Software for Synthesis of Radiation Patterns by Linear Antenna Arrays

JOSÉ LUIS RAMOS Q., MARTÍN J. MARTÍNEZ S., M. SUSANA RUIZ P.
Departamento de Electrónica, CUCEI
Universidad de Guadalajara
Av. Revolución 1500, Puerta 10, Módulo O, S. R., C.P. 44840, Guadalajara, Jalisco
MÉXICO

Abstract: - This paper presents the characteristics of a software developed on MatLab Scripts to obtain the synthesis of radiation patterns, using linear antenna arrays. Several classical techniques have been considered such as Schelkunoff method, Fourier series and transform methods, Woodward-Lawson method, Taylor line-source methods. In all those cases, algorithms were developed and the results agree with the literature. In order to improve some characteristics of the radiation pattern, an algorithm based on the gradient method has been included for optimization purposes.

Key-Words: - Antenna Arrays, Radiation Pattern Synthesis, Optimization, Gradient Method.

1 Introduction

A single antenna has a limited radiation pattern. But with the use of several antennas working together (array), it is possible to improve the radiation according to some specifications. In general the characteristics of the array are controlled by the proper choice of the element (dipole, horn, patch, etc.), the geometry of the array and the excitation (amplitude and phase) of each element [1-5].

The antenna arrays considered in this work, are restricted to linear arrays where the radiated fields depend only on the θ coordinate. The electric field in far zone is represented by [6]

$$E_\theta \approx E_{\theta 1} \cdot AF \qquad (1)$$

where $E_{\theta 1}$ is the electric field produced by the element at the origin of the coordinate system and AF is known as the array factor and is given by

$$AF = \sum_{k=1}^{N} a_k e^{j(kd_k \cos\theta + \beta_k)} \qquad (2)$$

a_k, β_k, and d_k, represent; amplitude, phase and position of the k-th element respect to the origin.

The process to obtain the position and excitation of the elements in order to get some desired radiation characteristics, is known as the synthesis of an array. In this work several techniques were considered. Special scripts were written for: Schelkunoff method, Fourier transform and series methods, Woodward-Lawson method, and Taylor methods.

2 Characteristics of the Software

The software consists on about 25 programs used by the main program "*sintesis.m*" [7]. This program is menu driven. In general all the techniques give good approximation to the desired characteristics but today with the help of personal computers, it is a good advice the use of software oriented to optimize the results. Here we present an algorithm to improve the radiation patterns, based on the gradient method. Fig.1 shows the main menu where each option opens a new menu to introduce data and to apply the desired technique of synthesis.

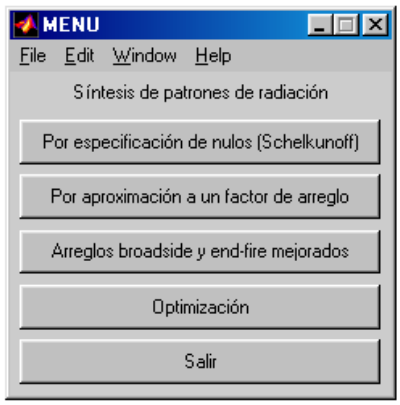


Figure 1. Main menu used to synthesize radiation patterns.

3 Schelkunoff Method

This method is useful when some nulls are desired in the array factor. The number of elements and their amplitude excitation are determined. As more elements are used, a better control of the shape of the array factor is obtained [8]. Let,

$$z = x + jy = e^{j\psi} = e^{j(kd\cos\theta + \beta)} \tag{3}$$

then (2) for N elements with uniform separation, non uniform amplitude and progressive phase, can be written as a polynomial in z of degree $N-1$,

$$AF = a_n (z - z_1)(z - z_2)(z - z_3)\cdots(z - z_{N-1}) \tag{4}$$

where $z_1, z_2, z_3, \cdots z_{N-1}$ are the roots of the polynomial. The magnitude of (4) is written as

$$|AF| = |a_n||z - z_1||z - z_2||z - z_3|\cdots|z - z_{N-1}| \tag{5}$$

From (3) it is seen that the magnitude of z is always one, and the angle depends on d, β and θ. When θ varies from 0º to 180° for fix values of d and β, there are true values for z (visible region). For any point on this region, the magnitude of the AF is obtained by (5). Fig.2 shows the synthesis of a pattern with $\beta = 0°$, nulls at $\theta = 0°$, $\theta = 90°$ and $\theta = 180°$, with a separation of $d = \lambda / 4$.

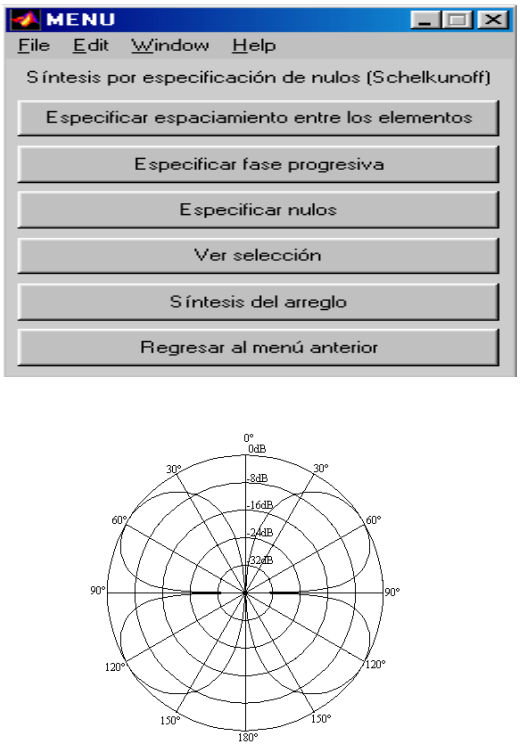


Figure 2. Menu and pattern for Schelkunoff method.

4 Fourier Transform Method

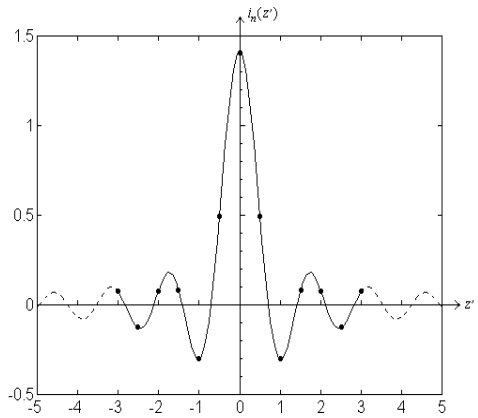
This method is useful to determine the amplitude distribution of a continuous line source, but can be adapted to determine the excitation amplitude of an antenna array. The relationship between a line source and the produced space factor is given by [1,2,8],

$$SF(u) = \int_{-\infty}^{+\infty} \boldsymbol{i_n}(z') e^{-juz'} dz' \tag{6}$$

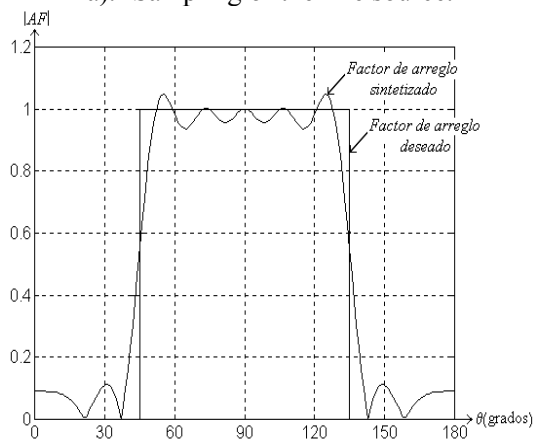
This is a Fourier transform equation. By other hand, the inverse Fourier transform allow us to calculate the amplitude distribution of the line source from the desired space factor as,

$$\boldsymbol{i_n}(z') = \frac{1}{2\pi} \int_{-\infty}^{+\infty} SF(u) e^{juz'} du \tag{7}$$

Once the excitation of the line source is obtained, it is possible to construct an amplitude distribution for the elements of an array by sampling the former as shown in Fig.3a) for an array of 13 elements. In Fig.3b) the desired and the synthesized array factors are shown.



a).- Sampling of the line source.



b).- Array Factors: desired and synthesized.

Figure 3- Synthesis by the Fourier transform method.

5 Fourier Series Method

The array factor for a linear array with $2M+1$ equally spaced elements, uniform phase and amplitude proportional to C_n, can be expressed as [6]

$$AF = \sum_{n=-M}^{M} C_n e^{jk(nd)\cos\theta} \tag{8}$$

Let $C_n = C_{-n}$, then we can write

$$AF(v) = C_0 + \sum_{n=1}^{M} 2C_n \cos nv \tag{9}$$

where

$$v = kd\cos\theta \tag{10}$$

Eq. (9) represents a cosine series of Fourier truncated to $M+1$ terms with unknown amplitudes.

When the desired pattern has been specified, the coefficients C_n are calculated using

$$C_n = \frac{\lambda}{l}\left[\frac{1}{2\pi}\int_{-kd}^{kd} AF_d(v)\cos nv\, dv\right] \tag{11}$$

An array factor synthesized by this method, with 13 elements and spacing of 0.5λ is shown in Fig.4, in the same figure the desired AF is also shown.

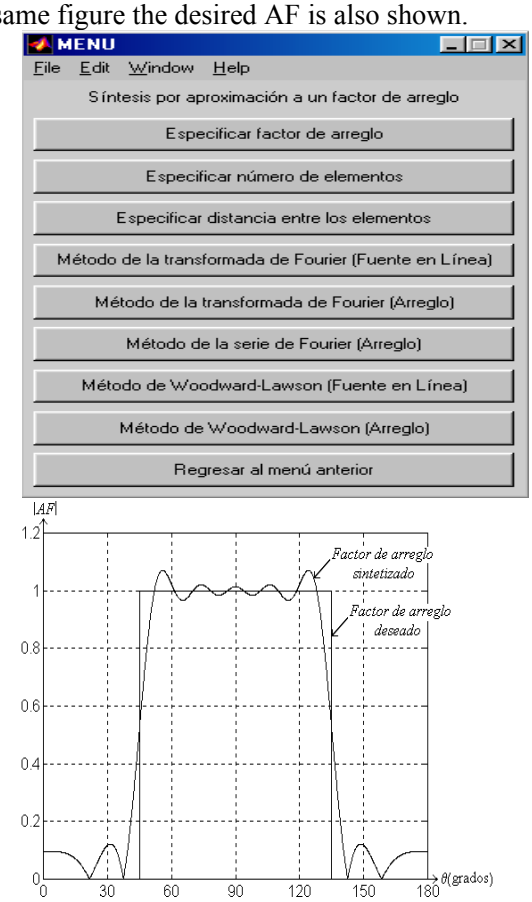


Figure. 4- Menu and array factor synthesized by the method of the Fourier series

6 Woodward-Lawson Method

This is a beam shaping method based on the sampling of the desired AF. For each sampling a set of uniform amplitude and progressive phase excitation signals are calculated. Such excitation produces a sinc(x) like pattern with the major lobe just on the direction of the sampling. Minor lobes and nulls are also produced, but sampling points are chosen so that the contribution in the sampled direction comes from one set of excitation only. Excitation of the array is obtained by adding all partial excitation signals.

The total array factor is given by [1]

$$AF(\theta) = \sum_{m=-M}^{M} b_m \frac{sen\left[\frac{N}{2}kd(\cos\theta - \cos\theta_m)\right]}{Nsen\left[\frac{1}{2}kd(\cos\theta - \cos\theta_m)\right]} \tag{12}$$

where the sampled directions are

$$\theta_m = \cos^{-1}\left(\frac{m\lambda}{Nd}\right) \tag{13}$$

The excitation of each element is obtained as

$$a_n = \frac{1}{N}\sum_{m=-M}^{M} b_m e^{-j(2\pi z_n' \cos\theta_m)} \tag{14}$$

where z_n' is the position of the element n.

The array factor synthesized with 10 elements spaced 0.5λ, is shown in Fig.5. There is also presented the desired AF and the sampling points.

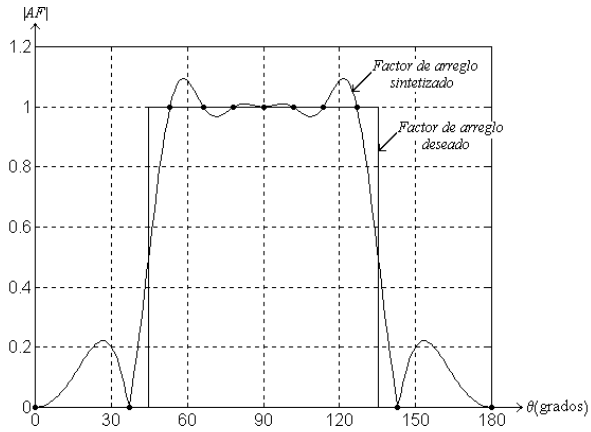


Figure 5- Array factor synthesized by the Woodward-Lawson method.

7 Taylor Method (Tschebysheff error)

This method has the characteristic to produce broadside space factor (or array factor) with minor lobes that decay monotonically. The space factor proposed by Taylor is [1]

$$SF(u, A, \bar{n}) = \prod_{n=1}^{\bar{n}-1} \left[\frac{1-\left(\frac{u}{u_n}\right)^2}{1-\left(\frac{u}{n\pi}\right)^2} \right] \cdot \frac{sen(u)}{u} \qquad (15)$$

where

$$u = \pi \frac{l}{\lambda} \cos\theta \qquad (16)$$

When $u = u_n$, the function in (15) has zeros (nulls), while when $u = n\pi$ the function has poles. u_n is given by (16) when $\theta = \theta_n$, i.e. the angles where the space factor has nulls.

The parameter $\bar{n}$ in (15) is an integer that has to be selected in order to fix the amplitude of some minor lobes (inner lobes) to $1/R_0$, where R_0 is the desired side lobe ratio. The remaining lobes (outer lobes) decay progressively at a rate of π/u.

In order to have a smooth transition between inner and outer lobes, Taylor introduced the parameter σ (scaling factor) given by

$$\sigma = \frac{\bar{n}}{\sqrt{A^2 + \left(\bar{n} - \frac{1}{2}\right)^2}} \qquad (17)$$

Using this parameter, the location of the nulls are obtained with

$$u_n = \pi \frac{l}{\lambda} \cos\theta_n = \begin{cases} \pm \pi\sigma \sqrt{A^2 + \left(n - \frac{1}{2}\right)^2} & 1 \le n < \bar{n} \\ \pm n\pi & \bar{n} \le n \le \infty \end{cases} \qquad (18)$$

where the constant A is related to R_0 as:

$$\cosh(\pi A) = R_0 \qquad (19)$$

The normalized line source distribution corresponding to the space factor of (15) is given by

$$I(z') = \frac{1}{l}\left[1 + 2\sum_{p=1}^{\bar{n}-1} SF(p, A, \bar{n}) \cos\left(2\pi p \frac{z'}{l}\right)\right] \qquad (20)$$

The coefficients $SF(p, A, \bar{n})$ represent samples of the Taylor pattern, and they can be obtained from (15) with $u = \pi p$.

In order to have the excitation coefficients of a linear array, it is possible to sample the distribution of the line source. Fig.6 shows the array factor synthesized by this method. This example consider 11 elements with 0.5λ of separation (the corresponding length of the line source is 5λ), side lobe level of 20 dB and $\bar{n} = 2$.

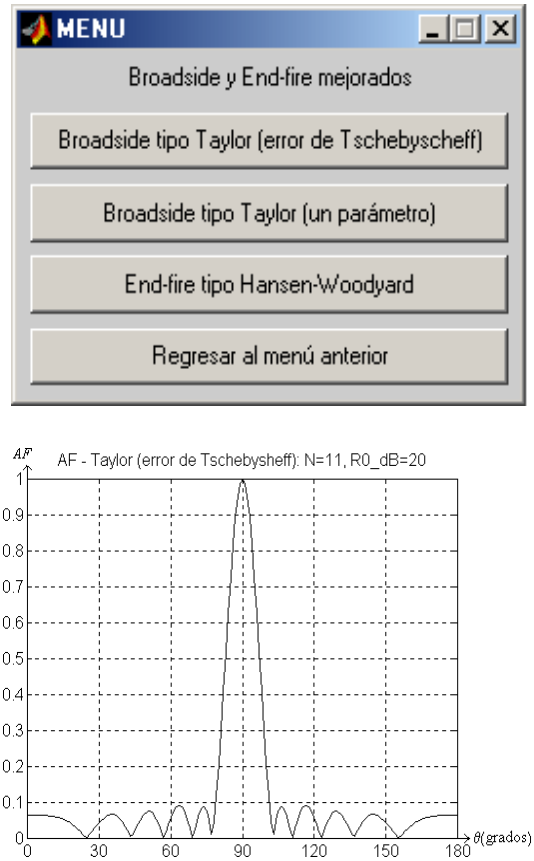


Figure 6- Menu and magnitude of the array factor synthesized by the Taylor Method.

8 Taylor Method (One-Parameter)

This method is useful to generate broadside space factors whose minor lobes always decay as they are away from the main lobe.

The level of lobes is controlled by a parameter that is the only one that can be modified (besides the length of the line source)

This method gives as a result the excitation of a line source from the specification of the desired level of lobes and from the length of the line source itself.

The source distribution proposed by Taylor is [1]

$$I_n(z') = \begin{cases} J_0\left[j\pi B\sqrt{1-\left(\frac{2z'}{l}\right)^2} \right] & -l/2 \le z' \le l/2 \\ 0 & \text{elsewhere} \end{cases} \tag{21}$$

where J_0 is the Bessel function of the first kind of order zero, l is the total length of the line source, and B is a constant to be determined according to the desired lobe level.

The space factor is given by Taylor as

$$SF(\theta) = \begin{cases} l\dfrac{senh\left[\sqrt{(\pi B)^2 - u^2}\right]}{\sqrt{(\pi B)^2 - u^2}}, & u^2 < (\pi B)^2 \\ l\dfrac{sen\left[\sqrt{(\pi B)^2 - u^2}\right]}{\sqrt{(\pi B)^2 - u^2}} & u^2 > (\pi B)^2 \end{cases} \tag{22}$$

where

$$u = \pi\frac{l}{\lambda}\cos\theta \tag{23}$$

The inequality $u^2 < (\pi B)^2$ in (22) represents the region closed to the main lobe. The minor lobes are in the region $u^2 > (\pi B)^2$.

When $u = 0$, i. e. when $\theta = \pi/2$, the amplitude of the pattern is maximum and is equal to

$$SF(\theta)_{\max} = \frac{senh(\pi B)}{\pi B} = H_0 \tag{24}$$

For $u^2 >> (\pi B)^2$, the space factor in (22) reduces to

$$SF(\theta) = \frac{sen\left[\sqrt{u^2 - (\pi B)^2}\right]}{\sqrt{u^2 - (\pi B)^2}} \cong \frac{sen(u)}{u} \qquad u >> \pi B \tag{25}$$

Using (24), the level of the side lobe is defined as

$$R_0 = \frac{1}{0.217233}\frac{senh(\pi B)}{(\pi B)} = 4.603\frac{senh(\pi B)}{(\pi B)} \tag{26}$$

The constant B has to be obtained from (26) and then the distribution of the line source is calculated using (21). Also it is possible to sample the excitation of the line source to obtain the excitation coefficients for a linear array.

As an example, we considered an array of 11 elements with a total length of 5λ to produce an array factor with a lobe ratio of 20 dB. The program "sintesis" generates the synthesized array factor shown in Fig.7.

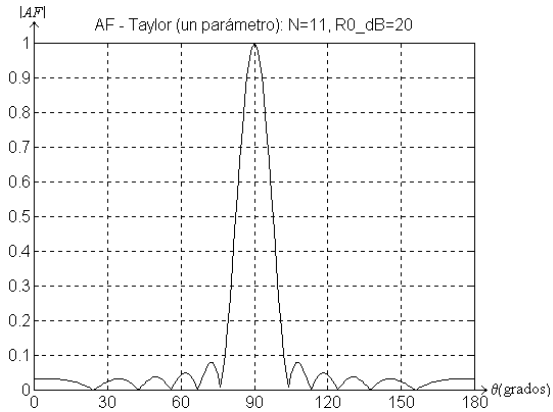


Figure 7- Array factor synthesized using the method of Taylor (one-parameter).

9 Optimization by the Gradient Method

The optimization is a procedure in which, the variables involved in the performance of a system are modified starting from a non optimum design, in such a way that one o several characteristics of the system are improved [9]

In this work a scrip for *matlab* was development to optimize some radiation characteristics of the array factor using the method of the gradient.

Basically the process is a loop searching for a optimum point. The optimum value could be a maximum or a minimum depending of the specific characteristic. For example the beamwidth is minimized while the directivity and the lobe ratio are maximized. A parameter "epsilon" is defined to determine the value of the norm of the gradient vector to be close enough to zero in order to stop the searching process. The routine "gradient" calculates the value of the gradient required to determine the direction of searching the optimum value in each iteration. The routine "salemarg" cheks that all the other radiation characteristics that are not been optimized, remain quite unaltered. This optimization method has been tested gives good results.

As an example, an array of 11 elements synthesized using the Fourier series method was optimized to improve the beamwidth. The restrictions were: for the direction $\pm 10°$ respect to the original,

for the directivity 50% respect to the original and a lobe ratio of at least 15 dB. The results are presented in Fig.8.

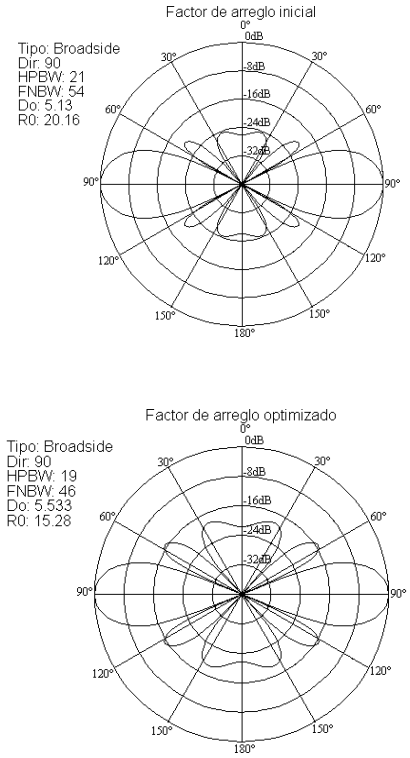


Figure 8- Original and optimized array factors.

10 Conclusions and Future Work

The most relevant contribution in this work is the development of a specialized software to design radiation patterns using antenna linear arrays since it can help researchers and teachers to test in a cheap and fast way their own ideas. At the moment to implement some of the algorithms the mathematical models were manipulated to get more convenient expressions.

The optimization routine is a widely known powerful tool. In this case we have used it successfully to antenna array optimization.

At the present, we are working to match this software with another one developed for the analysis of antenna linear array, which includes: Uniform arrays, non-uniform arrays, non-uniform separation arrays, etc. [10, 11]. The software is being improved by including other elements besides de dipole, as: loops, horns, patches, etc.

Referencias:

1. C. A. Balanis, *Antenna Theory Analysis and Design*, John Wiley, 1997.
2. W. L. Stutzman & G. A. Thiele, *Antenna Theory and Design*, John Wiley, 1981.
3. T. A. Milligan, *Modern Antenna Design*, Mc Graw-Hill, 1985.
4. J. L. Ramos, M. J. Martínez, Ma. Susana Ruíz, G. A. Vega, Análisis de Arreglos Lineales de Antenas apoyados por Computadora y Cálculo Numérico de la Impedancia Propia y Mutua basada en el Método de Momentos, *Memorias del Congreso de Instrumentación SOMI XIII*, Sociedad Mexicana de Instrumentación, (Ensenada, B.C.), 1997, 560-565.
5. J. Ramos, G. Vega, M. Martínez, J. Hernández, M. Ruíz, Arreglos Lineales de Antenas y Dispositivos de Excitación, *Memorias del Congreso de Instrumentación SOMI XI*, Sociedad Mexicana de Instrumentación, (Morelia, Mich.), 1996, 662-669.
6. R. E. Collin, *Antennas and Radiowave Propagation*, Mc Graw-Hill, 1985.
7. MatLab Manuals.
8. J.D. Kraus, *Antennas*, Mc Graw Hill, 1950.
9. J. S. Arora, *Introduction to Optimum Design*, McGraw-Hill, pag. 1989.
10. J. L. Ramos, G. A. Vega, M. J. Martínez, Arreglos Lineales de Antenas con Separacion No Uniforme, *Memorias del Congreso de Instrumentación SOMI XII*, Sociedad Mexicana de Instrumentación (San Luis Potosí, S.L.P.), 1997, 748-754.
11. José Luis Ramos Q., Martín J. Martínez S., Gustavo A. Vega G., M. Susana Ruiz P., Software for Calculating Radiation Patterns for Linear Antenna Arrays, to be presented at the WSEAS Int. Conf. On Simulation, Modelling and Optimization, Cancún, México, May 2002.