Abstract: Morphological image processing has been widely used to process binary and grayscale images. To extend the concept to color images, an ordering of the data is required. In this work, an algorithm for color morphology is proposed and called the Mahalanobis Color Distance-based Morphological Ordering Algorithm. The proposed algorithm implements the Mahalanobis measure to replace the angle-valued pixels by a scalar, and, is based on a combination of reduced and conditional ordering of the underlying data. The proposed algorithm has been implemented with the main morphological operators as well as with other operators such as: image smoothing (noise suppression), gradient and Laplacian operators.

Keywords: Mathematical Morphology, Color Processing and Vectorization

1 Introduction

Mathematical morphology (MM) [1-3] is a relatively new, rapidly growing, nonlinear theory for image processing and based on set theory, with a strong geometric orientation. Originally developed for binary images, it was later generalized for grayscale images as well. MM facilitates quantitative analysis or description of geometric structures of images and concerns mainly nonlinear image transforms such as erosion, dilation, opening, closing, order statistic filters, skeletonization, etc [1-3]. The geometric description depends on small synthetic images called SEs (SE).

Extending morphological operators to color image processing has been problematic because it is not easy to define geometry of vector-valued function and ordering of vectors is not straightforward. Serra [4] provided a rigorous mathematical treatment of extending morphological operators to vector-valued functions. Trahaisas and Venetsanopoulos [5] summarized several techniques for ordering of multivariate data that were originally presented by Barnett [6]. The most common techniques are marginal ordering (MO), reduced ordering (RO), conditional ordering (CO) and partial ordering (PO). The analysis of these filters using the multivariate probability function is complex. Comer and Delp [7] presented a component-wise ordering which is the same as MO. Peters [8] proposed CO of color vectors using hue, saturation, and luminance. Here, the Luv color space was implemented. The method does not specify how the infimum or supremum is chosen. Finally, Surtor and Weeks [9] have implemented a combination of RO & CO where the concept of a reference color is introduced. However, the proposed operators were restricted to red erosion, green opening etc.

The arrangement of this paper is as follows: in section 2 binary, gray as well as color morphology are discussed. In section 3, the proposed MCD-MOA is introduced. Section 4 presents results
obtained and includes comparisons with other existing techniques. Section 5 summarizes the results and gives conclusions.

2 Image Morphology

Let X represents a binary image. X is transformed by the SE. The shape and size of SE determine the resultant image [10]. There are four basic morphological operators: dilation, erosion, opening and closing, represented by the symbol \( \oplus \), \( \ominus \), \( \odot \) and \( \bullet \), respectively and are defined as:

\[
X \oplus H = \{ (x,y) : H_{(x,y)} \cap X \neq \emptyset \} \tag{1} \\
X \ominus H = \{ (x,y) : H_{(x,y)} \subseteq X \} \tag{2} \\
X \odot H = (X \ominus H) \oplus H \tag{3} \\
X \bullet H = (X \odot H) \ominus H \tag{4}
\]

where \( \{ X \subseteq \mathbb{R}^2 \text{ or } Z^2 \} \), \( \{ H \subseteq \mathbb{R}^2 \text{ or } Z^2 \} \) is the used SE, and \( H_{(x,y)} \) is the translate of the set H by \( (x,y) \).

For grayscale operations, the image will be represented by the function \( f(x,y) \), and SE will be \( h(x,y) \). Grayscale dilation, erosion, opening and closing become, respectively:

\[
(f \oplus h)(x,y) = \sup_{(r,s) \in H} \{ f(x-r, y-s) + h(r,s) \} \tag{5} \\
(f \ominus h)(x,y) = \inf_{(r,s) \in H} \{ f(x+r, y+s) - h(r,s) \} \tag{6} \\
f \odot h = (f \ominus h) \oplus h \tag{7} \\
f \bullet h = (f \oplus h) \ominus h \tag{8}
\]

where sup\{\} & inf\{\} - the supremum and infimum operators, respectively.

A special class of grayscale morphological filters, referred to as a function -and-set-processing (FSP) filters, results when \( h(x,y) = 0 \) \( \forall (x,y) \in H \).

To perform a morphological operation on a colored (vector-valued) image, an ordering of the vector field is required. The problem of ordering of multivariate data is not unique to MM. However, based on Barnet classification mentioned earlier [9], in MO, the components in each spectral band are ordered independently of the components of other bands. For example, the component-wise dilation and erosion of \( f(x,y)=[f_{G}(x,y),f_{B}(x,y)]^{T} \) by the SE \( h(x,y)=[h_{G}(x,y),h_{B}(x,y)]^{T} \) can be as:

\[
(f \oplus h) = [(f_{G} \oplus h_{G}), (f_{B} \ominus h_{B})]^{T} \tag{9} \\
(f \ominus h) = [(f_{G} \ominus h_{G}), (f_{B} \ominus h_{B})]^{T} \tag{10}
\]

However, since image components are processed independently, an alteration of the color balance and object boundaries accompanies this type of ordering.

In RO each multivariate observation is reduced to a single value, which is a function of the component values for that observation, with the multivariate samples ranked according to this single value. Let \( x_{1}, x_{2}, ..., x_{n} \) be a collection of multivariate samples, where each \( x_{i} \) is a vector in \( \mathbb{R}^{p} \). The first step is to map each \( x_{i} \) to a scalar value \( d_{i}=d(x_{i}) \), where \( d: \mathbb{R}^{p} \rightarrow \mathbb{R} \). Then, the vectors \( x_{1}, x_{2}, ..., x_{n} \) are ordered according to their \( d \) values: \( d_{1}, d_{2}, ..., d_{n} \) and angle-valued (vector) dilation becomes:

\[
(f \oplus H)(x,y) = a \in \{ f(r,s) : (r,s) \in H_{(x,y)} \} \\
d(a) \geq d(f(r,s)) \forall (r,s) \in H_{(x,y)} \tag{11}
\]

Vector erosion can be analogously defined:

\[
(f \ominus H)(x,y) = b \in \{ f(r,s) : (r,s) \in H_{(x,y)} \} \\
d(b) \leq d(f(r,s)) \forall (r,s) \in H_{(x,y)} \tag{12}
\]

Evidently, vector opening (closing) is defined as vector erosion followed by vector dilation (dilation followed by erosion).

Finally, in CO, the samples are ordered using one component initially. In the case where multiple samples have the same initial component value, a secondary component is used to order those samples, and so on. In this work, data ordering is performed using the Mahalanobis Color Distance (MCD) to replace the angle-valued pixels by a scalar and, hence it is called Mahalanobis Color Distance-based Morphological Ordering Algorithm (MCD-MOA).

3 The MCD-MOA Algorithm

The Mahalanobis Color Distance (MCD) is commonly used in pattern recognition analysis [11]. The MCD can be applied to the RGB color space as follows:

\[
\Delta d = \begin{bmatrix} \Delta R \\ \Delta G \\ \Delta B \end{bmatrix} \begin{bmatrix} \Delta R \\ \Delta G \\ \Delta B \end{bmatrix}^{-1} \tag{13}
\]

The covariance matrix \( \mathbf{M} \) in (13) can be as:

\[
\mathbf{M} = \begin{bmatrix} \sigma_{RR} & \sigma_{RG} & \sigma_{RB} \\ \sigma_{GR} & \sigma_{GG} & \sigma_{GB} \\ \sigma_{BR} & \sigma_{BG} & \sigma_{BB} \end{bmatrix} \tag{14}
\]

Assuming \( \mathbf{L} = \mathbf{M}^{-1} \) and considering that \( \mathbf{L}_{RG} = \mathbf{L}_{GR} \), \( \mathbf{L}_{RB} = \mathbf{L}_{BR} \) and \( \mathbf{L}_{GB} = \mathbf{L}_{BG} \), then we obtain:
\[ \Delta d = \sqrt{L_{RR} \Delta R^2 + L_{GG} \Delta G^2 + L_{BB} \Delta B^2 + 2L_{RG} \Delta R \Delta G + 2L_{GB} \Delta G \Delta B} \]  

(15)

Here, \( \Delta \) requires a reference color, for example, \( \Delta R = R_i - R_o \), where \( R_i \) is the R current value and \( R_o \) is the R-value of the reference color. To avoid incompatibility, the reference color was taken to be the pure black with RGB=(0,0,0).

The MCD-MOA implements RO and CO techniques and takes place in the following steps:

(i) The algorithm starts by selecting a suitable structuring element \( W \).

(ii) For the given input RGB image, the matrix \( M \) is calculated, then for each input vector pixel \( \text{RGB}(i,j) \) (\( \forall i,j \subset W \)) the value \( \Delta d(i,j) \) is found.

(iii) For dilation operation, the current vector pixel \( \text{RGB}(i,j) \) (\( \forall i,j \subset W \)) is replaced by the vector pixel \( \text{RGB}(r,s) \) (\( \forall r,s \subset W \)) that exhibits the maximum \( \Delta d(r,s) \) as:

\[
\text{RGB}_{\text{out}}(i,j) = \{ \text{RGB}(r,s) : \Delta d(r,s) = \text{max}, \forall (r,s) \subset W \} \]

(16)

Analogously, for erosion operation:

\[
\text{RGB}_{\text{out}}(i,j) = \{ \text{RGB}(r,s) : \Delta d(r,s) = \text{min}, \forall (r,s) \subset W \} \]

(17)

(iv) If it happens that more than one vector pixel \( \text{RGB}_{\text{in}} \) has maximum \( \Delta d \) value (for dilation) or minimum \( \Delta d \) (for erosion), then \( \Delta d \) is recalculated in respect to the intensity value:

\[
\Delta d(r,s) = 0.299 R(r,s) + 0.578 G(r,s) + 0.110 B(r,s), \forall (R(r,s), G(r,s), B(r,s)) \subset W \)

(18)

However, this situation occurs very rarely (typically 0.1-0.2%).

(v) Here, steps (i-v) can be also repeated using other color models such as: \( YIQ, Luv, HSI \), etc.

### 4 Simulations and Results

In this section, the performance of the above proposed algorithm will be explored. Here, several tests have been carried out using a variety of test color images. Since the analysis of different morphological operators with different color images is not a simple task, hence, in order to simplify this issue the test images have been grouped into distinct classes varying from simple color geometrical shapes on different color backgrounds to complex color images containing texture, edges, shapes, etc. In Fig.1, typical examples of some implemented image classes are depicted. The experimental procedure is organized in the following subsections.

![Examples of the implemented test true-color images: a) class 1, b) class 2, and c) class 3](image)

#### 4.1. Psychophysical Evaluation

To evaluate the performance of the proposed algorithm and compare with other algorithms, it is usually referred to perceptual image quality assessment (Psychophysical Experiment) which was adopted from image compression applications and adapted to suit image morphology [10]. The psychophysical procedure has been organized as follows:

- The used original resulting images were subjectively evaluated by 30 observers: 5 specialists in image processing, 10 engineers with some background in image processing, and, 15 undergraduate students that have passed a course in "image processing". Before starting the procedure, the participants have been shortly explained the concept of image morphology (mainly: dilation, erosion, opening and closing), then they have been
shown some results with grayscale as well as binary morphology. After which the original and the resulting images were displayed side by side on the center of a calibrated CRT display in a dark environment.

A 5-scale judgment has been implemented: 5 - excellent quality, 4 - very good, 3 - good, 2 - acceptable, 1 - unacceptable. Here, the images were displayed for 30 sec. A period of 15 sec was given to the observers to give their judgment (between 1 and 5).

Thus, the Average value for the Psychophysical Evaluation (APE) of the i-th resulting image can be:

\[
APE_i = \frac{1}{MN} \sum_{m=1}^{M} \sum_{n=1}^{N} QS_{mn}(i)
\]

Where:
- N - Number of observers (in our case N=30)
- M - Number of times the image is being displayed.
- QS_{mn}(i) - Quality Step given by the n-th observer for the i-th reconstruction at the m-th displaying time.

Fig.3. APE results comparison for dilation
Fig.4. APE results comparison for dilation
Fig.5. APE results comparison in the YIQ space: MCD-MOA, CC-MOA, T&V, Ptrs and S&W Techniques
4.2. Experimental Results

In the proposed color dilation and erosion operators, the SE is based on FSP. Various shapes and dimensions of SE have been implemented such as: line, square, rectangular, and pseudo-circular. Several color models have been also tested including: RGB, YIQ/YUV, HSI. Fig.2-4 plot results of implementing the MCD-Based morphological dilation and erosion. For comparison purposes, in Fig.5, the morphological opening and closing are applied to various existing algorithms. Here, for simplicity, the following notations have been implemented:

- T&V: the algorithm proposed by Trahanias and Venetsanopoulos [5].
- Ptrs: the algorithm proposed by Peters in [8].
- S&W: the algorithm proposed by Sartor and Weeks in [9].
- The prefixes Dlt, Erd, Opn & Cls (see Fig.5 denote dilation, erosion, opening and closing respectively.

Analyzing obtained results, one can draw the following notes:

1. MCD-MOA algorithm gave an improvement in the output image quality. For example, with YIQ model, its APE was improved between 0.62-0.76 (12.3-15.2%) over what is possible using T&V, Ptrs and S&W algorithms.

2. MCD-MOA gave better shape conservation, according to viewers notices.

3. The implementation of MCD-MOA with HSI model gave lower results than YIQ. This is due to the fact that erosion or dilation employ minimum/maximum operations. It is evident that two pixels having a hue of, for example, 3° and 375° are close to each other even they have a large numerical distance.

4. Results obtained from images taken from class 2 (images I3, I4 in Fig.1) are comparable in all cases (MCD-MOA gave 2-3% improvement). This is evident since the objects (edges, contours) are well defined in this class and, thus, simple morphological operators can be applied.

4.3. Extending MCD-MO Algorithm

The MOA-MCD can be used to produce morphological algorithms for smoothing, gradient, Laplacian filter, etc. All are constructed from the primitives for the defined vector dilation and vector erosion.

4.3.1. Morphological Smoothing Operator

The smoothing operator can be obtained as cascaded opening and closing for grayscale images. It can be extended to color images using MOA-MCD as:

\[
S_{\text{MCD}}(A, SE) = C_{\text{MCD}}(O_{\text{MCD}}(A, SE), SE)
\]

where,
\(A\) - the input color image.
\(S_{\text{MCD}}\) - Smoothing operator using MCD-MOA.
\(O_{\text{MCD}}\), \(C_{\text{MCD}}\) - MCD-MOA opening and closing respectively.

To evaluate the performance of the smoothing operator, an additive Gaussian noise has been added to the input images with zero mean and a variance \(\sigma^2=0.01, 0.015, \) and 0.02 (denoted as SG1, SG2 and SG3 respectively). The evaluation of the image reconstruction quality can be performed using the total PSNR:
PSNR = \frac{1}{3}(\text{PSNR}_R + \text{PSNR}_G + \text{PSNR}_B) \quad (21)

Where: \text{PSNR}_R, \text{PSNR}_G, \text{PSNR}_B - \text{PSNR} for the reconstructed red, blue and green components of the image respectively.

Results of implementing the smoothing operator are shown in Fig.6. For better illustration and visual comparison, examples of the filtered images are shown in Fig.7.

4.3.2. Morphological Gradient and Laplacian

For linear filters the gradient filter yields a vector representation with a magnitude and direction. The version presented here generates a morphological estimate of the color gradient magnitude [10]:

\[
\text{Grd}_{\text{MCD}} = \frac{1}{2} |\text{D}_{\text{MCD}}(A,SE) - \text{E}_{\text{MCD}}(A,SE)| \quad (22)
\]

Where, 
\text{D}_{\text{MCD}}, \text{E}_{\text{MCD}} - \text{Dilation and erosion operators using MCD-MOA}.

The color morphological Laplacian filter can be obtained as an extension of the grayscale Laplacian [10]:

\[
\text{Lap}_{\text{MCD}} = \frac{1}{2} |\text{D}_{\text{MCD}}(A,SE) - \text{E}_{\text{MCD}}(A,SE) - 2A| \quad (23)
\]

5 Conclusions

MM has been widely used to process binary and grayscale images, with morphological techniques being applied to noise reduction, image enhancement, and feature detection. To perform a morphological operation on a vector-valued image an ordering of the vector field is required. The problem of ordering of multivariate data is not unique. However, a possible classification of such ordering (sub-ordering) can be as: marginal ordering MO, RO, CO and PO. In this work, an algorithm for color morphology is proposed and based on a combination of reduced and conditional ordering of the underlying data. The proposed algorithm implements the Mahalanobis Color Measure (MCD) to perform the basic four morphological operators: dilation, erosion, opening and closing. Here, several psychophysical tests have been carried out and have shown that MCD-MOA (in the YIQ model) improved APE between 0.62-0.76 (12.3-15.2%) over what is possible using T&V, Pts and S&W algorithms.

References