Thermionic Current Modeling and Equivalent Circuit of a III-V MQW P-I-N Photovoltaic Heterostructure

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Abstract: - We explore the relation between photogeneration and recombination currents in an illuminated III-V p-i-n multiple quantum well (mqw) solar cell, via an improved p-n junction equivalent circuit. P-i-n solar cells with their intrinsic region replaced by a multilayered heterostructure (e.g. GaAs-AlGaAs thin successive layers) are devices of exceptional performance, when illuminated, for two main reasons (a) they provide wider optical windows to incident solar photons, compared to their bulk counterparts, and therefore higher number of photo-excited carriers and (b) they reduce recombination in the intrinsic region, thus leading to more pronounced thermal currents out of the quantum wells. In the present communication, incident photons are modeled as two current sources representing photogenerated (I_{ph}) and recombining electrons (I_{r}) respectively, along with the shunt and series resistance of such devices. General relations are developed for open circuit voltage \( V_{oc} \), short circuit current \( I_{sc} \), incident power, all as functions of net (after recombination) thermionic currents \( I_{th} = I_{ph} - I_{r} \). The basic device equivalent circuit is improved by including (a) a photogeneration current-source and (b) a recombination (due to losses) current-source, the net effect of which is a third thermionic current source \( I_{th} \). The latter circuit element strongly depends on the incident photon flux and on the selected geometry of the p-i-n heterostructure device.

Key-Words: - Solar cells, equivalent circuit, photogeneration, recombination

1. Introduction
Photo-carriers in GaAs-AlGaAs quantum well systems are subject to recombination mechanisms that may lead to a reduction of overall collected currents. Single or multiple quantum wells are useful probes for the study of trapped carriers that may escape recombination and eventually contribute to thermionic currents of various types of devices. General theoretical expressions are developed here, for trapped excess carriers in such quantum wells, when illuminated through incident photon flux \( G(\lambda) \). The situation of intrinsic and/or lightly doped quantum wells based on III-V semiconductor layers (e.g. low-gap GaAs layers grown on wide-gap AlGaAs layers) has become a common p/i/n solar cell design aiming to better understanding [1,2] and overall device improvement. In general, two types of quantum well designs are of interest: (a) quantum wells are far from each other (AlGaAs layers are much thicker compared to narrow GaAs layer widths) ensuring negligible tunneling current contribution and (b) thin potential barriers that may sustain tunneling. In either case, when the device is illuminated, excited carriers (coming from generated electron-hole pairs (ehp’s)) may find themselves in the quantum wells,
where they face two options: either recombine or contribute to current. Now, depending on the geometry design mentioned above, these carriers might either escape thermionically to the continuum of the conduction band or tunnel through thin potential barriers to be collected at the end of the device. In the absence of any type of tunneling, it is desired to analytically calculate any non-negligible thermionic current density (*in mA/cm²/per quantum well*), by considering low background doping of the GaAs layers (thus, minimizing impurity scattering).

2. **Theory**

2.1 **Carrier diffusion**

The device structure in mind is a p/i/n GaAs-AlGaAs solar cell with the intrinsic region (length $L_i$) comprised of a sequence of quantum wells and potential barriers made out of low and wide gap GaAs and AlGaAs layers respectively. Miniband solutions are expected to exist in these finite quantum wells, so that they may serve as traps of photogenerated carriers arising from the valence band after optical excitation. It is expected that illumination of the intrinsic region will cause direct generation of electron-hole pairs (ehp’s), thus contributing to increased carrier-concentration in each quantum well. Such population increase is then expected to escape from the wells into the conduction band continuum leading to prospective current density components. The task here is to propose a method of (a) determining photo-carrier concentration $\delta N$ (in cm$^{-2}$) in each well, and (b) evaluating thermal currents out of each quantum well. In the process, two recombination mechanisms are taken into account, namely, Auger and radiation recombination, while Shockley-Read-Hall (SRH) recombination is not included, for not being dominant in direct-gap systems as the one adopted here [3,4,5].

A general method of dealing with the diffused photo-carriers has been provided elsewhere [6]. Photo-carriers are separated by the built-in field and then diffuse along the growth direction of the device. The latter event is described via the diffusion equation solution:

$$\delta n(x) = n_1 e^{x/L_n} + n_2 e^{-x/L_n} + n_3 e^{-\alpha x} + n_4 e^{-\alpha x/2}$$  \hspace{1cm} (1)

Thermionic currents are typically expressed in terms of excess carriers $\delta n(x)$ is the per-volume excess carrier concentration $\{v(E)$ is the carrier’s thermal velocity as function of its kinetic energy, $g(E)$ and $f(E)$ are the density of states (DOS)\} and Maxwellian distribution respectively

$$J_{th} = q \int dx E \delta n(x) \left( g(E) f(E) / v(E) \right)$$  \hspace{1cm} (2)

Final excess carrier concentrations $\delta N$ (cm$^{-2}$) depend strongly on a number of parameters, such as the geometry of the device, the absorption coefficient and the diffusion length of electrons. The term $g(E)f(E)$ represents the “availability” factor including (a) the 2-d density of states (DOS) of the systems involved and (b) the occupation probability (Maxwell-Boltzmann distribution as long as $E_c - E_F > 3kT$). The last term in the same line of current density represents the velocity of electrons escaping from the lowest energy level in the quantum wells $E_1$ (to infinity). The integrals involved sweep the extents of
quantum wells and a continuum of energy values from lowest miniband to very high energies. Thermal current densities are found from (2):

\[ J_{th} = A^{**} \times L_w \times \delta n \times T^{3/2} e^{-\delta E_1/kT} e^{-(E_c-E_F)/kT} \]  

Where \( A^{**} = 1.462 \text{mA/cm/K}^{3/2} \), \( \delta n \) is the net excess carrier (after recombination) per well, \( \delta E_1 \) is the energy miniband per quantum well, \( E_c-E_F \) energy distance of the Fermi level from the conduction band of each GaAs layer (bottom of the quantum well).

2.2 Equivalent Circuit

Equivalent circuit modeling of any solar cell includes the following “circuit” elements:

(a) independent current source \( I_{ph} \), representing photocurrents directly connected to the incoming solar photon flux
(b) independent current source \( I_{rec} \) due to Auger and radiation recombination losses
(c) diode current \( I_d \)
(d) series resistance \( R_s \)
(e) shunt resistance \( R_{sh} \)

The net effect of \( I_{ph} \) and \( I_{rec} \) is thermionic current \( I_{th} \) (or thermal current density \( J_{th} \)). The latter is due to carriers (electrons) thermally escaping from quantum wells in the intrinsic region. The latter region’s geometry includes a series of quantum wells (multiple quantum wells: mqw’s) and generates electric current when illuminated: this current comes from escaping electrons (under no tunneling) and contributes to overall device conduction. Total load current density then reads as follows:

\[ J = J_{th} - J_o \left( e^{q\beta_n(V + J_r) - 1} \right) \frac{V + J_r}{r_{sh}} \]  

Where \( J \) is the total load current density, \( J_o \) is the reverse saturation current of the diode (p-n junction), \( V \) is the output-load voltage, \( \beta_n=(nkT)^{-1} \) is the temperature factor with the quality factor \( n \) included, \( r_s \), \( r_{sh} \) (\( \Omega \text{ cm}^2 \)) represent series and shunt resistance values, essentially resistivity times length. These ohmic elements are due to (i) bulk semiconductor resistance (ii) contact(s) resistance (iii) current leakage and (iv) imperfections in total device structure. Under open-circuit conditions (OC), the expression above becomes:

\[ 0 = J_{th} - J_o \left( e^{q\beta_nV_{oc} - 1} \right) \frac{V_{oc}}{r_{sh}} \]  

From which the T-dependence of open circuit voltage \( dV_{oc}/dT \) is found to be:
Expression (5) is certainly more complicated and in more detail, than what is usually taken for granted in the wider literature [7].

Under short-circuit (SC)-conditions, (4) leads to \((V=0, J=J_{sc})\):

\[
J_{th} = J_{sc}(1 + \frac{r_s}{r_{sh}}) + J_o(e^{q\beta n J_{sc} r_s} - 1) \tag{7}
\]

Temperature variation of \(J_{sc}\) is found from (7) above:

\[
\frac{dJ_{sc}}{dT} = \frac{dJ_{th}}{dT} \frac{1}{(1 + \frac{r_s}{r_{sh}}) + q\beta n J_o r_s e^{q\beta n J_{sc} r_s}} + \frac{qJ_o J_{sc} r_s}{nkT^2} \times \frac{e^{q\beta n J_{sc} r_s}}{1 + (q\beta n J_o r_s e^{q\beta n J_{sc} r_s}} \tag{8}
\]

Where again, (8) provides a more detailed expression of temperature dependence of \(J_{sc}\) than the one usually taken for granted for single p-n junction solar cells. Both (6) and (8) express, in detail, the temperature dependence of the two characteristic parameters of any photovoltaic structure: \(V_{oc}\) and \(J_{sc}\) (or \(I_{sc}\)). In both (6) and (8), resistive elements are explicitly involved via \(r_s\) and \(r_{sh}\), which in turn may be related in the following way: on one hand \(r_{sh}\) can be found from (5), given \(V_{oc}\) and \(J_{th}\), and on the other, \(r_s\) may be computed from a connection between expressions (5) and (7). It is found that, for mqw-based p-i-n photovoltaic heterostructures, values of \(r_{sh}/r_s\) ratios are well in excess of 100, in other words \(r_s\) can always be determined.

### 2.3 Power Considerations

Revisiting (4), one may re-write the J-V characteristic as follows:

\[
J + \frac{(V)}{r_{sh}} = \frac{J_{th} + J_o}{1 + (r_s / r_{sh})} \frac{J_o}{1 + (r_s / r_{sh}) e^{q\beta n(V + r_s J_o)}} \tag{9}
\]

The voltage \(V\) of a typical mqw-pin solar cell does not exceed 0.85V [Aperathitis, Varonides et al], and the shunt resistance of such devices is well in excess of 3.4kΩ, so that the ratio \((V/r_{sh})\) represents current density values varying from 0.033mA/cm\(^2\) to 0.3mA/cm\(^2\), which makes the term negligible compared to total device current in the region of maximum current production from the device. In other words, under optimum or maximum power conditions, the second term in (9) may be neglected, so that (9) leads
to expressing voltage as an (easier to deal with, in most practical applications) function of total $J$:

$$V = \frac{n k T}{q} \ln \left( 1 + \frac{J_n}{J_o} \right) - \left( 1 + \frac{r_s}{r_{sh}} \right) \frac{J}{J_o} - J_r$$  \hspace{1cm} (10)$$

Incident power density $P_{in}$ is calculated as follows:

$$P_{in} = \frac{J_{sc}}{J_o} V(J) \frac{dJ}{dJ} = \frac{n k T}{q} (J_{sc} - \frac{J_o + J_{th}}{1 + (r_s / r_{sh})}) \left\{ \ln \left( \frac{J_{th} - (1 + (r_s / r_{sh}) J_{sc})}{J_o} + 1 \right) - 1 \right\}$$

$$+ \left( \frac{J_{th} + J_o}{1 + (r_s / r_{sh})} \right) \left( \frac{n k T}{q} \right) \left\{ \ln \left( \frac{J_o + J_{th}}{J_o} \right) + 1 \right\} - \frac{1}{2} r_s J_{sc}$$  \hspace{1cm} (11)$$

Expression (11) is used for computation of $r_s$ [sroeder et al], under $r_{sh} >> r_s$ conditions, but in the present context, (11) could be used for incident power density evaluations within, specific limits of $J_{sc}$, namely from $J_{sc}/2$ to $J_{sc}$ (within which the maximum power point may fall into).

3 Results

A number of parameters of importance in these devices, namely, quantum well width $L_w$, photo-generated carriers $\delta n$ (cm$^{-2}$), temperature as $T^{3/2}$, ground state $dE_1$ depending on the well width, and quasi-Fermi level position in reference with the conduction band of the GaAs layer (bottom of the quantum well). Computations of $J_{th}$ from (2) via (1) show that 10nm GaAs layers interfaced with thick AlGaAs layers in the intrinsic region are expected to contribute thermal current density values between 0.1 to 0.9 mA/cm$^2$ per quantum layer under illumination levels of the order of $10^{17}$ incident photons/cm$^2$. Also, current of 9mA/cm$^2$/quantum well predicted at 30$^\circ$C (10nm quantum well and $10^{12}$ cm$^{-3}$ doping of the GaAs layer). Photo-generated carrier concentrations at levels of $\delta n=10^{12}$ cm$^{-2}$/L$w$ ($= 10$nm), may lead to current densities of values at 4.5mA/cm$^2$/well, at 30$^\circ$C, and 4mA/cm$^2$/quantum well at -10$^\circ$C. For low doping levels in the intrinsic region (so that impurity scattering is minimized) multiplicity of quantum wells in the illuminated intrinsic region of $p/n(qw)/n$ solar cells will improve overall collected current densities and hence efficiencies. Thermal current components $J_{th}$ as computed via (3) become important current sources as circuit elements in the solar cell equivalent circuit. This component includes (1) temperature effects (2) loss mechanisms (via recombination) (3) device specific geometry (quantum well width, and therefore energy bands in the quantum wells) and (4) incident photon flux. Expression (6) provides an explicit temperature variation of the open circuit voltage $V_{oc}$ in terms of the resistive elements of the device and in terms of the temperature variation of the thermal current $J_{th}$. In a similar
fashion expression (8) provides a detailed description of the temperature dependence of the short circuit current $J_{sc}$. Finally, incident power $P_{in}$ is calculated from (11), thus leading to a general relation for the series resistance of the device (area method of series resistance determination). It is found that a 40-quantum well (10nm) p-i-n solar cell (GaAs-AlGaAs) exhibits shunt resistance of the order of $3,400\Omega$, for series resistance values at usual cell operation at less than $5\Omega$.

4 Conclusions
The aim of this communication is two-fold: (a) to provide an explicit method of thermal current calculation out of the intrinsic region of a multi-quantum well pin-solar cell, when illuminated and (b) to deal with its circuit behavior under the above conditions. It is found that such a device may provide substantial current values, which may be appropriately interpreted as current sources. Such currents have been derived from general device physics considerations that have included losses (due to recombination mechanisms) and in the following sequence: first, given the incident power of solar photons, excited carrier concentrations in the quantum wells are being calculated, and the corresponding thermal current is provided as a function of the incident radiation wavelength and device geometry. Next, an equivalent circuit is proposed with net current sources involved and two resistive elements (shunt and series) computed. Finally, temperature variation of open-circuit voltage and short circuit current is provided by means of an improved generalized formula that includes temperature, both shunt and series resistive elements, and thermionic currents.

References