Novel Wideband Cascaded Sigma-Delta Modulator with Digital on-Line Calibration

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Abstract: - This work presents a novel noise-shaping cascaded sigma-delta modulator that incorporates two stages. The first stage is realized by low-order single bit architecture, while the second stage is realized by a high-order feed-forward multi-bit architecture. Moreover, the error cancellation schemes can be added in the digital circuit part to cancel the coarse quantization errors and thus effectively reduce the non-ideal effects such as DAC mismatch. The mismatches between the two stages such as in the gain error and pole error may seriously degrade performance. The blind on-line calibration technique is used to eliminate these imperfect analog circuit errors in the digital circuit. Accordingly, this architecture can reduce the over sampling ratio (OSR) and the in-band noise can be significantly suppressed to achieve a high resolution in Matlab and Switcap2 simulations. Simulation results indicate this sigma-delta modulator is very efficient in wide bandwidth applications.

Key-Words: - Sigma-Delta Modulator, wideband, digital calibration, xDSL

1 Introduction

One significant advantage of sigma-delta modulators (SDM) is that they can achieve high-resolution signal conversion without high precision component matching as required by the conventional signal converters such as flash type A/D converters or those A/D converters based on sub-ranging or successive approximation techniques [1]. However, the sigma-delta modulators are usually limited in lower or mid bandwidth applications due to their oversampling nature. According to our study, the conventional MASH structure has lower quantization noise power in lower frequency and the feedforward single-loop high-order structure has lower quantization noise power in higher frequency. To overcome these problems, an improved cascaded sigma-delta data converter is proposed. This architecture offers a novel noise shaping function that has the advantages of both conventional MASH and feedforward SDM (FFSDM) and can reduce the nonlinearity effects effectively. However, the mismatch between two stages of the proposed architecture should be considered because of the cascaded characteristic of our proposed architecture. The blind on-line digital calibration technique [3], [4] can be used to overcome this problem. The simulation shows that the new SDM architecture is suitable for higher bandwidth applications such as CDMA, xDSL,…etc.

2 The Conventional NTF

When we design a sigma-delta modulator, the noise transfer function (NTF) is an important factor. In general, we design it as a high-pass function, and thus noise errors can be moved to higher frequency bands. In general, the NTF can be classified into two categories as follows:

2.1 Type I: nth-Order Pure Differentiation

The *n*th-order pure differentiation noise-shaping function can be expressed as:

$$NTF = (1 - z^{-1})^n \quad (1)$$

The NTF of the conventional SDM belongs to this type, and the first order SDM architecture is shown in Fig. 1. The NTF and signal transfer function (STF) of the first-order SDM are expressed as follows:

$$NTF = (1 - z^{-1}) \quad (2)$$

$$STF = z^{-1} \quad (3)$$

$$Y(z) = X(z)STF + E(z)NTF \quad (4)$$

When the order of the NTF becomes higher, more noise error power will move to high frequency bands, and then the power of the noise error in the lower frequency bands will be reduced and the SNR in base band is thus increased.

2.2 Type II: Noise Shaping Using Filters with Non-Monotonic Transfer Function

The problem encountered in type I noise-shaping function is the large high-frequency noise shaping gain for higher order. The inverse Chebyshev high-pass function is to modify the Butterworth high-pass response to move the real stopband zeros at the (1, 0j) point out of the unit circle to generate nulls in the NTF at frequencies other than dc [1]. This filter exhibits greater attenuation over the desired signal passband range. The architecture is used in the feedforward sigma-delta modulator (FFSDM) [2], [5] and is shown in Fig. 2.

3 Novel NTF and Architecture

3.1 The Novel Noise-Shaping Function

In the previous section, we find that the NTF of type I has better low frequency band noise suppression. However, when the frequency increases, the power of the noise error will increase. Therefore, this architecture cannot be applied to the wide bandwidth application such as CDMA and xDSL. Because the NTF of type II can decide its bandwidth, it can be applied to higher bandwidth applications. However, it cannot get better noise suppression in low frequency. Generally, type I NTF has better SNR in lower bandwidth application but type II NTF has better SNR in higher bandwidth application. Therefore, we combine type I NTF and type II NTF to obtain a new architecture to have the advantages of both types, and the equation is expressed as follows:

$$NTF = (1 - z^{-1})^n \frac{N(z)}{D(z)}, \tag{5}$$

where

$$N(z) = \prod_{i=1}^{m} \left[(z-1)^2 + r_i \right] \tag{6}$$

$$D(z) = \prod_{i=1}^{m} \left[(z-1)^2 + r_i \right] + \sum_{i=1}^{m} \left(\left[a_{2i-1}(z-1) + a_{2i} \right] \bullet \prod_{j=i+1}^{m} \left[(z-1)^2 + r_j \right] \right) \tag{7}$$

Figure 3 shows the frequency responses of various types of NTF, and we can find that the proposed NTF has the characteristics of both type I NTF and type II NTF. The architecture of this realization is shown in Fig. 4. Since this architecture combines the characteristics of feedforward SDM and low-order SDM, we call it as cascaded low-order and feedforward sigma-delta modulator (CLFSDM).

Let us use Mason's gain theorem to analyze Fig. 4. From Fig. 4 we can find the equations expressed as:

$$Y_1 = z^{-n} X(z) + (1 - z^{-1})^n E_1(z) \tag{8}$$

$$Y_2 = -(E_1(z) + E_D(z)) + \frac{1}{1 + G(z)} E_2(z) \tag{9}$$

Assuming that:

$$H(z) = (1 - z^{-1})^n \tag{10}$$

then

$$Y(z) = z^{-n} X(z) + (1 - z^{-1})^n \frac{1}{1 + G(z)} E_2(z) - E_D(z)(1 - z^{-1})^n. \tag{11}$$

When we design the second stage as a feedforward SDM, the $\frac{1}{1+G(z)}$ will be an inverse-Chebydhev high-pass function. That is

$$Y(z) = z^{-n} X(z) + (1 - z^{-1})^n \frac{N(z)}{D(z)} E_2(z) - E_D(z)(1 - z^{-1})^n. \tag{12}$$

3.2 Blind On-line Digital Calibration

There are some nonlinearity effects such as pole errors, DAC mismatch, and coefficient variations that may degrade the performance. Let us discuss these nonlinearity effects and see how they can affect our new architecture.

(A) Pole Errors:

The pole error is an important nonlinearity that can degrade the performance of the CLFSDM. This error is due to the finite integrator gain at dc (A_{dc}), and can further shape the quantization noise. The transfer function of a delay and leaky integrator is:

$$H(z) = \frac{1}{z - (1 - \frac{1}{A_{dc}})}. \tag{13}$$

This effect may change the NTF and degrade the SNR.

(B) Coefficient Variations:

The coefficient variations may deviate the desired values, and thus change the desired NTF to an unknown status. However, in CLFSDM the coefficient variation only causes the SNDR to degrade slightly due to the characteristic of this architecture.

(C) DAC Mismatch:

Due to the use of the multi-bit quantizer, the multi-bit DAC in the feedback is required and the DAC mismatch will cause the SNDR to degrade seriously. However, in CLFSDM the DAC mismatch will be reduced due to the digital cancellation shaping function that is shown in equation (12). Therefore, the SNDR will be improved by this architecture compared to the conventional FFSDM with DAC mismatch.

Blind on-line digital calibration was proposed by Cauwenberghs [3], [4] and can be used to correct the gain and pole errors of the multi-stage SDM. The proposed SDM would benefit from the application of a modification of this technique. The approach requires a band-limited input signal, with a sampling frequency, f_s, strictly above the Nyquist rate, $f_N = 2f_b$, where f_b is the base-band frequency. This requirement is very reasonable for the proposed CLFSDM because of the over-sampling of our architecture. Figure 5 illustrates the stop band [f_b , $f_s - f_b$] that is reserved for calibration. No additional cost is incurred since an anti-aliasing band-limited filter is present for perfect reconstruction of the input.

First, the highpass filter, $H_c(z)$, is applied to Eq. (12) to eliminate the band-limited input, X. Thus, the coefficients of the digital cancellation filter, $H(z)$, is estimated by minimizing the variance of the quantization noise, $E_2(z)$, assuming a white and uniform power spectrum over the calibration band:

$$|H_c E_2(z)|^2 = \left|\frac{D(z)}{N(z)}\frac{\alpha_1}{z-\beta_1}\frac{\alpha_2}{z-\beta_2}H_c Y\right|^2 \quad (14)$$

$$\approx |N_c Y_1 + H(z) N_c Y_2|^2, \quad (15)$$

where $z \equiv e^{j\Omega}$ with $\Omega = 2\pi f / f_s$; $\frac{N(z)}{D(z)}$ is the NTF of the second stage in CLFSDM, and the digital emphasis filter,

$$N_c(z) = \frac{D(z)}{(z-1)^2 N(z)} H_c(z) \approx \frac{\alpha_1}{z-\beta_1}\frac{\alpha_2}{z-\beta_2}\frac{D(z)}{N(z)} H_c(z), \quad (16)$$

serves to equalize the newly proposed noise-shaping of the spectrum of $E_2(z)$ in the estimation and removes the signal band of X. The error generated in the approximation of the unknown noise-shaping in Eq. (16) does not influence the accuracy of the results to first order. Figure 6 shows the output spectrum of the CLFSDM with a 1% ~ 2% gain error and a 50 dB finite op-amp gain, using blind on-line calibration.

4. Simulation Results

Consider the circuit specifications of high-speed SDM's for xDSL system, the bandwidth is equal to 2.5 MHz and SNDR is more than 84 dB. The coefficients of the second stage in CLFSDM can be found using the HOST synthesis tool [5] and are shown in Table 1. The output spectra of CLFSDM form Switcap2 simulator [6] is shown in Fig. 7. Figure 8 shows the SNDR versus input level with various conditions. Table 2 shows the comparisons of the proposed and conventional sigma-delta modulators.

5. Conclusion

In this paper, a novel architecture is proposed. By using this architecture, the in-band noise power can be improved and the nonlinearity effects such as pole error, DAC mismatch and coefficient variations can be improved. Therefore, this architecture is very suitable in wide bandwidth applications.

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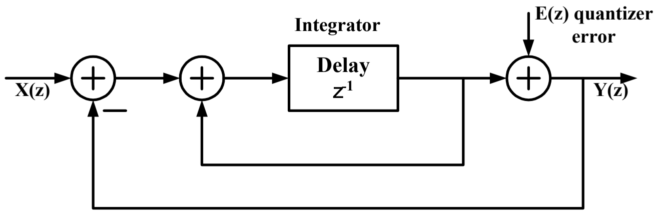

Fig.1: The linear model of First-order SDM

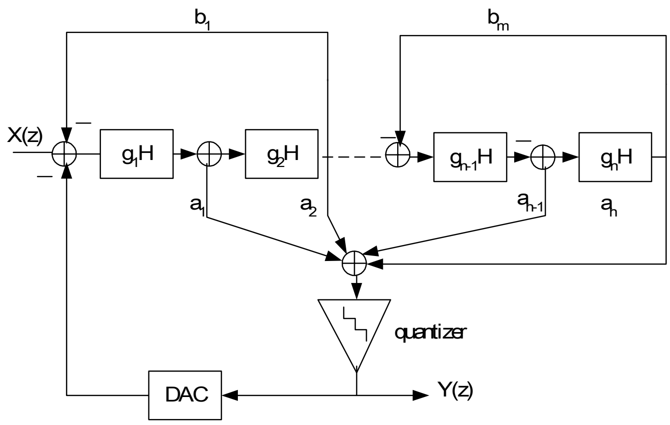

Fig. 2: The block diagram of the fourth-order FFSDM

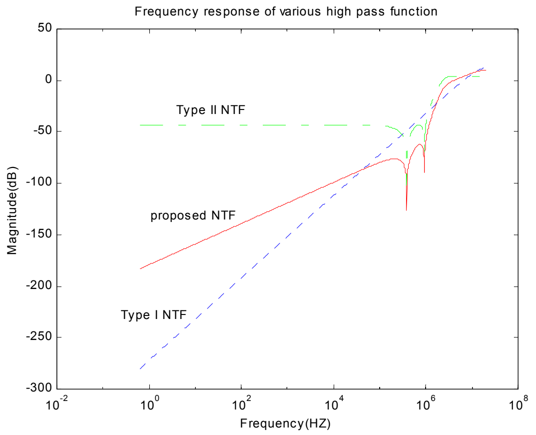

Fig.3: The frequency responses of various NTF's

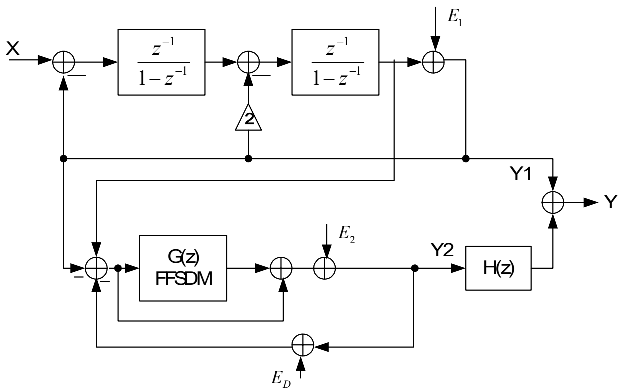

Fig.4: The block diagram of CLFSDM

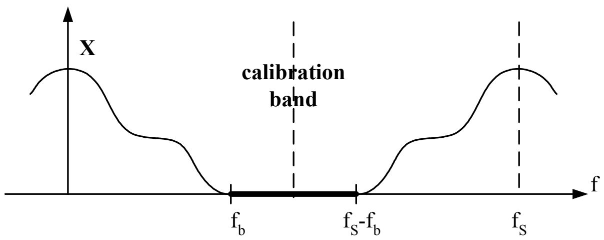

Fig. 5: Signal stop band for blind calibration.

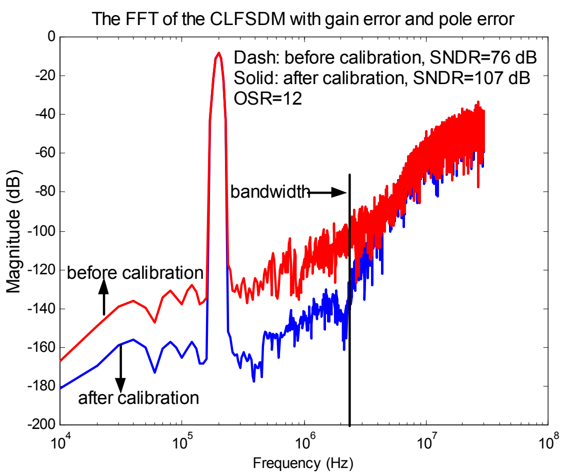

Fig. 6: The output spectrum of the CLFSDM with blind on-line calibration.

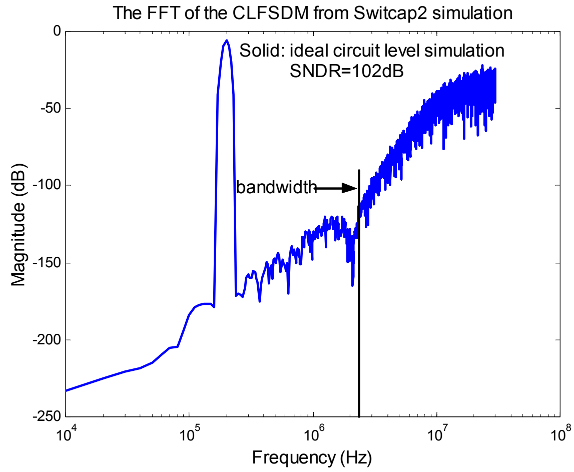

Fig. 7: The output spectrum of the CLFSDM from the Switcap2

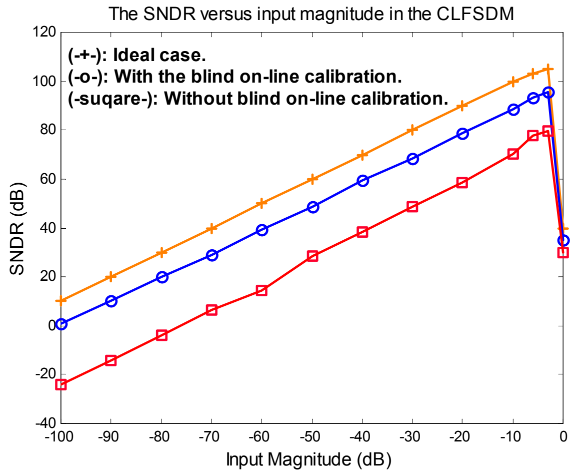

Fig. 8: The SNDR versus input magnitude in the CLFSDM with the OSR=12 and order=5.

Table 2: The coefficients of the second stage FFSDM in CLFSDM.

Coefficient	Value	Coefficient	Value
g1	4.4073	a2	0.24814
g2	1.17837	a3	0.32322
g3	0.1589	b1	0.258
a1	0.41052	b2	0.311397

Table 1: Comparisons of the various architectures

Architectures / Specifications	MASH 2-2-1	FFSDM	CLFSDM
Order	5	5	5
Quantizer bits	1.5@4	4	1.5@4
OSR	12	12	12
Sampling ratio (MHz)	60	60	60
SNDR with ideal case (dB)	89	97	107
SNDR with A_{dc}=60dB (dB)	72	95	105
SNDR with 2% DAC mismatch (dB)	80	68	96
SNDR with 2% Coefficient variation (dB)	70	95	104