Fuzzy Preference Relations in Procedures of Multicriteria Decision Making

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Abstract: - Results of research into the use of fuzzy sets for multicriteria decision making are presented. Two classes of problems that need the application of a multicriteria approach are classified. According to this, $< X, M >$ and $< X, R >$ models may be constructed. Analysis of $< X, R >$ models is considered as part of a general approach to solving a wide class of optimization problems with fuzzy coefficients. This approach consists in formulating and analyzing one and the same problem within the framework of interrelated models. It allows one to maximally cut off dominated alternatives. The subsequent contraction of the decision uncertainty region is based on reduction of the problem to multicriteria decision making in a fuzzy environment with applying two techniques based on fuzzy preference relations. The first technique (lexicographic procedure) consists in step by step comparison of alternatives, that provides the sequential contraction of the decision uncertainty region. The second technique is associated with constructing and analyzing membership functions of a subset of nondominated alternatives obtained as a result of simultaneous considering all criteria. The results of the paper are of a universal character and are already being used to solve problems of power engineering.

Key-Words: - Uncertainty Factor, Multicriteria Optimization, Fuzzy Coefficients, Fuzzy Preference Relations.

1 Introduction

In the process of posing and solving a wide range of problems related to the design and control of complex systems, one inevitably encounters diverse kinds of uncertainty. Taking into account the uncertainty factor in shaping the mathematical models serves as a means for increasing their adequacy and, as a result, the credibility and factual efficiency of decisions based on their analysis. Investigations of recent years show the utility of applying fuzzy set theory [1] for considering diverse kinds of uncertainty. Its use in problems of optimization character offers advantages of both fundamental nature (the possibility of validly obtaining more effective, less "cautious solutions") and computational character [2].

The uncertainty of goals is the notable kind of uncertainty that is associated with a multicriteria character of many optimization problems. It is possible to classify two types of problems, which need the use of a multicriteria approach [3]:

- problems in which solution consequences cannot be estimated on the basis of a single criterion. These problems are associated with the analysis of models including economic as well as natural indices (when alternatives cannot be reduced to the comparable form) and also by the need to consider indices whose cost estimates are hampered or impossible;

- problems that, from the substantial point of view, may be solved on the basis of a single criterion. However, if the uncertainty of information does not allow one to obtain a unique solution, it is possible to reduce these problems to multicriteria decision making: the use of additional criteria can serve as convincing means to contract the decision uncertainty regions [4].

In accordance with these types of problems, two classes of models (so-called $< X, M >$ and $< X, R >$ models) may be constructed.

When analyzing $< X, M >$ models, a vector of objective functions $F(X) = \{F_1(X),...,F_q(X)\}$ is considered, and the problem consists in simultaneous optimizing all objective functions (local criteria), i. e.,

$$F_p(X) \to \underset{X \in L}{\text{extr}}, \quad p = 1,...,q, \tag{1}$$

where L is a feasible region in R^n.

The lack of clarity in the concept of "optimal solution" is the basic methodological complexity in solving multicriteria problems. When applying the Bellman-Zadeh approach [5] for analyzing $< X, M >$ models, this concept is defined with reasonable validity: the maximum degree of implementing goals serves as a criterion of optimality. This conforms to the principle of guarantee result and provides a constructive line in obtaining harmonious solutions [6]. Furthermore, the approach permits one to realize an effective (from the computational standpoint) as well as rigorous (from the standpoint of obtaining solutions from a set of Pareto optimal solutions) method of analyzing multicriteria optimization models [6,7]. Finally, its use allows one to preserve a natural measure of uncertainty in decision making and to take into account indices, criteria, and constraints of qualitative character. These considerations justify the advisability of using the Bellman-Zadeh approach to analyze $< X, M >$ models. Some specific questions of using the Bellman-Zadeh approach in multicriteria optimization, including its practical applications to power engineering problems, are considered in [6,8].

Taking the above into account, attention in this paper is given to the analysis of $< X, R >$ models.

2 Optimization Problems with Fuzzy Coefficients

Numerous problems related to the design and control of complex systems [4,9] may be formulated as follows:

$$\text{maximize } \tilde{F}(x_1,\ldots,x_n) \tag{2}$$

subject to the constraints

$$\tilde{g}_j(x_1,\ldots,x_n) \subseteq \tilde{B}_j, \quad j = 1,\ldots,m\,. \tag{3}$$

The objective function (2) and constraints (3) include fuzzy coefficients, as indicated by the ~ symbol.

Given the problem (2), (3), we can state a problem of minimization with fuzzy coefficients:

$$\text{minimize } \tilde{F}(x_1,\ldots,x_n) \tag{4}$$

subject to the constraints (3).

A possible approach to handling constraints of the form (3) is proposed in [4]. This approach involves approximate replacement of each of the constraints of the form (3) by a finite set of deterministic (nonfuzzy) constraints, represented in the form of inequalities; these can be formulated readily, but with considerable increase in the dimension of the problem being solved. However, the principle of explicit domination [4] substantially reduces the dimensionality of the resulting equivalent nonfuzzy analog before solution of the problem commences. According to the physical essence of the problem solved, we may go over the constraints with fuzzy coefficients (3) to constraints

$$g_j(x_1,\ldots,x_n) \le b_j, \quad j = 1,\ldots,d' \ge m \tag{5}$$

or to constrains

$$g_j(x_1,\ldots,x_n) \ge b_j, \quad j = 1,\ldots,d'' \ge m\,. \tag{6}$$

The solution of problems with fuzzy coefficients in the objective functions alone is possible by a modification of traditional optimization methods [2,4]. In particular, it is possible to solve the problem (2) with satisfying the constraints (5) as well as the problem (4) with satisfying the constraints (6). The algorithms of solving the discrete fuzzy optimization problems (2), (5) and (4), (6), based on modifying generalized methods of discrete optimization [10,11], are proposed in [4,9]. When using these algorithms, the need arises to compare alternatives (in essence, to compare or rank fuzzy numbers) on the basis of relative fuzzy values of the objective function. This may be done with the use of the corresponding methods classified in [12]. In particular, one of the groups of the methods is based on building fuzzy preference relations, that provides [13] the most justified and practical way to compare alternatives. Taking this into account, it is necessary to distinguish the choice function or fuzzy number ranking index introduced by Orlovsky [14]. It is based on the conception of a membership function of a generalized preference relation.

If the membership functions corresponding to the natural or relative values $\tilde{F}_1$ and $\tilde{F}_2$ of the objective function to be maximized are $\mu(f_1)$ and $\mu(f_2)$, the quantity $\eta\{\mu(f_1),\mu(f_2)\}$ is the degree of preference $\mu(f_1) \succcurlyeq \mu(f_2)$, while $\eta\{\mu(f_2),\mu(f_1)\}$ is the degree of preference $\mu(f_2) \succcurlyeq \mu(f_1)$. Then the membership functions of the generalized preference relations $\eta\{\mu(f_1),\mu(f_2)\}$ and $\eta\{\mu(f_2),\mu(f_1)\}$ take the following form:

$$\eta\{\mu(f_1),\mu(f_2)\} = \sup_{f_1,f_2 \in F} \min\{\mu(f_1),\mu(f_2),\mu_R(f_1,f_2)\}, \tag{7}$$

$$\eta\{\mu(f_2),\mu(f_1)\} = \sup_{f_1,f_2 \in F} \min\{\mu(f_1),\mu(f_2),\mu_R(f_2,f_1)\}, \tag{8}$$

where $\mu_R(f_1,f_2)$ and $\mu_R(f_2,f_1)$ are the membership functions of the corresponding fuzzy preference relations.

If F is the numerical axis on which the values of the maximized objective function are plotted, and R is the natural order (≥) along F, then (7) and (8) reduce to the following expressions:

$$\eta\{\mu(f_1),\mu(f_2)\} = \sup_{\substack{f_1, f_2 \in F \\ f_1 \geq f_2}} \min\{\mu(f_1),\mu(f_2)\}, \quad (9)$$

$$\eta\{\mu(f_2),\mu(f_1)\} = \sup_{\substack{f_1, f_2 \in F \\ f_2 \geq f_1}} \min\{\mu(f_1),\mu(f_2)\}, \quad (10)$$

which agree with the Baas-Kwakernaak [15], Baldwin-Guild [16], and one of the Dubois-Prade [17] fuzzy number ranking indices.

On the basis of the relations between (9) and (10), it is possible to judge the preference (and the degree of preference) of any of the alternatives compared. Utilization of this approach is justified, that is confirmed by the results of [18]. However, experience shows that in many cases the membership functions of the alternatives $\mu(f_1)$ and $\mu(f_2)$ form flat apices (for example, [2,4]), i.e., they are so-called flat fuzzy numbers. In view of this, using (9) and (10), we can say that the alternatives $\tilde{F}_1$ and $\tilde{F}_2$ are indistinguishable if

$$\eta\{\mu(f_1),\mu(f_2)\} = \eta\{\mu(f_2),\mu(f_1)\}. \quad (11)$$

In such situations the algorithm of [4,9] does not allow one to obtain a unique solution because they "stop" when conditions like (11) arise. This occurs also with other modifications of mathematical programming methods because combination of the uncertainty and the relative stability of optimal solutions can produce these so-called decision uncertainty regions. In this connection, other choice functions or indices (for example, [13,19-21]) may be used as additional means for the ranking of fuzzy numbers. However, these indices occasionally result in choices which appear inconsistent with intuition, and their use does not permit one to close the question of constructing an order on a set of fuzzy numbers [4]. Besides, from the substantial point of view, these indices have been proposed with the aspiration for obligatory distinguishing the alternatives, that is not natural because the uncertainty of information creates the decision uncertainty regions There actually is another approach that is better validated and natural for the practice of decision making. This approach is associated with transition to multicriteria choosing alternatives in a fuzzy environment because the application of additional criteria (including the criteria of qualitative character, such as "comfort of maintenance", "flexibility of operation", etc.) can serve as convincing means to contract the decision uncertainty region.

3 Multiciteria Choice Procedures in Fuzzy Environment

Before starting to discuss multicriteria decision making in a fuzzy environment, it is necessary to note that considerable contraction of the decision uncertainty region may be obtained by formulating and analyzing one and the same problem within the framework of mutually interrelated models:

a) the model of maximization (2) with satisfaction of the constraints (5) interpreted as convex down;

b) the model of minimization (4) with satisfaction of the constraints (6) interpreted as convex up.

In this case, solutions dominated by the initial objective function are cut off from below as well as from above to the greatest degree [4]. It should be stressed that this is a universal approach and my also be used in solving continuous optimization problems, for example, by modifying the zero-order optimization methods.

Assume we are given a set X of alternatives, which are to be examined by q criteria of quantitative and/or qualitative nature to make a choice among alternatives. The problem of decision making is presented by a pair $<X, R>$, where $R=\{R_1,...,R_q\}$ is a vector fuzzy preference relation. In this case, we have

$$R_p = [X \times X, \mu_{R_p}(X_k, X_l)],\ p = 1,...,q,\ X_k, X_l \in X, \quad (12)$$

where $\mu_{R_p}(X_k, X_l)$ is a membership function of fuzzy preference relation.

It is supposed in [20,22] that the matrices R_p, $p=1,...,q$ are given on the basis of expert estimation. However, with the availability of fuzzy or linguistic estimates of alternatives $\tilde{F}_p(X_k)$, $p=1,...,q$, $X_k \in X$ (constructed on the basis of expert estimation or on the basis of aggregating information arriving from different sources of both formal and info1rmal character [2]) with the membership functions $\mu[f_p(X_k)]$, the matrices R_p, $p=1,...,q$ may be obtained as follows, using (9) and (10):

$$\mu_{R_p}(X_k, X_l) = \sup_{\substack{X_k, X_l \in X \\ f_p(X_k) \geq f_p(X_l)}} \min\{\mu[f_p(X_k)], \mu[f_p(X_l)]\}, \quad (13)$$

$$\mu_{R_p}(X_k, X_l) = \sup_{\substack{X_k, X_l \in X \\ f_p(X_l) \geq f_p(X_k)}} \min\{\mu[f_p(X_k)], \mu[f_p(X_l)]\}. \quad (14)$$

If the *p*th criterion is associated with minimization, then (13) and (14) are written for regions $f_p(X_k) \leq f_p(X_l)$ and $f_p(X_l) \leq f_p(X_k)$.

Considering that the fuzzy preference relations R_p, $p = 1,...,q$ play a role identical to the objective functions $F_p(X)$, $p = 1,...,q$ in $< X, M >$ models, it should be noted that the fuzzy preference relations may be introduced in the analysis of these models as well. For example, for $F_p(X)$, which is to be maximized, it is possible to construct

$$\mu_{R_p}(X_k, X_l) = \alpha[F_p(X_k) - F_p(X_l)] + \beta. \quad (15)$$

Following [23], it is possible to demand the fulfillment of the condition $\mu_{R_p}(X_k, X_k) = 0.5$, which leads to $\beta = 0.5$ and $\mu_{R_p}(X_k, X_l) + \mu_{R_p}(X_l, X_k) = 1$. This permits one to write $\alpha[\max_{X \in L} F_p(X) - \min_{X \in L} F_p(X)] + 0.5 = 1$ to obtain

$$\alpha = \frac{1}{2[\max_{X \in L} F_p(X) - \min_{X \in L} F_p(X)]}. \quad (16)$$

Thus, the correlation (15) may be presented as

$$\mu_{R_p}(X_k, X_l) = \frac{F_p(X_k) - F_p(X_l)}{2[\max_{X \in L} F_p(X) - \min_{X \in L} F_p(X)]} + 0.5, \quad (17)$$

providing $0 \leq \mu_{R_p}(X_k, X_l) \leq 1$.

Let us consider the situation of setting up a single preference relation R. In a nonfuzzy case, we may be given a nonstrict preference in one of the following forms [22]:

a) $(X_k, X_l) \in R$ or $X_k \succcurlyeq X_l$ that means "X_k is not worse than X_l",

b) $(X_l, X_k) \in R$ or $X_l \succcurlyeq X_k$ that means "X_l is not worse than X_k",

c) $(X_k, X_l) \notin R$ or $(X_l, X_k) \notin R$ that means "X_k and X_l" are not comparable.

The nonstrict preference relation R can be presented by a strict preference relation R^s and indifferent relation R^I [22,23]. We can say that "X_k is strictly better than X_l" if $(X_k, X_l) \in R$ and $(X_l, X_k) \notin R$. The subset of all these pairs is the strict preference relation R^s, and it is possible to use the inverse relation R^{-1} ($(X_k, X_l) \in R^{-1}$ is equivalent to $(X_l, X_k) \in R$ [22]) to obtain

$$R^s = R \setminus R^{-1}. \quad (18)$$

If $(X_k, X_l) \in R^s$, then X_k dominates X_l, i.e., $X_k \succ X_l$. The alternative $X_k \in X$ is nondominated in $< X, R >$ if $(X_k, X_l) \in R^s$ for any $X_l \in X$.

If we have $\mu_R(X_k, X_l)$ as a nonstrict fuzzy preference relation, then the value $\mu_R(X_k, X_l)$ is the degree of preference $X_k \succcurlyeq X_l$ for any $X_k, X_l \in X$. The membership function, which corresponds to (18) in this case (considering that $\mu_{R^{-1}}(X_k, X_l) = \mu_R(X_l, X_k)$ [22]) is the following:

$$\mu_R^s(X_k, X_l) = \max\{\mu_R(X_k, X_l) - \mu_R(X_l, X_k), 0\}. \quad (19)$$

The use of (19) permits one to carry out the choice of alternatives. In particular, $\mu_R^s(X_l, X_k)$ for any X_l describes a fuzzy set of alternatives, which are strictly dominated by X_l. Therefore, the complement of this fuzzy set by $1 - \mu_R^s(X_l, X_k)$ gives the fuzzy set of alternatives, which are not dominated by other alternatives from X. To choice the set of all alternatives, which are not dominated by other alternatives from X [22], it is necessary to find the intersection of all $1 - \mu_R^s(X_l, X_k), X_k \in X$ on all $X_l \in X$. This intersection is a subset of nondominated alternatives and has a membership function

$$\mu_R'(X_k) = \inf_{X_l \in X}[1 - \mu_R^s(X_l, X_k)] = 1 - \sup_{X_l \in X} \mu_R^s(X_l, X_k). \quad (20)$$

Because $\mu_R'(X_k)$ is the degree of nondominance, it is natural to obtain alternatives providing

$$X' = \{X_k' \mid X_k' \in X, \mu_R'(X_k') = \sup_{X_k \in X} \mu_R'(X_k)\}. \quad (21)$$

If $\sup_{X_k \in X}(X_k) = 1$, then the alternatives $X'' = \{X_k'' \mid X_k'' \in X, \mu_R'(X_k'') = 1\}$ are [22] nonfuzzy nondominated and can be considered as the nonfuzzy solution of a fuzzy problem.

If the fuzzy preference relation R is transitive, then $X'' \neq \varnothing$. Taking this into account, it should be noted that when $\tilde{F}_p(X_k)$ is quantitatively expressed, $X'' \neq \varnothing$. With qualitative $\tilde{F}_p(X_k)$ it is possible to have $X'' = \varnothing$ under intransitivity of R, that permits one to detect contradictions in an expert estimates.

The expressions (19)-(21) may be used to solve the choice problem as well as ranking problem [20] with the single preference relation. If we have the vector fuzzy preference relation, the expressions (19)-(21) can serve as the basis for constructing a lexicographic procedure associated with step by step introduction of criteria for comparing the alternatives. This procedure permits one to obtain a sequence $X^1, X^2, ..., X^q$ so that $X \supseteq X^1 \supseteq X^2 \supseteq ... \supseteq X^q$ with the use of the following expressions:

$$\mu_R'^p(X_k) = \inf_{X_l \in X^{p-1}} [1 - \mu_{R_p}^s(X_l, X_k)]$$
$$= 1 - \sup_{X_l \in X^{p-1}} \mu_{R_p}^s(X_l, X_k), \quad p = 1,...,q, \quad (22)$$

$$X^p = \{X_k'^p \mid X_k'^p \in X^{p-1}, \mu_k'^p(X_k'^p)$$
$$= \sup_{X_k \in X^{p-1}} \mu_R'^p(X_k)\}, \quad (23)$$

obtained on the basis of (20) and (21), respectively.

It should be noted that if R_p is transitive, we can bypass the pairwise comparison of alternatives at the pth step. In this situation, the comparison can be done on a serial basis (the direct use of (13) and (14)) with memorizing the best alternatives.

It is natural that the lexicographic procedure is applicable if criteria can be arranged in order of their importance. If the construction of the uniquely determined order is difficult, it is possible to apply another choice procedure. In particular, the expressions (19)-(21) are applicable if we take $R = \bigcap_{p=1}^{q} R_p$, i.e.,

$$\mu_R(X_k, X_l) = \min_{1 \le p \le q} \mu_{R_p}(X_k, X_l), \quad X_k, X_l \in X. \quad (24)$$

When using this procedure, the application of (19)-(21) leads to the set X' that fulfils [22,23] the role of the set of Pareto optimal solutions. Its contraction is possible on the basis of differentiating the importance of R_p, $p = 1,...,q$ with the use of the following convolution [22]:

$$\mu_T(X_k, X_l)$$
$$= \sum_{p=1}^{q} \lambda_p \mu_{R_p}(X_k, X_l), \quad X_k, X_l \in X, \quad (25)$$

where λ_p, $p = 1,...,q$ are weights (importance factors) of the corresponding criteria ($\lambda_p > 0$, $p = 1,...,q$, $\sum_{p=1}^{q} \lambda_p = 1$).

The construction of $\mu_T(X_k, X_l)$, $X_k, X_l \in X$ allows one to obtain the corresponding membership function $\mu_T'(X_k)$ of the subset of nondominated alternatives according to an expression similar to (20). The intersection of $\mu_R'(X_k)$ and $\mu_T'(X_k)$ defined as

$$\mu'(X_k) = \min\{\mu_R'(X_k), \mu_T'(X_k)\}, \quad X_k \in X \quad (26)$$

provides us with

$$X' = \{X_k' \mid X_k' \in X, \mu'(X_k') = \sup_{X_k \in X} \mu'(X_k)\}. \quad (27)$$

4 Applications

The results of the paper are of a universal character and can be applied to the design and control of systems and processes of different nature as well as the enhancement of corresponding CAD/CAM systems and intelligent decision support systems. In practical aspect, the results of the paper have served as a basis for solving problems of power engineering, including substation planning in power systems [8] and optimization of reliability (optimization of reliability indices while meeting restrictions on resources or minimization of resource consumption while meeting restrictions on reliability levels) in distribution systems.

5 Conclusion

Two classes of problems that need the application of a multicriteria approach have been classified. According to this, $< X, M >$ and $< X, R >$ models may be constructed. The use of $< X, R >$ models, which allow one to combine considering different types of uncertainty, is associated with applying a general approach to solving optimization problems with fuzzy coefficients. This approach is based on a modification of traditional optimization methods and consists in formulating and analyzing one and the same problem within the framework of interrelated models. The subsequent contraction of a decision uncertainty region is associated with reduction of the problem to multicriteria selecting alternatives in a fuzzy environment with applying two techniques based on fuzzy preference relations. The first technique consists in step by step comparison of alternatives, that provides the sequential contraction of the decision uncertainty region. The second technique is associated with constructing and analyzing membership functions of a subset of nondominated alternatives obtained as a result of simultaneous considering all criteria. The results of the paper are of a universal character and can be applied to the design and control of systems and processes of different nature as well as the enhancement of corresponding CAD/CAM systems and intelligent decision support systems. In practical aspect, the results of the paper have served as a basis for solving problems of power engineering.

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