# A Robust Continuous State Feedback Current Control for Induction Motor Drives

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***Abstract**: In this paper a current controller based on space voltage vector PWM scheme is presented for induction motor drives. The design consists of a modified continuous state feedback control. This controller guarantees global asymptotic stability for the system as well as robustness against parameters variations and external torque disturbances. Complete theoretical analysis and a simulation example are given to illustrate our approach.*

**Keywords**: robust control, induction motor, current control, PWM.

## 1. **Introduction**

Since the induction motor is modeled as a nonlinear multi-input single output dynamic system, many researchers have proposed the use of nonlinear state-feedback theory in order to design advanced controllers with requirements of high performance in terms of precision and operation efficiency [1]. This has motivated some researchers to use robust controllers such as adaptive approaches e.g. [1], sliding mode controllers on the basis of variable structure control theory (VSS) e.g.[2] or some advanced optimal-like controllers such as $H_2/H^\infty$ e.g. [3]. In this paper we propose a new different approach for current fed controller design, which consists of a continuous state feedback control. The proposed scheme will be proved to guarantee a continuous solution of the system output and an asymptotic convergence of the statoric current of the machine even in the presence of system uncertainties. Because the rotor flux linkage $\lambda_{dr}$ can not be easily measured, our design will be on the basis of the only knowledge of statoric currents in the *d-q* frame $i_{ds}$ and $i_{qs}$, whereas we consider the transient of the state of rotor flux linkage $\lambda_{dr}$ in the *d*-axis as a time varying bounded disturbance.

## 2. **Induction motor state equations**

The electromagnetic dynamics of induction motor in the synchronously *d-q* frame, when applying the field oriented control as in [4], is given by:

$$\frac{d}{dt}\begin{bmatrix} i_{dr} \\ i_{qr} \\ \lambda_{dr} \end{bmatrix} = \begin{bmatrix} -(\frac{R_s}{L_\sigma}+\frac{R_r L_m^2}{L_r^2 L_\sigma}) & \omega_e & \frac{R_r L_m}{L_r^2 L_\sigma} \\ -\omega_e & -(\frac{R_s}{L_\sigma}+\frac{R_r L_m^2}{L_r^2 L_\sigma}) & -\frac{\omega_r L_m}{L_r L_\sigma} \\ \frac{R_r L_m}{L_r} & 0 & \frac{R_r}{L_r} \end{bmatrix} \begin{bmatrix} i_{dr} \\ i_{qr} \\ \lambda_{dr} \end{bmatrix} + \frac{1}{L_\sigma}\begin{bmatrix} V_{dr} \\ V_{qs} \\ 0 \end{bmatrix} \quad (1)$$

where

$i_{ds}, i_{qs}$ : *d*-axis and *q*-axis stator currents

$\lambda_{dr}, \lambda_{qr}$ :*d*-axis and *q*-axis rotor flux linkages. $\lambda_{qr} = 0$, $\lambda_{dr} = \lambda_r =$ constant.

$V_{ds}, V_{qs}$ : *d*-axis and *q*-axis stator voltages

$R_s, R_r$ : stator and rotor resistances

$L_s, L_r$ : stator and rotor inductances

$L_m, L_\sigma$ : mutual and leakage inductances; $L_\sigma := L_s - \frac{L_m^2}{L_r}$

$\omega_e, \omega_r$ : electrical angular speed and rotor angular speed

**3. Robust current controller design**

This equation represents the decoupled current control form for the induction machine. In general, the rotor flux linkage $\lambda_{dr}$ can not be easily measured. For this purpose, our design will be on the basis of the only knowledge of $i_{ds}$ and $i_{qs}$, whereas we consider the transient of the state $\lambda_{dr}$ as a time varying bounded disturbance under controlled current $i_{ds}$ and has a constant disturbance component in the steady state. The state space equation can then be written as

$$\frac{d}{dt}\begin{bmatrix} i_{ds} \\ i_{qs} \end{bmatrix} = \begin{bmatrix} -\alpha_d & 0 \\ 0 & -\alpha_q \end{bmatrix}\begin{bmatrix} i_{ds} \\ i_{qs} \end{bmatrix} + \frac{1}{L_\sigma}\begin{bmatrix} V_{ds} + D_d \\ V_{qs} + D_q \end{bmatrix} \tag{2}$$

where

$$\alpha_d = \alpha_q = \left(\frac{R_s}{L_\sigma} + \frac{R_r L_m^2}{L_r^2 L_\sigma}\right)$$

$$D_d = \frac{R_r L_m}{L_r^2}\lambda_{dr} + L_\sigma \omega_e i_{qr} - L_\sigma \Delta\alpha_d \; ; \; D_d = \frac{R_r L_m}{L_r^2}\lambda_{dr} + L_\sigma \omega_e i_{qr} - L_\sigma \Delta\alpha_d$$

$\Delta\alpha$ represents variations in values of statoric and rotoric resistances and inductances.

**Assumption 1**

Without loss of generality, we suppose that each uncertainty is bounded by a certain known function as

$$\|D_d\| \le \rho_d$$

and

$$\|D_q\| \le \rho_q$$

Under the above formulation, $\lambda_{dr}$ is not required to be measured or estimated. But the transient dynamics of the induction motor can be obtained as well. Define now the state variable errors as

$$\tilde{\xi} = \begin{pmatrix} \tilde{\xi}_d \\ \tilde{\xi}_q \end{pmatrix} = \begin{pmatrix} i_{ds} - i_{dsr} \\ i_{qs} - i_{qsr} \end{pmatrix}$$

where $i_{dsr}$ and $i_{qsr}$ represent the reference currents commands in *d-q* axis.

The state error dynamics can be then obtained as

$$\frac{d}{dt}\begin{bmatrix} \tilde{\xi}_d \\ \xi_q \end{bmatrix} = \begin{bmatrix} -\alpha_d & 0 \\ 0 & -\alpha_q \end{bmatrix}\begin{bmatrix} i_{ds} \\ i_{qs} \end{bmatrix} + \frac{1}{L_\sigma}\begin{bmatrix} V_{ds} + E_d \\ V_{qs} + E_q \end{bmatrix} \tag{3}$$

where

$$E_d = D_d - L_\sigma \alpha_d i_{dsr}$$

$$E_q = D_q - L_\sigma \alpha_d i_{qsr}$$

In this equation, $E_d$ and $E_q$ are seen as disturbance terms.

Consider the following continuous state feedback control

$$u = \begin{cases} L_\sigma \alpha i_{sr} - \dfrac{\tilde{\xi}}{\|\xi\|}\rho_d & \text{if } \tilde{\xi} \notin \Omega \\ L_\sigma \alpha i_{sr} - \dfrac{\tilde{\xi}}{\varepsilon\varphi(t)}\rho_d & \text{if } \tilde{\xi} \in \Omega \end{cases} \tag{4}$$

where

$$u = \begin{bmatrix} V_{ds} & V_{qs} \end{bmatrix}^T,\ i_{sr} = \begin{bmatrix} i_{qsr} & i_{qsr} \end{bmatrix}^T$$

and the subset $\Omega$ is defined as

$$\Omega = \left\{ \tilde{\xi} : \left\|\tilde{\xi}\right\| \le \varepsilon\varphi(t) \right\}$$

In (4) $\varepsilon$ is a positive sufficiently small real and $\varphi$(t) is a class of uniformly continuous functions such that $0 \prec \varphi(t) \le 1$ and $\omega(t) := \int \varphi(t)dt$ satisfying $\omega(t) \le 0$ .

The continuity of the control is derived from the fact that the switching sphere defined by $\left\|\tilde{\xi}\right\| = \varepsilon\varphi(t)$ keeps shrinking in the state space as the time increases.

**Lemma 1**

The control (5) is continuous and stabilizes asymptotically the uncertain dynamical system if there exist a Lyapunov function candidate verifying $V(.) : \Re^2 \times \Re \to \Re^+$ such that

i. $\gamma_1(\left\|\tilde{\xi}\right\|) \le V(\xi, t) \le \gamma_2(\left\|\tilde{\xi}\right\|) \qquad \forall(\tilde{\xi}, t) \in \Re^2 \times \Re$

ii. $\dot{V}(\tilde{\xi}, t) \le -\gamma(\left\|\tilde{\xi}\right\|) + \gamma(\eta)\varphi(t) \qquad \forall(\tilde{\xi}, t) \in \Re^2 \times \Re$

where $\eta$ is a positive constant, $\lim_{r\to\infty} \gamma_i(r) = \infty$ , $i$=1, 2 and $\gamma$ a positive definite function, such that $\gamma(0) = 0$ . If $\gamma, \varphi(t)$, and $\omega(t)$ verify $\gamma(y) - \gamma(\eta) > 0$ for any $y > \eta$ , $0 < \varphi(t) \le 1$ , $\omega(t) \le 0$ and if $\eta$ is a positive definite function in $\xi_o$ ($\xi_0 = \xi(0)$), then every solution $\tilde{\xi}(t; \tilde{\xi}_0, t_0) : [t_0, \infty) \to \Re^n$ of the system is globally asymptotically stable equilibrium.

**Theorem 1**

In the current control PWM drive, the control (4) asymptotically stabilizes the uncertain dynamical system (3) and guarantees a zero-error convergence for the state statoric currents.

**Proof**: By replacing $V_s$ by the control $u$, where $V$s represents the combined voltage vector such that $V_s = \begin{bmatrix} V_{ds} & V_{qs} \end{bmatrix}^T$ yields to the following error dynamics equation

$$\dot{\tilde{\xi}} = -A\xi + \frac{1}{L_\sigma}(u + E) \tag{5}$$

where $E = \begin{bmatrix} E_d & E_q \end{bmatrix}^T$ and $A = \begin{bmatrix} \alpha_d & 0 \\ 0 & \alpha_q \end{bmatrix}$

Choosing a Lyapunov candidate function

$$V = \tfrac{1}{2}\tilde{\xi}^2 \tag{6}$$

it is easy to verify that this candidate Lyapunov function satisfies i. In Lemma 1 [6].

Taking the derivative of $V$ with respect to time,

$$\dot{V} = \tilde{\xi}\dot{\tilde{\xi}}$$

if $\tilde{\xi} \notin \Omega$, then

$$\tilde{\xi}\frac{1}{L_\sigma}(u + E) = \tilde{\xi}\frac{1}{L_\sigma}(-\rho\frac{\tilde{\xi}}{\|\tilde{\xi}\|} + D)$$

$$\leq -\tilde{\xi}\frac{1}{L_\sigma}(\rho - \|D\|) \leq 0$$

then

$$\dot{V} \leq -A\tilde{\xi}^2 \leq 0 \tag{7}$$

If $\tilde{\xi} \in \Omega$, from the Caushy-Shwartz inequality, we can write

$$\tilde{\xi}\frac{1}{L_\sigma}(u + E) \leq \tilde{\xi}\frac{1}{L_\sigma}(D + \rho\frac{\tilde{\xi}}{\|\tilde{\xi}\|})$$

$$= \tilde{\xi}\frac{1}{L_\sigma}(-\frac{\rho}{\varepsilon\varphi(t)}\tilde{\xi} + \rho\frac{\tilde{\xi}}{\|\tilde{\xi}\|})$$

This last term achieves a maximum value of $\varepsilon\rho\varphi(t)/2$ when $\|\tilde{\xi}\| = \varepsilon\varphi(t)/2$

then,

$$\dot{V} \leq -A\tilde{\xi}^2 + \varepsilon\rho\varphi(t)/2 \tag{8}$$

From (7) and (8), in both cases we have

$$\dot{V} \leq -A\tilde{\xi}^2 + \varepsilon\rho\varphi(t)/2$$

which satisfies ii. In Lemma 1. It follows from Lemma 1 that every solution $\tilde{\xi}$ is asymptotically stable in large, and then $\lim_{t\to\infty}\tilde{\xi} = 0$ .

4. **Simulation example**

In this section we present simulation results obtained from a 3 kW induction motor drive. The characteristics for the induction motor are given below

$P$ : 3 kW

$V$ : 220 V (rms)

$\omega_{\text{nom}}$ : 310 rad/s

$L_\sigma = L_r = L_m = 150\,\text{mH}$

$R_s = 1.1\Omega$; $R_r = 1.4\Omega$

The function $\varphi(t)$ is taken as $\varphi(t) = \dfrac{1}{1+t^2}$, while $\varepsilon$ is taken as 1.

Figs.1-4 represent the current waveforms and the current errors of the $d$-$q$ frame statoric currents $i_{qs}$ and $i_{ds}$ with the control (4). To test the behavior of the proposed robust control under parameters changes, a variation of 50 % of rotor resistance is considered.

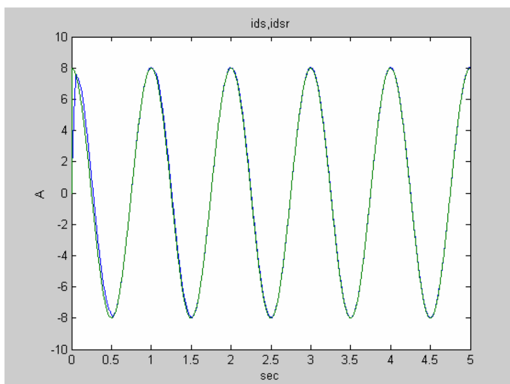


Figure 1. Waveform of the statoric current $i_{ds}$ under control (4)

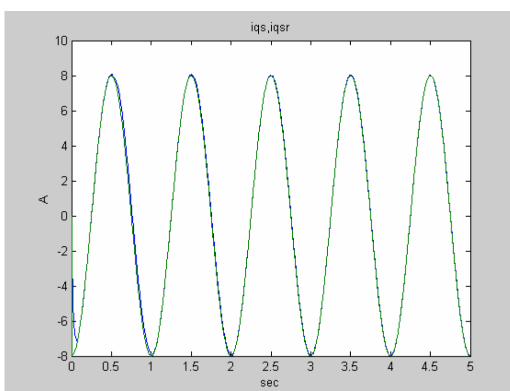


Figure 2. Waveform of the statoric current $i_{qs}$ under control (4)

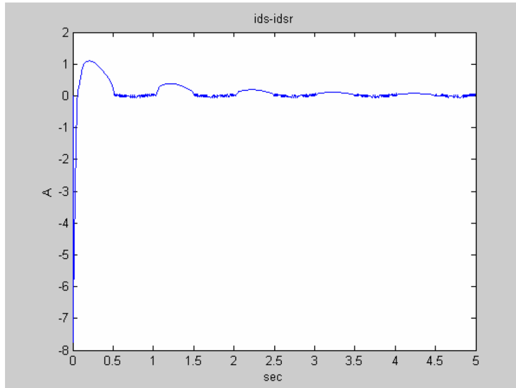


Figure 3. error convergence of the statoric current $i_{ds}$

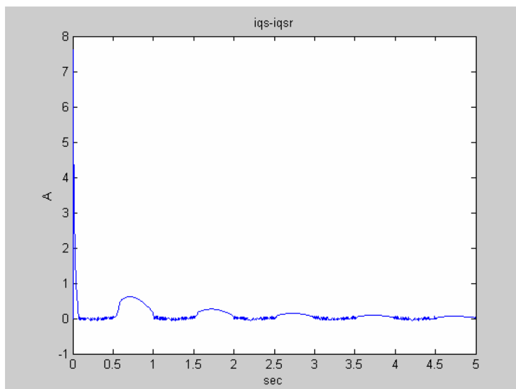


Figure 4. error convergence of the statoric current $i_{qs}$.

It can be seen from the above results that the control objective to ensure asymptotic convergence of the statoric currents in *d-q* frame is mainly achieved. This asymptotic convergence is maintained even in the presence of system parameters variations such as the rotoric resistances and inductances.

## 5. Conclusions

In this paper, a robust current control based on space voltage vector PWM scheme is proposed to improve dynamic and static performances of the drive system. The proposed control is simple to be implemented in a practical setup and possesses the advantage to be continuous and ensuring global asymptotic stability to the statoric currents in *d-q* frame.

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