Symmetrical Experiments to Test the Clock Paradox

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Abstract: The classical issue of clock paradox is revisited in a symmetrical context. It has been shown that the argument based on asymmetry is invalid. The clock paradox is restated in the symmetrical twin brothers (2-J) and quadruplet brothers (4-J) experiments. These experiments show that the clock paradox is inherent in the Lorentz transformation, or any transformation in which time is dependent on space coordinates and velocity. The 4-J experiment also shows that the relativity of simultaneity may lead to a paradox of occurrence. It has been shown that the around-the-world atomic clock experiment can serve as a different version of the 2-J experiment, and that the paradox holds in the presence of gravity and acceleration.

Key-words: clock paradox, relativity, time dilation, twin.

1 Introduction

The relativistic clock paradox was first mentioned by Einstein [1], and later discussed in more detail by Langevin [2], Laue [3], Lorentz [4] and Pauli [5]. The issue has attracted great attention of the scientific community ever since [6-15]. Indeed, nothing is more important and fundamental than the concept of space and time that defines our whole paradigm of physics, and no logical flaw on this matter should be allowed to escape our scrutiny. It may take a million experiments to build our confidence in a theory, but it takes only one paradox to forfeit it. It is in this spirit that the issue of clock paradox is revisited.

2 The Clock Paradox

The well known clock paradox can be briefly stated as follows: One of the twin brothers flies into space with a speed comparable to the speed of light, while another brother stays home on the Earth. According to the theory of relativity, A moving clock runs slow and the traveling brother should be younger than his twin brother at the time of their reunion. On the other hand, in the reference frame of the traveling brother, the sitting brother is moving and should be younger at the time of reunion!

The efforts to resolve the above clock paradox fall, by and large, into two categories: the kinetic school that ignores the time delay of the accelerating period [12-14], and the dynamic school that stakes the whole business on the effect of acceleration[15]. The two Genies are empowered by the same magic lamp: the asymmetry of the experiment, and try to fulfill the same wish: The traveling brother comes home younger than his twin brother by a factor of γ. It is argued that the traveling brother has to accelerate and decelerate to return, while the sitting brother gets to sit on Earth and twist his thumbs, the situations for the twins are different, asymmetrical, and therefore the differential aging. It turns out, magically, that the poor hard working traveler, who has to suffer all the ordeal of mechanical shock, long term fatigue and loneliness, lives happier and stays younger!

3 The Symmetrical Twin and quadruplet Experiments

To ultimately expel the phantom of asymmetry, we here design two completely symmetrical thought experiments: one employing twin brothers, which shall be referred to as the 2-J (Jack and John) experiment, and one employing quadruplet brothers, which shall be referred to as the 4-J (Jack, Jim, John and Joe) experiment.

3.1 The 2-J Experiment

In this experiment we let both twins start their journey from a space station far from any heavenly bodies, so that the whole experiment can be carried out without the influence of gravity.

As depicted in Fig.1, the twin brothers Jack and John are equipped with identical "twin" shuttles and "twin" clocks synchronized at the departure O. The twins travel in opposite directions along the same straight line. Other than the direction, their accelerating and cruising processes are preprogrammed to be identical as measured by their own speedometers, clocks and accelerometers. Thus, they start their journeys with the same preset acceleration for the same time period t_1 as read from their own clocks to reach a relativistic speed v at the points A and A', and then cruise for a preset long period t_2 to the points B and B'. They start turning back with the same deceleration for a time $2t_1$ to return back to points B and B' with their velocities reversed and cruise back home for a time t_2. The deceleration for the landing is also symmetrical.

The perfectly symmetrical experiment allows no filibustering argument based on asymmetry. The special theory of relativity predicts that if Jack is considered to be at rest, John should be younger than Jack at the time of reunion. If, however, John is considered to be at rest, then Jack should be younger than John. The theory of relativity gives two contradictory predictions, depending on who is mentally taken to be the rest reference system. But we know both predictions are wrong, since the symmetrical arrangement of the experiment dictates that the twins must have aged the same by the reunion.

Could the acceleration cause a compensating effect to cancel the Lorentz transformational time dilation so that the twin brothers age the same by the reunion? The answer is no. To examine this, let us assume the total differential aging Δt consists of two parts, $\Delta\tau_u$ and $\Delta\tau_a$:

$$\Delta t = \Delta\tau_u + \Delta\tau_a$$

where $\Delta\tau_u$ is the differential aging resulted from Lorentz transformation during the cruising periods, and $\Delta\tau_a$ is the differential aging during the accelerating periods. Since the two brothers must have the same age after taking a symmetrical journey, we must have

$$\Delta\tau_u + \Delta\tau_a = 0$$

i.e., $\Delta\tau_u$ and $\Delta\tau_a$ must be opposite in sign and equal in magnitude. But this is impossible. First, the two components can not be equal in magnitude since $\Delta\tau_u$ is proportional to the arbitrarily long cruise time, while $\Delta\tau_a$ should depend solely on the mathematical structure of the turning curve preset by the acceleration program. One is therefore forced to commit the same sins of Tolman's theory[15], unless both $\Delta\tau_u$ and $\Delta\tau_a$ vanish. Second, the two components

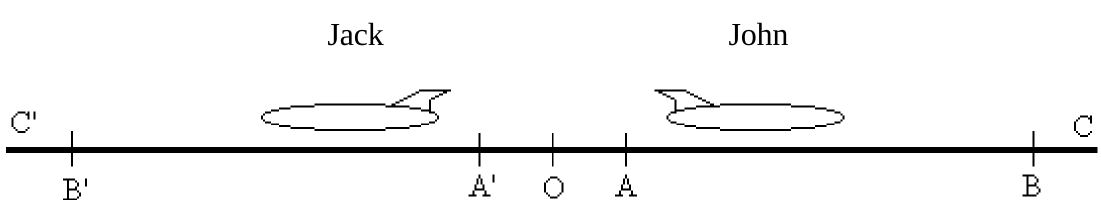

Figure 1

can not be opposite in sign because it violates Einstein's clock hypothesis which states that the instantaneous rate of a clock depends only on its instantaneous speed but not on its acceleration. The time dilation of an accelerating system is assumed to be the same as that of a co-moving system with the same instantaneous velocity u. The time along the relevant part of the world line with acceleration is given by the integral

$$t' = \int_0^t g \; dt \tag{1}$$

where t is the proper time of the accelerating traveler, and t' is the time measured by the observer. Eq. (1) shows that t' is always greater than t and the time dilation during the acceleration period can not possibly be negative. Moreover, it shows that the time dilation is solely determined by the integrand γ a certain function independent of the acceleration and the past history.

3.2 The 4-J Experiment

We can formulate another experiment that keeps acceleration entirely out of the picture. We now enlist quadruplets, Jack, Jim, John and Joe, with four identical clocks and two identical long space shuttles. Jack and Jim shall ride on one shuttle, while John and Joe on the other. Jack and John shall sit in the front cockpits , while Jim and Joe in the rear cabins. Each of the quadruplet brothers shall carry a clock, and the two clocks in each shuttle are spaced at certain distance L. All four clocks are synchronized at the moment of departure. The two teams shall take the same preprogrammed symmetrical travel and return home as described in the 2-J experiment, except for the landing portion. When the two teams come back home, they do not reduce their speed but continue cruising with the same speed and pass each other. At the moment when the two pilots Jack and John meet, they synchronize all four clocks and define this moment as the time origin, i.e., t = t' = 0. The synchronization of clocks within the same inertial system is always allowed by the theory of relativity. As a matter of fact, they may mot even need to do any physical synchronization, but merely check their clock readings, which are expected to be the same since all four clocks were synchronized at departure, and the trip is symmetrical. At the moment when Jack meets John in flight, the four brothers must have the same age due to symmetry. See Fig.2a.

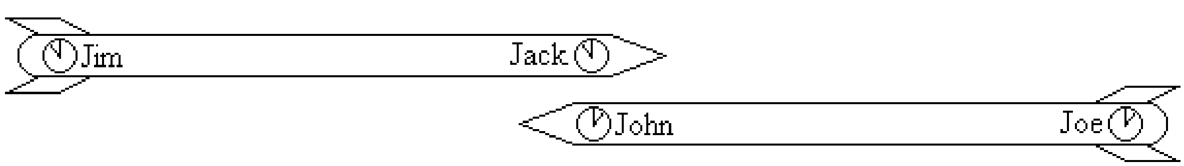

Figure 2a

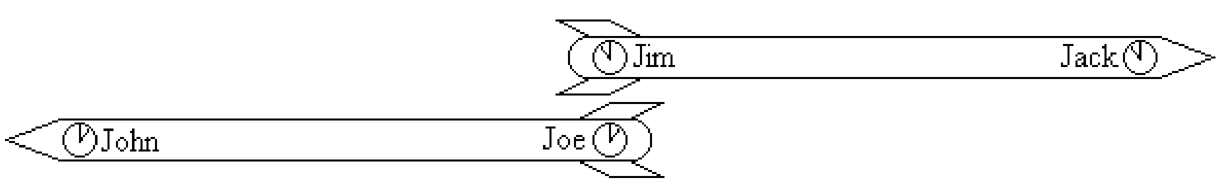

Figure 2b

After a certain amount of time the two brothers sitting at the rear, Jim and Joe, will meet. At this moment Jim's clock reads t' and Joe's t. The symmetry dictates that their clocks read the same time from their own clocks in their own reference frames. Namely,

$$t = t' \quad (2)$$

which is to say that all the quadruplet brothers must have aged the same [Figure 2b]. But if John and Joe try to calculate and compare their times according to the theory of special relativity, they should have

$$t = \gamma t' \quad (3)$$

Likewise, Jack and Jim shall insist

$$t' = \gamma t \quad (4)$$

The results (2), (3) and (4) contradict each other and manifest the same clock paradox.

4 Simultaneity Paradox

According to the theory of relativity, two events occurring simultaneously in one reference frame are in general not simultaneous to the observers in another moving frame. This is usually explained by saying that the signals sent simultaneously from different places do not reach a moving observer simultaneously. Such explanation does not touch the real issue. If simultaneity means that the observer must detect the signals at the same time, then we can not even speak of any simultaneity at all even within the same inertial system! As a matter of fact, signals of simultaneous events reach an observer in the same system simultaneously only when these events take place on a circle with the observer at the center. But we know things can happen simultaneously even when they are not on the circle. All the events on any line parallel to the x axis in a space-time diagram are simultaneous to observers anywhere in that reference frame, but the light signals of these events will not reach any observer simultaneously. The non-unique simultaneity of events is entirely different from the difference in time of arrival. The 4-J experiment allows us to explore the consequences of the non-unique simultaneity of special relativity.

Let us examine the event E_A when pilot John meets passenger Jim, and the event E_B when pilot Jack meets passenger Joe. Classically, these two events should take place simultaneously because the two space shuttles have identical length. Relativistically, however, the two events are not simultaneous due to Lorentz contraction. To John and Joe, Jack and Jim's shuttle is shorter and John should meet Jim before Joe meets Jack, namely, E_A should take place before E_B. On the other hand, Jack and Jim should expect E_B taking place before E_A since the motion is relative. This result is considered "paradox of simultaneity" by Newtonists, and "relativity of simultaneity" by Einsteinists.

To settle the argument, some detailed calculation is in order. At the event E_A, John's coordinates in his own system are $(0,t_1)$, and Jim's coordinates in Jim's own system are $(-L, t_1')$. Likewise, at the event E_B, Joe's coordinates are (L,t_2), and Jack's coordinates are $(0, t_1')$, as measured in their respective reference systems. These coordinates are related by Lorentz transformation:

$$0 = \gamma(-L + u\, t_2') \quad (5)$$

$$t_1 = \gamma(-L\, u/c^2 + t_2') \quad (6)$$

$$L = \gamma\, u\, t_1' \quad (7)$$

$$t_2 = \gamma\, t_1' \quad (8)$$

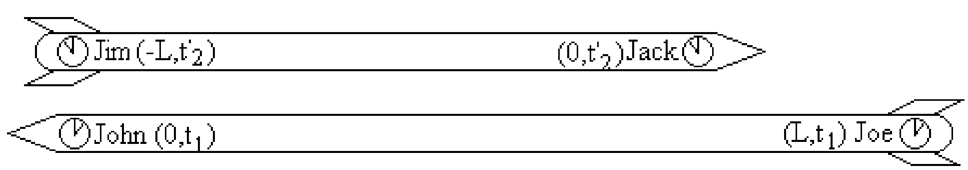

Figure 3

We obtain

$$t_1 = L/(\gamma\, u) \qquad (9a)$$

$$t_2 = L/u \qquad (9b)$$

$$t_1' = L/(\gamma\, u) \qquad (10a)$$

$$t_2' = L/ u \qquad (10b)$$

Eqs. (9a) and (9b) predict that John meets Jim before Joe meets Jack as observed by John and Joe, i.e., at the event E_A, Joe is on the right side of Jack, waiting to meet him, See Fig.3.

But John is too good a seasoned pilot to forget double checking things. He wants to make sure that Jack's coordinate x in John's system is less than Joe's coordinate L at the event E_A:

$$x < L \qquad (11)$$

At the moment of E_A, Jim's clock reads t_2', and Jack's clock must read the same t_2' because they stay in the same reference frame. Jack's coordinates in the Jack-Jim frame are therefore $(0, t_2')$, where $t_2' = L/u$ as given by Eq. (10b). Jack's coordinates as transformed to John-Joe's reference frame should be

$$x = \gamma\,(0 + u\, t_2') = \gamma\,(u\, L/u)$$

$$x = \gamma\, L > L\ ! \qquad (12)$$

Whoops! Somehow Jack has sneaked through, relativistically, to the right side of Joe without meeting him! The paradox of simultaneity and causality manifests itself as an occurrence paradox, as stated in the contradicting inequalities (11) and (12).

5 The Around-the-World Atomic Clock Experiment

The symmetrical experiment described above is quite a challenge at the current stage of technology. But the power of logic is that we do not even have to actually carry out the experiments to see the paradoxical. However, many have claimed that they have tested the relativistic time dilation experimentally, the most noteworthy being the much celebrated around-the-world atomic clocks experiment by Hafele and Keating [16]. They flew four portable cesium clocks around the world, once eastward and once westward, and claimed that their results "provide an unambiguous resolution of the famous clock 'paradox' with macroscopic clocks".

Using a non rotating Schwartzchild metric, Hafele obtained the relative difference δ of the times recorded by the flying and the sitting clocks

$$\delta = (\Delta\tau - \Delta\tau_0)/\Delta\tau_0$$

$$= gh/c^2 - (2R\Omega v + v^2)/(2c^2) \qquad (13)$$

where g, h, R, v and Ω are, respectively, the gravitational acceleration on Earth, the flying height and the ground speed of the, the radius and the angular speed of the Earth [17].

Hardly anyone realizes that Eq. (13) offers a direct proof, not resolution, of the clock paradox. Thus, we arrange two jets carrying two synchronized identical clocks flying along the equator at the same height h, see Fig.4.

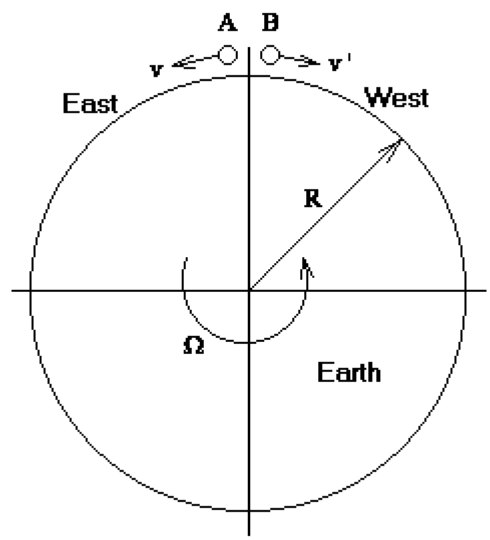

Figure 4

If clock A flies eastward with ground velocity v and clock B flies westward with ground velocity

$$v' = -\,(v + 2\,R\,\Omega) \qquad (14)$$

then $\delta = \delta'$ (15)

according to Eq. (13). Namely, whenever condition (14) is satisfied, the two clocks flying in the opposite sense shall have the same time rate and register the same time (or age) when the two meet again. This directly contradicts the relativistic prediction that clock B will lose if clock A is TAKEN AS the rest frame, and clock A will lose if clock B is TAKEN AS the rest frame -- a logical paradox. This experiment is a different version of the 2-J experiment. Introduction of gravity and general theory of relativity does not change the logic of the clock paradox.

One might contend that none of the flying clocks can be taken as the rest frame, the rest frame has to be taken with respect to the North Star. Such argument actually demands an absolute coordinate system, a notion in direct conflict with the principle of relativity. We know the North Star is moving, so is the Milky way, and even the Universe.

It should be noted that Eq. (13) represents the theory of relativity faithfully. If these equations lead to a paradoxical result, it is not Hafele's fault. The paradox is deeply rooted in the theory of relativity, and it is logical. As a matter of fact, Hafele has made a contribution to the discussion of the clock paradox by demonstrating that the time dilation involving gravity and acceleration is amenable to calculation, and it is not related to the past history, as some hoped to happen in order to cancel the effect during an arbitrarily long cruise.

7. Conclusion

The symmetrical 2-J and 4-J experiments have removed the asymmetry argument from the discussion of the clock paradox. Both experiments have shown that the clock paradox is inherent to the Lorentz transformation, or any transformation in which time depends on space coordinates. The 4-J experiment demonstrates that non-unique simultaneity can lead to an occurrence paradox.

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