition. If the wireless capacity usage $\alpha$ is satisfying, the isolation condition doesn't hold
If the wireless usage $\alpha$ is not satisfying, the maintained fragmentation into inhomogeneous isolated areas for this simple case gives an intuitive example for an application of the inhomogeneity model. Refer to the related work [1] for extensions of the introduced inhomogeneity model (15) to a simple, a multiple and a strict inhomogeneity model.
In the following section, typical larger distributions of network nodes termed very small village, cities or china town are analyzed in a similar manner and are briefly reviewed.

### 5.2 Typical Larger Distributions

To illustrate how to determine inhomogeneity of distribution within typical larger ad hoc networks, some examples have been chosen (refer to fig. 2-4). The general connectivity of the given network topology cities (fig. 2) can be expressed by the simple connected network model (5). Obviously, areas with more than the basic connectivity of the net do exist. In the lower left part, an isolated area with a connectivity degree three $\left(N_{1}^{* 3}\right)$ of the multiple network model (11) can be found. For every pair of different nodes inside the isolated area, three independent paths exists.
If the wireless usage $\alpha$ is below 9 , the second isolation condition $\left(\left|N_{j}\right|<\left|N^{+} \backslash N_{j}\right|:\left(\alpha \operatorname{deg}\left(W_{N_{j}, N \backslash N_{j}}\right) \ll\left|N_{j}\right|\right)\right)$ for the maintained fragmentation among the isolated region $N_{1}^{* 3}$ $\left.\left(\left|N_{1}^{* 3}\right|=67, \operatorname{deg}\left(W_{N_{1}^{* 3}, N^{*} \backslash N_{1}^{* 3}}\right)=8\right)\right)$ and the rest of the network $\left(\left|N^{*} \backslash N_{1}^{* 3}\right|=133\right)$ is satisfied (Def. 5).
In the second example, termed china town (fig. 3) an isolated region is identified that is less intuitive. In the upper right part of figure 3, the maintained fragmentation of the isolated region $N_{1}^{* 2}$ is chosen. An area almost similar to the basic connectivity


Figure 2: Distribution cities and identified isolated region $N_{1}^{* 3}$
of the net do exist. For every pair of different (almost all) nodes inside the isolated area, two independent paths exist.
If the wireless usage $\alpha$ is below 5 , the second isolation condition $\left(\left|N_{j}\right|<\left|N^{+} \backslash N_{j}\right|:\left(\alpha \operatorname{deg}\left(W_{N_{j}, N \backslash N_{j}}\right) \ll\left|N_{j}\right|\right)\right)$ for the maintained fragmentation among the isolated region $\left.\left.N_{1}^{* 2}=26, \operatorname{deg}\left(W_{N_{1}^{* 2}, N^{*} \backslash N_{1}^{* 2}}\right)=5\right)\right)$ and the rest of the network $\left(\left|N^{*} \backslash N_{1}^{* 2}\right|=174\right)$ is satisfied (Def. 5). Finally, the


Figure 3: Distribution china town and identified region $N_{1}^{* 2}$
distribution termed village (fig. 4) shows twenty very small groups of five network nodes. In the right part of figure 4, an isolated region $N_{1}^{* 5}$ can be identified. For every pair of different nodes inside the isolated area and almost in the whole network, five independent paths exist.
If the wireless usage $\alpha$ is below 3 , the second isolation condition $\left(\left|N_{j}\right|<\left|N^{+} \backslash N_{j}\right|:\left(\alpha \operatorname{deg}\left(W_{N_{j}, N \backslash N_{j}}\right) \ll\left|N_{j}\right|\right)\right)$ for the maintained fragmentation among the isolated region $\left.\left.N_{1}^{* 5}=55, \operatorname{deg}\left(W_{N_{1}^{* 2}, N^{*} \backslash N_{1}^{* 2}}\right)=15\right)\right)$ and the rest of the network ( $\left|N^{*} \backslash N_{1}^{* 5}\right|=45$ ) is satisfied (Def. 5).
Among the four example distributions (the first out of previous section), the satisfaction of the isolation condition in the last example is very weak.
For each of the presented examples, the isolated areas are either equal to the underlying type of net model or a shift ahead in the range of net model connectivity. The extended versions of the model are derivatives of the inhomogeneity model based on the connected model introduced and applied to the simple example in the preceding section and the larger examples in this section.


Figure 4: Distribution village and isolated region $N_{1}^{* 5}$

## 6 Conclusion

A number of ad hoc network models have been described. Finalized by a model of inhomogeneity, certain measurements for ad hoc network connectivity have been specified. Based on a specific physical layer ratio of wireless capacity usage, the defined condition of isolation gives the opportunity to recognize isolated areas within a given ad hoc network.
To forestall possible bottlenecks in ad hoc networks in a given environment is the main purpose of this approach. The approaches to ad hoc networks in the last decades have focused mainly on simulation studies of routing algorithms or theoretical comparative studies on routing complexities due to a changing ad hoc environment or an added hierarchy type for real time service, mobility registry and maximized frequency reuse. The approach here suggested explicates inhomogeneity for any given distribution of ad hoc nodes, including covering even the moving nodes within low connectivity.
With the help of the models presented, ad hoc node distributions causing capacity bottlenecks can be distinguished from more homogeneous node distributions. Almost homogeneous ad hoc networks are a challenge due to optimized routing. The specified model gives the opportunity to distinguish between bottlenecks in capacity and difficulties due to routing optimization in any given specification and distribution of an ad hoc network. Based on a simple intuitive example and larger ad hoc network node distributions, a characterization of inhomogeneity have been presented.

## 7 Work in Progress

Instead of a common constant ratio for the usage of wireless capacity in the isolated condition, the task is to find a certain system specific characterization, depending on a detailed system specification. Another task is to determine exactly the scale of the necessary number of alternative paths, if the number of nodes in the isolated areas and the rest of the network becomes enormous.

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