A Comparison of Iterative Feedback Tuning and Classical PID Tuning Schemes

MAGNUS MOSSBERG
Dept. of Electrical Engineering, Karlstad University
SE-651 88 Karlstad, SWEDEN

MICHEL GEVERS
Centre for Systems Engineering and Applied Mechanics (CESAME)
Université Catholique de Louvain
B-1348 Louvain-la-Neuve, BELGIUM

OLIVIER LEQUIN
DCRT-ACE/Process Control Optimization - Plastics, Solvay S.A.
Rue de Ransbeek, 310, B-1120 Brussels, BELGIUM

Abstract: Iterative Feedback Tuning (IFT) is used for tuning PID controllers for the case when it is of interest to reach a new set point level as quickly as possible. A special variant of the IFT criterion is used, where zero weighting is applied during the transient response. The result of the IFT method is compared with the results of three tuning schemes commonly used in industry. It turns out that the IFT method always performs as good as or better than the other methods.

Keywords: Iterative Feedback Tuning, PID controllers, optimal control

1 Introduction

Iterative Feedback Tuning (IFT) was first derived in [3] and is a model free technique for tuning the parameters of a fixed structure controller. The facts that no model is needed and that the method works with closed loop data are important reasons why it has gained a lot of interest and is used in industry, see e.g. [4, 5].

The original version of IFT [3] was derived for a general controller with two degrees of freedom for an unknown, linear and time-invariant system as in Fig. 1.

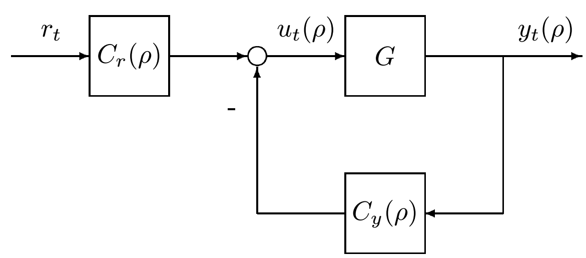

Fig. 1: The controller C_r, C_y applied to the system G, where r, u, y and v are the reference signal, the control signal, the output signal and a disturbance.

The criterion to be minimized is

$$J(\rho) = \frac{1}{2N}\mathrm{E}\Big\{\sum_{t=1}^{N} w_y(t)\big(L_y(y_t(\rho) - r_t\big)^2 + \lambda \sum_{t=1}^{N} w_u(t)\big(L_u u_t(\rho)\big)^2\Big\}, \tag{1}$$

where N is the number of data points, ρ is the controller parameter vector, L_y and L_u are frequency weighting filters, λ is a penalty factor, and $w_y(t)$ and $w_u(t)$ are time weighting functions. Note that all signals, except the reference signal r, are dependent of the controller parameters ρ and that the optimal controller parameter vector

ρ^* is the one that minimizes the cost function (1),

$$\rho^* = \arg\min_{\rho} J(\rho). \tag{2}$$

When finding (2) using IFT, a feature is that an estimate of the gradient $\nabla J(\rho)$ of the cost function (1) is given by using signals from an experiment on the closed loop system, without knowing the true system. It is now possible to reach the minimum value (2) by using an iterative algorithm

$$\rho_{i+1} = \rho_i - \gamma_i R_i^{-1} \nabla J(\rho_i), \tag{3}$$

where γ_i is a positive real scalar that determines the step size and R_i is a positive definite matrix.

In [6], a modification of the criteria function (1) was presented:

$$\begin{aligned} J_m(\rho) = \frac{1}{2N}\mathrm{E}\Big\{&\sum_{t=t_0}^{N}\big(L_y(y_t(\rho) - r_t\big)^2 \\ &+ \lambda\sum_{t=1}^{N}\big(L_u u_t(\rho)\big)^2\Big\}. \end{aligned} \tag{4}$$

Zero weighting is put on the transient part of the output error and this will be interpreted as a mask of length t_0. When it is desired to tune the controller parameters in such a way that the output signal responds to a set point change as quickly as possible, a minimization of the cost function (4) can be fruitful. The reason for this is that it most often is of no interest how the new set point is reached as long as a large overshoot and an oscillatory behavior is avoided. This means that the controller parameters do not have to compromise between reaching the new set point and following a desired transient response that might not be natural for the closed loop system. Therefore, all effort is put on achieving the fastest possible settling time.

In the paper the problem of tuning PID parameters in order to minimize the settling time in the case of a set point change is considered. It is assumed that

$$C_r(\rho) = K(1 + \frac{1}{T_i s}), \tag{5}$$

$$C_y(\rho) = K(1 + \frac{1}{T_i s} + T_d s) \tag{6}$$

in Fig. 1 and that the PID parameters

$$\rho = [K, T_i, T_d]^T. \tag{7}$$

The IFT criteria (4) is used with the choices $L_y \equiv 1$, $L_u \equiv 1$, $\lambda = 0$ and the scaling factor $1/(2N)$ omitted, i.e.

$$J_m(\rho) = \mathrm{E}\Big\{\sum_{t=t_0}^{N}(y_t(\rho) - r_t)^2\Big\}. \tag{8}$$

This approach was proposed in [6] where it also was suggested that one should start with a large zero weighting time interval, i.e. a big mask (a large value of t_0) in combination with PID parameters that give a slow response with no overshoot. The mask size should then be reduced until an overshoot starts to appear. This idea of IFT is applied in the paper and the method is compared with three well known tuning methods that are widely used in industry.

2 The tuning methods

Apart from the IFT method described in Section 1, three other methods are considered for tuning the parameters of a PID controller:

- Ziegler-Nichols (ZN)
- Integral Square Error (ISE)
- Internal Model Control (IMC)

In the literature, numerous methods are found and these three are not necessarily the best possible choices. The motivation for choosing these techniques is that they are among the most commonly used methods and it is therefore interesting to compare the IFT method with them. Short descriptions of the methods follow next.

The Ziegler-Nichols (ZN) tuning method is perhaps the most well known tuning method. In this method, the gain is increased until the closed-loop system starts oscillating, and the controller gain K_{cr} and the oscillation period P_{cr} are registered. The controller parameters are then given as

$$K = \frac{K_{cr}}{1.7}, \quad T_i = \frac{P_{cr}}{2}, \quad T_d = \frac{P_{cr}}{8}. \tag{9}$$

In the Integral Square Error (ISE) method the criterion function is

$$\mathrm{ISE}(\rho) = \int_0^{\infty} e_t^2 \mathrm{d}t = \frac{1}{2\pi\mathrm{i}}\int_{-\mathrm{i}\infty}^{\mathrm{i}\infty} E(s)E(-s)\mathrm{d}s. \tag{10}$$

The last integral is calculated recursively using Åström's integral algorithm [1], and minimized with respect to the PID parameters.

The Internal Model Control (IMC) method applied here is described in e.g. [2]. A basic assumption is that the system can be modeled as

$$G(s) = \frac{K_p}{1+sT} e^{-sL}. \tag{11}$$

In [2] it is also shown how models of other forms can be approximated by the form (11). In the case of unknown model parameters, they can be estimated from an open loop step response. The controller given by this method can be interpreted as a PID controller with the parameters

$$K = \frac{2T+L}{2K_p(T_f+L)},\ T_i = T + \frac{L}{2},\ T_d = \frac{TL}{2T+L}, \tag{12}$$

where T_f is a design parameter.

3 Simulation examples

The four tuning methods (ZN, ISE, IMC and IFT) are tested on the three systems

$$G_1(s) = \frac{1}{1+20s} e^{-5s}, \tag{13}$$

$$G_2(s) = \frac{1}{(1+10s)^8}, \tag{14}$$

$$G_3(s) = \frac{1-5s}{(1+10s)(1+20s)}. \tag{15}$$

A sampling time of 0.01 s is used in the simulations.

3.1 A study of the system $G_1(s)$

In Fig. 2, the results of the four tuning schemes applied to the system $G_1(s)$ in (13), that has a time delay, are shown in terms of step responses and the corresponding input signals for the closed loop systems, and the obtained PID parameters are listed in Table 1. With shorter settling times and smaller control signals, the IFT and IMC methods clearly perform better than the ZN and ISE methods.

The IMC method is tailored for the system considered here, since (13) is of the form (11), i.e. the assumption made in Section 2 is fulfilled. The design parameter $T_f = 1.3$ gives the best result for IMC, and the mask size t_0 for the IFT method was decreased from 70 s to 10 s in steps of 20 s. In this simulation, a Padé approximation of order three is used for the time delay.

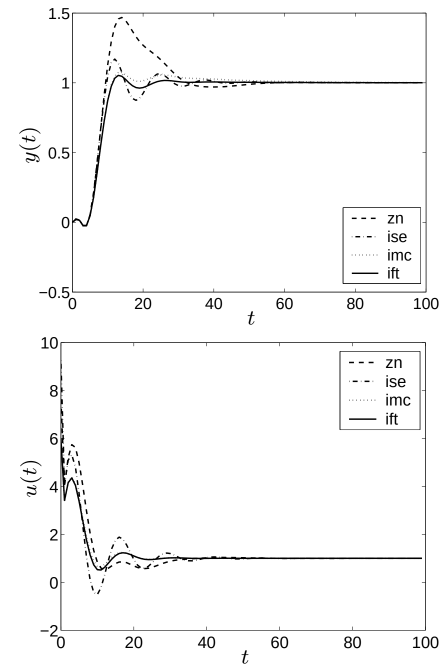

Fig. 2: The step responses and corresponding input signals for the closed loop systems with $G_1(s)$ and controllers tuned with the four methods.

Method	K	T_i	T_d
ZN	4.06	9.25	2.31
ISE	4.46	30.5	2.32
IMC	3.62	22.4	2.18
IFT	3.67	27.7	2.11

Table 1: The PID parameters obtained from the simulation with system $G_1(s)$.

3.2 A study of the system $G_2(s)$

For the system $G_2(s)$ in (14) with a single pole of order eight, the step responses for the closed loop systems after tuning of the controller parameters, see Table 2, are shown in Fig. 3 together with the corresponding input signals. Also in this case, the IFT and IMC methods perform better than the ZN and ISE methods, but the settling time for the IFT method is now smaller than for the IMC method.

For the IFT method, the mask size t_0 was reduced from 280 s down to 130 s in steps of 30 s, and $T_f = 42$ was the best choice of the design parameter in the IMC method.

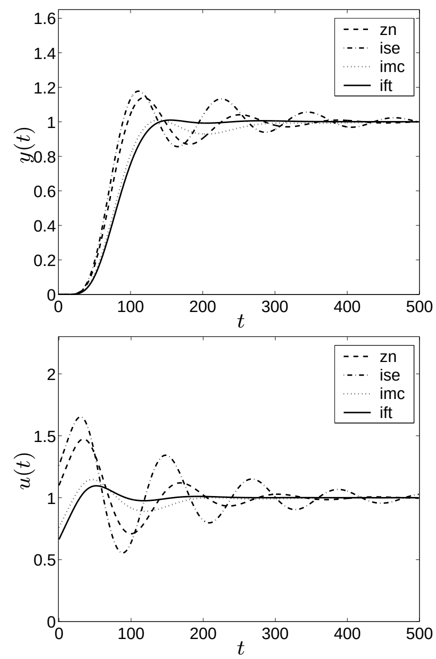

Fig. 3: The step responses and corresponding input signals for the closed loop systems with $G_2(s)$ and controllers tuned with the four methods.

Method	K	T_i	T_d
ZN	1.10	75.9	19.0
ISE	1.26	74.1	26.3
IMC	0.76	64.7	14.4
IFT	0.66	54.0	18.2

Table 2: The PID parameters obtained from the simulation with system $G_2(s)$.

3.3 A study of the system $G_3(s)$

The step responses and corresponding input signals for the closed loop system with the nonminimum phase system $G_3(s)$ in (15), where the PID parameters in Table 3 are tuned by the four methods, are shown in Fig. 4. For this system, the IFT method outperforms the other methods. In this case, the IFT criterion (4) is used as in (8) with the exception that the penalty on the control effort is now included with $\lambda = 1 \cdot 10^{-7}$, i.e.

$$J_m(\rho) = \mathrm{E}\Big\{\sum_{t=t_0}^{N}(y_t(\rho) - r_t)^2 + \lambda \sum_{t=0}^{N}\big(u_t(\rho)\big)^2\Big\}. \tag{16}$$

As a consequence, the initial control effort is reduced a factor two and the closed loop step response is slightly improved compared to the case when (8) is used.

The mask size t_0 was decreased from 110 s to 30 s in steps of 20 s in the IFT method, and the design parameter $T_f = 0.2$ in the IMC method.

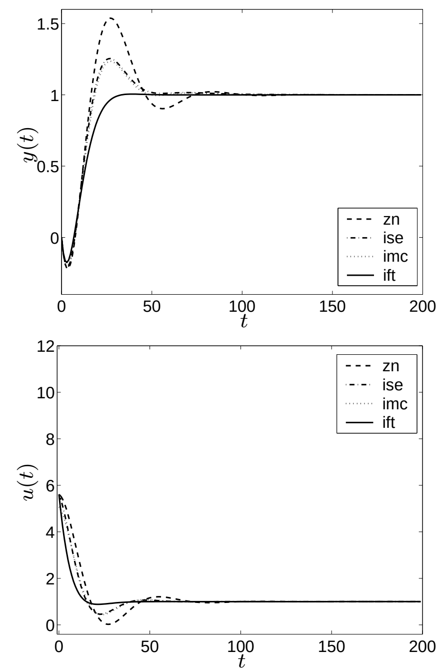

Fig. 4: The step responses and corresponding input signals for the closed loop systems with $G_3(s)$ and controllers tuned with the four methods.

Method	K	T_i	T_d
ZN	3.53	16.8	4.20
ISE	3.53	28.7	4.20
IMC	3.39	31.6	3.90
IFT	3.03	46.3	6.08

Table 3: The PID parameters obtained from the simulation with system $G_3(s)$.

3.4 A study of robustness

In order to investigate the robustness to model errors for the four tuning methods, the closed loop step responses with the controllers in Table 3 were registered when the model $G_3(s)$ in (15) was changed to

$$G_{3\alpha}(s) = \frac{1.5(1-5s)}{(1+10s)(1+20s)} \tag{17}$$

and

$$G_{3\beta}(s) = \frac{(1-5s)}{(1+10s)(1+20s)}e^{-3s}, \tag{18}$$

respectively. Compared to $G_3(s)$, the steady state gain is increased by 50% in $G_{3\alpha}(s)$ and $G_{3\beta}(s)$ contains a time delay of 3 s.

The closed loop step responses and corresponding input signals with the systems $G_{3\alpha}(s)$ in (17) and $G_{3\beta}(s)$ in (18) with the PID parameters in Table 3 are shown in Figs. 5 and 6. From these figures, it is clear that the IFT method is the one that is most robust to model errors.

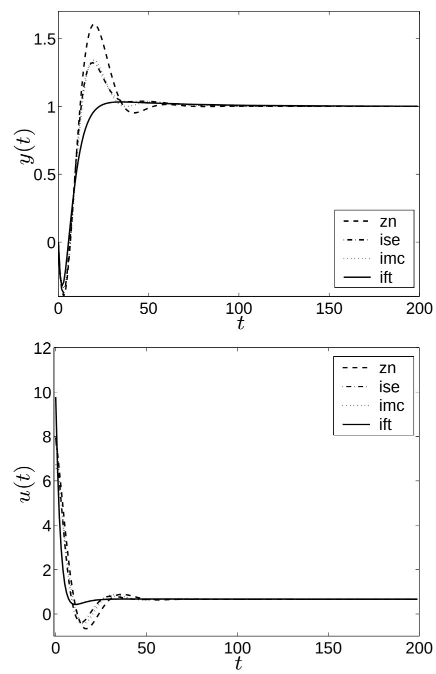

Fig. 5: The step responses and corresponding input signals for the closed loop systems with the perturbed system $G_{3\alpha}(s)$ and the controller parameters as in Table 3.

3.5 A study of noise sensitivity

The IFT method takes account of the presence of noise and makes a trade off between noise rejection and tracking, which is not the case for the other tuning methods considered here. In order

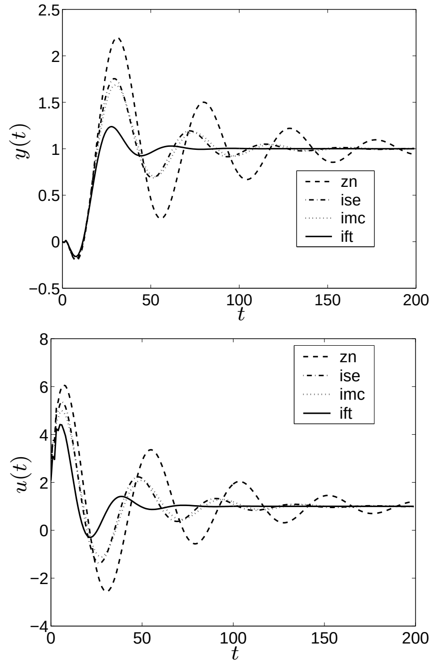

Fig. 6: The step responses and corresponding input signals for the closed loop systems with the perturbed system $G_{3\beta}(s)$ and the controller parameters as in Table 3.

to study the noise sensitivity of the IFT method, white Gaussian distributed noise with standard deviation $\sigma = 0.05$ was added to the output of the closed loop system with system $G_1(s)$ in (13) during the tuning procedure. The resulting controller parameters are

$$\rho = [2.73, 28.3, 1.34]^T, \tag{19}$$

to be compared with the ones obtained under noise free conditions in Table 1.

The closed loop response given by the IFT-tuned PID parameters in (19), under the same noise conditions is shown in Fig. 7 together with the closed loop response of the IMC-tuned PID parameters in Table 1. Needless to say, the results in Fig. 7 can vary quite a lot if the simulations are repeated with other noise realizations.

Since the IFT method takes the presence of a noise disturbance into account, the IFT controller performs better than the IMC controller.

4 Conclusions

Three classical PID tuning schemes that are often used in industry are compared with a variant of the IFT method, in which zero weighting is

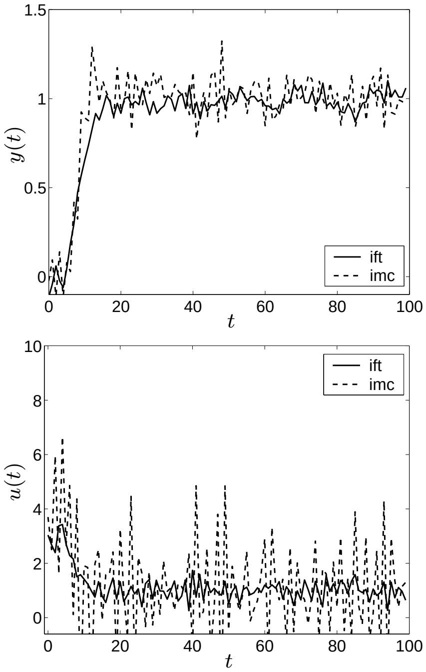

Fig. 7: The step responses and corresponding input signals for the closed loop systems with $G_1(s)$ and a noise perturbation, and the controllers tuned by the IFT method as (19) and the IMC method as in Table 1.

applied to the transient part of the output error. The IFT method performs at least as good as and in some cases much better than the other three methods. The IFT method has two advantages. First, it is not needed to open the loop and second, a noise rejection objective is built into the design process. In addition, it is a model free technique.

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