Analysis of Polyphase Filter Section with CFAs

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Abstract: - A new RC active polyphase filter section is presented. The section uses three conventional current feedback amplifiers (CFAs). The transfer function of the section has a single pole and optionally a single zero. With a cascade of this sections, any polyphase filter can be easily realized. The paper also introduces the leakage caused by element deviation and the effects of the amplifier in non-ideal case. Furthermore, non-ideal performances of the proposed filter section are tested with computer simulations.

Key-Words: - Analog filter, polyphase filter, CFA filter, leakage, mismatch, simulation

1 Introduction
Since the polyphase filters provide an efficient way for wideband quadrature signal generator with reduced sensitivity to components mismatch, they are recently gained a renewed interest in the design of low intermediate frequency (IF) receivers. Moreover, polyphase filters are applied passive implementation of complex analog filters with their distinct property of discriminating between positive and negative sequences[1],[2].

Integrated Circuit (IC) implementations of polyphase filters are the state of art[3]-[6]. A design methodology is therefore proposed in this paper for an RC network polyphase filter section with conventional CFAs.

Section 2 deals with the formulation of polyphase filter transfer function. Section 3 discusses the analysis of element deviation, and the effects of amplifier gains in non ideal case.

2 Problem Formulation
The analog polyphase signal, as it is discussed here, consists of two phases of a signal with a relative phase shift of 90°. In this paper, we use the complex notation so that the voltage in each phase is a complex number and the phase difference appears as a factor of j. Let the voltage in phase 0 be $U_{a,0}$ or $U_{a,1}$ deviates from $U_a$ or $jU_a$ and a description applies in terms of the polyphase signal of $U_a$ and an error signal of $\Delta U_a$ (see Fig. 1) This gives

$$U_{a,0} = U_a + \Delta U_a$$
$$U_{a,1} = jU_a - j\Delta U_a$$

As to the filter, the two phase polyphase filter is a four-port. The input to output voltage transfer functions are given by

$$\begin{bmatrix} U_{b,0} \\ U_{b,1} \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} U_{a,0} \\ U_{a,1} \end{bmatrix}$$

where $U_{b,0}$ and $U_{b,1}$ are the filter outputs, $U_{a,0}$ and $U_{a,1}$ are the filter inputs. Application of (1) leads to

$$\begin{bmatrix} U_b \\ \Delta U_b \end{bmatrix} = \begin{bmatrix} H_p(s) & H_m^*(s^*) \\ H_m(s) & H_p^*(s^*) \end{bmatrix} \begin{bmatrix} U_a \\ \Delta U_a \end{bmatrix}$$

The polyphase filter function $H_p$ is equal to $(H_{11} + H_{22}) / 2 + j(H_{12} - H_{21}) / 2$ and the mismatch function of $H_m$ equals $(H_{11} - H_{22}) / 2 + j(H_{12} + H_{21}) / 2$. In the case of a true polyphase filter, $H_{11} = H_{22}$ and $H_{12} = -H_{21}$ so that $H_m = 0$.

2.1 Current Feedback Amplifier (CFA)
The circuit symbol of a CFA is shown in Fig. 2. An ideal CFA can be defined by the following equations:

$$V_x = V_y, \quad V_0 = V_z, \quad I_x = 0, \quad I_z = I_x$$

Ideally a CFA has a zero x-input resistance, whereas the y and z terminal resistances are infinite and capacitances of these terminals are zero. In practice however, these values can be obtained as with nonzero and finite appropriate amounts. Note that both plus and minus signs or the letters y and x are used in literature to denote the inputs of a CFA. In this study, corresponding to the
current conveyor terminology, y and x are preferred for the inputs of the commercially available CFA, AD844.

Taking also the active element non-idealities into account, the terminal equations of CFA can be written as follows [7]:

\[
V_x = \beta V_y, \quad V_0 = \gamma V_z, \quad I_y = 0, \quad I_z = \alpha I_x
\]

(5)

Here \(\alpha = 1 - \varepsilon_i\), and \(\beta = 1 - \varepsilon_v\) denote the current and voltage gain of the current conveyor, and \(\gamma = 1 - \varepsilon_0\) denotes the voltage gain of the voltage buffer, where \(\varepsilon_i\), \(\varepsilon_v\), and \(\varepsilon_0\) are the current tracking error, the voltage tracking error of the input buffer, and the voltage tracking error of the output buffer, respectively.

Furthermore, the low input impedance of the buffer enables easy cascading in voltage mode operation.

### 3 Filter Section

Fig. 3 shows the basic diagram of the filter section, which is derived from a frequency-shifted CFA integrator. The circuit consists of two integrators with an input admittance of \(Y_a\) and a feedback admittance of \(Y_b\).

The conductors of \(G_a\) and \(G_b\) realize the frequency shift of the zero and pole. The coupling of phase 0 to phase 1 has an opposite sign relative to the coupling of phase 1 to phase 0 and a third amplifier is used.

In Fig. 3, the admittances of \(Y_a\) and \(Y_b\) are the parallel circuit of a capacitor and a resistor so that \(Y_a = sC_a + G_{ar}\) and \(Y_b = sC_b + G_{br}\). In the case of ideal CFA, the transfer function of the filter section obeys

\[
H_p(s) = \frac{(sC_a + G_{ar} + jG_a)(sC'_b + G'_{br} - jG_b)}{(sC'_b + G'_{br} + jG_b)(sC'_b + G'_{br} - jG_b)} \]

(6)

where \(C'_b\) and \(G'_{br}\) are equal to \(2C_b\) and \(2G_{br}\) respectively. There are two poles and two zeros; one of the pole is cancelled by a zero. The remaining zero and pole are given by

\[
S_0 = -\frac{G_{aw} + jG_a}{C_a}, \quad S_x = -\frac{G'_{aw} + jG_{b}}{C'_{b}} \]

(7)

Thus, the real parts of \(s_0\) and \(s_x\) relate to \(G_{aw}\) and \(G'_{b}\) and imaginary parts relate to \(G_a\) and \(G_b\), respectively.

#### 3.1 Element Deviation

For the calculation of the effects of the element deviation, the circuit of Fig. 4 is used. Calculation of \(H_p(s)\) gives (8), and for \(H_m(s)\), (9) holds shown at the bottom of the page.

The leakage function of \(H_m = -\Delta U_b / \Delta U_a\) gives the error signal in the output that results from element deviation. Let \(E_{x,0}\) and \(E_{x,1}\) be two corresponding filter element of branch 0 and branch 1. \((E_{x,0} + E_{x,1}) / 2\) is then used as the nominal value of \(E_{x}\) and \((E_{x,0} - E_{x,1}) / 2\) as the deviation of \(\Delta E\), and (9) results in (10), shown at the bottom of the page. The mismatch \(C_a\), \(G_{ar}\), or \(G_{a}\) of the input part of the section gives an error signal in the output according to

\[
H_m(s) = -\frac{s\Delta C_a - G_{ar} + jG_a}{2sC_b' + G'_{br} - jG_b} \]

(11)

and the mismatch of the elements of the pole part of the section leads to

\[
H_m(s) = \frac{-(sC_a + G_{ar} + jG_a)(sC'_b + G'_{br} + jG_b) - (sC_a + G_{ar} + jG_a)(sC'_b + G'_{br} + jG_b)}{(sC'_b + G'_{br} - jG_b)(sC'_b + G'_{br} + jG_b)} \]

(9)

\[
H_m(s) = \frac{-(sC_a + G_{ar} + jG_a)(s\Delta C'_b + \Delta G'_{br} + j\Delta G_b) - (sC'_b + G'_{br} + jG_b)(s\Delta C_a + \Delta G_{ar} + j\Delta G_b)}{(sC'_b + G'_{br})^2 + G_b^2 - (s\Delta C_b + \Delta G_{br})^2 - \Delta G_b^2} \]

(10)
\[ H_m(s) = -\frac{(sC_a + G_a + jG_a)(s\Delta C_b' + \Delta G_b' + j\Delta G_b)}{(sC_b' + G_b' + jG_b)(sC_b' + G_b' + jG_b)} \]  

(12)

From (11) and (12), it follows that the pole of \( S'_z \) amplifies the effects of mismatch. This is a disadvantage when the inverse of the passband frequency has to be suppressed. In that case, the discrimination between the stopband and passband increases with quality factor \( Q \). This is also the quality factor of the hidden pole at \( S'_p \) that boosts the effects of mismatch so that leakage will become dominant with increasing \( Q \). For that reason, an active RC-passive polyphase filter suits better when more than 50 dB, is required [8].

### 3.2 Non-ideal Amplifier Case

The calculation of the effect of non-ideal amplifier starts from the circuit of Fig. 3. It assumes that all amplifiers have the same voltage and current gains. Elimination of \( U_c \) (Fig. 3) reveals that \( U_{b,0} \) and \( U_{b,1} \) are affected differently. Elimination of \( U_{b,0} \) and \( U_{b,1} \) with (1) results in (13) and (14).

\[ H_p(s) = -\frac{AY_b G_b + \frac{j(1 + B)}{2} (-Y_b G_b + A G_a Y_b)}{\Delta} \]  

(13)

\[ H_m(s) = \frac{j(1 - B)(-Y_b G_b + A G_a Y_b)}{2\Delta} \]  

(14)

where \( \Delta = A^2 Y_b^2 + B G_b^2 \), \( A = \frac{1}{\gamma} \) and \( B = 2 / A \).

In order to calculate the effect on \( H_p(s) \), (13) is simplified in a conventional way. It is supposed that \( B \) is approximately equal to 1 \((B \leq 1)\). Under this condition, by using (6), (13) reduced to

\[ H_p(s) \approx \frac{(Y_b + jG_b)(AY_b - jG_b)}{(AY_b + jG_b)(AY_b - jG_b)} \]  

(15)

There are also two poles and two zeros; one of the poles is canceled by a zero. The remaining zero and poles are given by

\[ S_{oo} = S_o = -\frac{G_a + jG_a}{C_a}, \quad S_{xx} = S_x = \frac{j(B - 1)G_b}{C_b} \]  

(16)

Finite value of \( B \) causes a shift of pole to \( S_x \) but no shift of zero. Pole change occurs along the imaginary axis and it has no effect on the filter stability.

### 3.3 Simulation Results

Frequency response of transfer function and mismatch function with varying parameter \( A \) are given in Fig. 5 and Fig.6 respectively. Assuming the numerical values as \( G_a = 1 \), \( G_b = 2 \), \( Y_a = (s + 1) \), and \( Y_b = (s + 1)/2 \), (13) and (14) are used for the simulations. It is evident that magnitude of transfer function slightly depends on parameter \( A \), but magnitude of mismatch function entirely depends on parameter \( A \), especially at high frequencies. Also SPICE simulations of the filter circuit given in Fig. 3 for the values presented above and AD844 are obtained and compared with the simulation results given in Fig. 5 and Fig.6, respectively.

### 4 Conclusion

The presented three-CFA version of the general polyphase section applies for implementation with commercially available components. A cascade of sections enables the implementation of an arbitrary polyphase transfer function.

In this paper, the polyphase signal has been treated as the sum of a true polyphase signal and an error. The error signal behaves as an inverse frequency signal and is transferred with \( H_p^*(s^*) \). The conversion of an input signal to an output error is given with \( H_m(s) \), where \( H_m^*(s^*) \) describes the conversion of an input signal to a true polyphase output.

The transfer function of the discussed filter section includes an additional pole and zero at \( S'_z \). The cancellation of pole and zero at \( S'_z \), is not affected by element deviation. The hidden pole at \( S_t \) occurs in \( H_m \) and enhances the leakage.

Finite DC gain of the amplifier causes a shift of the frequency of the pole but it doesn’t cause a shift of the zero. Filter stability is not affected by the shift of pole, because pole shift takes place along the imaginary axis. Finite amplifier gain also causes an additional contribution to the leakage.

In non-ideal case, computer simulations of the proposed filter section are performed. According to the
simulation results transfer function amplitude is not independent of the variations on parameter A. Also amplitude of mismatch function fully depends on parameter A, especially at high frequencies. Both the phases of transfer and mismatch functions are not change with parameter A.

References:
Fig. 5  Frequency response of $H_p(s)$

Fig. 6  Frequency response of $H_m(s)$,