A Fuzzy Model For Change Intervals At Signalized Intersection

J. Lin and K.Y. Kuo

Ching Yun Institute of Technology
229, Chien-Hsin Rd., Jung-Li City, Taiwan 320, R.O.C.

Abstract: The main object of this paper is to develop a new procedure to calculate the change and clearance intervals of a traffic signal from a rule-based fuzzy logic system. This procedure is based on the theory that driver’s decision making at signalized intersections is based on imprecise or fuzzy information. The procedure requires no analytical or mathematical modeling of the system’s phenomena and provides the flexibility for modification to improve the performance of the system.

Key Words: Fuzzy, Signalized, Intervals, Dilemma, Congestion Factor

1. Introduction

One of the most important criteria in traffic is safety. Motorists determine safety from the prevailing environmental conditions surrounding them. A typical example is the yellow indication at a signalized intersection. During the change interval at a signalized intersection, approaching drivers are faced with the decision of either proceeding to cross or preparing to stop. The decision to stop or go has fuzziness associated with it.

Driver behavior and decision at change interval is a complex nonlinear system, which is not easily modeled mathematically. Therefore, it becomes difficult to mathematically model such systems.

The main goal of the research is to improve driver safety through the new decision-making at signalized intersections for the change interval. Aside from the benefits of reduced accidents, increased efficiency and reduced environmental costs are also the immediate benefits as a direct result of increased safety. Moreover, this study will recognize the fuzziness of the information and will apply sets and logic to analyze this information.

2. System Definition

2.1 Design of System

2.1.1 Simplicity of System

The inputs of the system are: speed of vehicles, traffic density, capacity, grade, width of intersection, and location of driver.

2.1.2 System Process

The concepts and illustration of the system process is shown in Figure 1.

Figure 1: Graphic Representation of System Process

2.2 Designing the Input Membership Function

2.2.1 The primary Inputs

a. Membership function of speed and CF

Vehicle speed is an important consideration in highway transportation because it has significant implication for the safety, comfort, and convenience of highway travel for motorist. The congestion factor (CF) in this research is treated as car density of street, which refers to the ratio of the actual number of vehicles to the...
design capacity of vehicles in the direction zone. The math model is expressed by following relationship:

\[ C = \frac{l \times n}{h \times v} \]

where \( C \) = capacity of the detection zone (number of vehicles), \( h \) = headway at saturation (sec/vehicle), \( v \) = posted speed (feet/second), \( l \) = length of the detection zone (feet), \( n \) = number of lanes in the zone (lane).

If \( N \) is defined as the actual number of vehicles detected in the zone, then the congestion factor could be defined as \( CF = \frac{N}{C} \).

b. Membership function of driver location

One critical variable is the driver location at the onset of the yellow interval. In determining the signal change, the distance from the intersection at the onset of the yellow interval is used as a surrogate variable for the true measurement of time from the intersection.

2.2.2 The Secondary Inputs Membership

It is reasonable to believe that the primary input cannot get optimum results due to ignorance of the analysis of the intersection. Therefore, we characterize the intersection by the grade and the width of membership as secondary input to modify the whole system.

a. The membership function for width

The range of intersection width for this study are designed on a number of factors including the following:

1. lane width (between 10 to 12 feet)
2. the presence (or absence) of a median (range from 2 to over 10 feet)
3. intersection curd radius
4. location of the stop line

b. The membership function for approach grade

The grade of the roadway affects acceleration and deceleration of all vehicles.

2.3 Designing the Output Membership Function

We define the three outputs membership function for Yellow Time, All-Red, and Green Extension. From the previous study, we know the level of vehicle supply at the yellow onset is a governing factor of the yellow interval requirement. The all-red interval is provided to clear the intersection before releasing vehicles from the approach which have the right of way. Moreover, the green extension increases the efficiency and safety of signalized intersections.

The developed system has three primary inputs and two secondary inputs. The primary inputs are: CFRATIO, DRIVER-LOCATION, and SPEED.

a. CFRATION

The rate of congestion factor is defined by the rate of the volume of traffic to the capacity of the approach or lane group.

b. DRIVER-LOCATION

The location of the driver influences driver anxiety and hence plays a major role in making decisions whether to stop or to go. Drivers located far away from the intersection will attempt to stop at the onset of the yellow indication.

c. SPEED

The SPEED of traffic is variable. It depends on the posted speed limit and environmental conditions. Based on functional classification of streets and highways, the appropriate ranges of speed were determined and used in designing the membership functions.

Each input is assigned linguistic values, and each value represented by a fuzzy set. Having defined these linguistic values, we must associate them with numerical values. One way of doing this is by assigning each of the linguistic values with a degree of membership function. Each linguistic value has its own fuzzy set. The shape of the fuzzy set determines the degree of membership.

To the fuzzy variable CFRATIO, we will assign three fuzzy values: HIGH, MEDIUM, and LOW.

To the fuzzy variable DRIVER-LOCATION we will assign three fuzzy rules: CLOSE, MEDIUM, and FAR.

To the fuzzy variable SPEED we will assign three fuzzy rules: SLOW, MEDIUM, and HIGH.

For the primary input variables we have 27 cases.

3.1.2 Assessment of Compatibility of Input Variables

All the 27 cases are assessed for compatibility and specific examples where such conditions might exist are summarized in Table 1.

<table>
<thead>
<tr>
<th>Group</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1,4,7</td>
<td>It is very difficult to define intersections with high speeds and high CFRATIO. The posted speed limit may reflect high speeds, but approaching speed would be very low due to interaction as a result of high traffic densities.</td>
</tr>
<tr>
<td>Case 10,13,16</td>
<td>At high-speed, isolated intersections in suburban areas during the peak periods.</td>
</tr>
<tr>
<td>Cases 19, 22, 25</td>
<td>At high-speed, isolated intersections in suburban and rural areas during the peak periods.</td>
</tr>
<tr>
<td>Cases 2, 5, 8</td>
<td>A signalized intersections on major</td>
</tr>
</tbody>
</table>
3.1.2  Secondary Inputs to Modify the Fuzzy-Rule-Based
The outputs are modified using the secondary factors: GRADE and WIDTH.

One equally important factor is the width of the intersection. An all-red or clearance interval is provided to clear all vehicles within the intersection before changing the right-of-way. The intersection within can be added to the DRIVER-LOCATION variable; however, this will be appropriate only for commuters and not for drivers unfamiliar with the intersection. The variable WIDTH is included as a secondary variable to adjust the yellow time or provide an all-red interval to clear vehicles within the intersection. To the fuzzy variable WIDTH we will assign two fuzzy values:

SMALL – three or less lanes across.
WIDE – more than four lanes across.

To the fuzzy variable GRADE we will assign two fuzzy values:
NORMAL – not likely to decrease speed of vehicle.
STEEP – likely to decrease speed of vehicle.

Each of the four combination cases from the secondary inputs is combined with the primary variables. This results in four scenarios producing 108 (4×27) rules.

3.2  Inference of the Fuzzy Logic System
After assigning the input and output values to define fuzzy sets, we must map each possible input condition to an output condition. The common expression of such mapping in this research is defined below:

<table>
<thead>
<tr>
<th>CF(H), SP(M)</th>
<th>arterial where traffic produce with little dispersion on arrival at downstream intersections.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 11, 14, 17 CF(M), SP(M)</td>
<td>On major and minor arterial and collectors with some platoon dispersion occurs before arrival at downstream intersections.</td>
</tr>
<tr>
<td>Cases 20, 23, 26 CF(L), SP(M)</td>
<td>At minor arterial and collectors during the off peak periods and also isolated intersections in suburban areas where traffic arrival is medium to random.</td>
</tr>
<tr>
<td>Cases 3, 6, 9 CF(H), SP(S)</td>
<td>On major and minor arterial in CBD, urban areas at peak periods and also on coordinated streets with high platoon flows.</td>
</tr>
<tr>
<td>Cases 12, 15, 18 CF(M), SP(S)</td>
<td>On major and minor arterial as well as major collectors with some platoon flows.</td>
</tr>
<tr>
<td>Cases 21, 24, 27 CF(L), SP(S)</td>
<td>On isolated intersections, major and minor arterial as well as major collectors during the off-peak period when traffic flow is medium to random.</td>
</tr>
</tbody>
</table>

IF (CONDITION OR ANTECEDENT) THEN (ACTION OR CONSEQUENCE)

For example, a yellow signal-controlling problem might contain the following expert rules:

**If the speed is very high, then**

**If the intersection width is wide, then** a long yellow time.

In this case restrictions are placed on actions taken by the controller; i.e., “a long yellow time”. The canonical form of a set of expert production rules is defined as a set of unconditional restrictions followed by a set of conditional restrictions as illustrated below:

<table>
<thead>
<tr>
<th>R_i</th>
<th>Restriction</th>
</tr>
</thead>
<tbody>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>R_{k+1}</td>
<td>If condition C_1, THEN restriction R_{k+1}</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>R_{k+r}</td>
<td>If condition C_{r+1}, THEN Restriction R_{k+r}</td>
</tr>
</tbody>
</table>

The restriction R_1, ..., R_r applies to output actions taken on the output or decisions made for desired performance. These restrictions are usually connected by linguistic connectives such as “and”, “or” or “else”. The canonical fuzzy rule-based expert system is comprised of a set of conditional rules put in the following form:

**Rule 1:** If condition C_1, THEN restriction R_1.

**Rule 2:** If condition C_2, THEN restriction R_2.

...  

**Rule R_r:** If condition C_{r+1}, THEN restriction R_r.

Put in a more compact form, the fuzzy rule-based expert system can be expressed as: (IF x is A_1, THEN y is B_1) ⊕ IF x is A_2, THEN y is B_2 ⊕ ... ⊕ IF x is A_r, THEN y is B_r. The symbol ⊕ represents linguistic connectives such as “and”, “or” or “else”.

The following section discusses compound rule structures to transform them into simple canonical forms. Suppose a system rule base is defined as:

IF x = A_i and A_j and A_k and ... THEN y is B_i.

Let the new fuzzy subsets

A_x = A_1 ∩ A_2 ∩ A_3 ∩ ... ∩ A_n,

then the membership function is defined as

U_{A_x}(x) = \text{Min}[U_{A_1}(x), U_{A_2}(x), ..., U_{A_n}(x)].

Which may be rewritten as

IF A_x THEN B_i.

In other cases we may have a fuzzy logic rule given by

IF x = A_i or x = A_j or x = A_k and ... THEN y is B_i.

Let A_x = A_1 ∪ A_2 ∪ A_3 ∪ ... ∪ A_n, then the fuzzy union operation is defined as

U_{A_x}(x) = \text{Max}[U_{A_1}(x), U_{A_2}(x), ..., U_{A_n}(x)].

Which may be rewritten as “IF A_x THEN B_i.”.

For conditional rules with “ELSE,” or “UNLESS,”
as in the following rules:

IF $A_1$ THEN $(B_1 \text{ ELSE } B_2)$, we may decompose this rule into two simple canonical form rule as

IF $A_1$ THEN $B_1$, OR IF NOT $A_1$ THEN $B_2$.

Sometimes in complex logic systems, the mechanism may require nested rules. In situations where two or more inputs may be required to produce a specific action, nested IF-THEN rules may be employed. For example:

IF $A_1$, THEN (IF $A_2$ THEN $(B_1)$), may be broken down into the following

IF $A_1$ AND $A_2$ THEN $B_1$.

Generalizing the above statement, we have

IF $A_1$, THEN (IF $A_2$, THEN (IF $A_3$, THEN (THEN IF $A_4$ THEN $(B_1)$)),...), which may be rewritten as

IF $A_1$ AND $A_2$ AND $A_3$ AND ... THEN $B_1$.

System with more than one output; e.g., $m$-outputs, may be described by a collection of fuzzy-rule-based systems where each rule-base set deals with only one output. This type of fuzzy rule base expert system is the most common form for identification and control problems. The $n$-input and single-output system, with non-interactive input fuzzy sets, could be put in the form as shown below:

$$R_i : \text{ IF } x_1 \text{ is } A_{i1} \text{ AND } x_2 \text{ is } A_{i2} \ldots \text{AND } x_n \text{ is } A_{in}, \text{ THEN } y_1 \text{ is } B_{i1}. $$

$$R_n : \text{ IF } x_1 \text{ is } A_{n1} \text{ AND } x_2 \text{ is } A_{n2} \ldots \text{AND } x_n \text{ is } A_{nn}, \text{ THEN } y_1 \text{ is } B_{n1}. $$

For multiple input ($n$-input) and multiple output ($m$-output) systems with non-interactive input fuzzy sets $A_1, A_2, A_3, ...$ defined on the universe $x_i$ for $i=1,2,3,n$ and also non-interactive fuzzy sets $B_1, B_2, B_3, ...$ defined on the universe $y_j$ for $j=1,2,3, m$, then the canonical rule-based expert system could be put in the form shown below:

$$R_i : \text{ IF } x_1 \text{ is } A_{i1} \text{ AND } x_2 \text{ is } A_{i2} \ldots \text{AND } x_n \text{ is } A_{in}, \text{ THEN } y_1 \text{ is } B_{i1} \text{ AND } y_2 \text{ is } B_{i2} \ldots \text{AND } y_m \text{ is } B_{in}. $$

$$R_n : \text{ IF } x_1 \text{ is } A_{n1} \text{ AND } x_2 \text{ is } A_{n2} \ldots \text{AND } x_n \text{ is } A_{nn}, \text{ THEN } y_1 \text{ is } B_{n1} \text{ AND } y_2 \text{ is } B_{n2} \ldots \text{AND } y_m \text{ is } B_{nn}. $$

### 3.3 Defuzzification Process

Defuzzification is the process of conversion of a fuzzy quantity represented by a membership function to a precise or crisp value. One of the ways to combine several is to take the maximum set membership values of the active consequents. This is called the Maximized technique. The maximized technique takes the maximum degree-of-membership values from the various triggered rules and performs a corresponding single action. Mathematically this can be expressed as:

$$U_i(y) = \max[U_{i1}(y), U_{i2}(y), \ldots U_{in}(y)]$$

For example, let the results of three rules be:

- Rule 2: $U_{\text{short}}(\text{yellow time}) = 0.9$
- Rule 8: $U_{\text{medium}}(\text{yellow time}) = 0.7$
- Rule 23: $U_{\text{long}}(\text{yellow time}) = 0.5$

Then, since Rule 2: $U_{\text{short}}(\text{yellow time}) = 0.9$ has the maximum degree of membership, it will be selected as the final result, and the output crisp value corresponding to this output degree of membership is determined. However, there are conflicting results, when two rules have the same degree of membership. The maximized technique is the simplest defuzzifier, but it suffers from cases where the aggregated union of the output memberships is not valid.

The weighted average technique averages all the rules or actions triggered after assigning weights based on the degree of membership values. For example, assuming the following three rules are fired:

- Rule 2: $U_{\text{short}}(\text{yellow time}) = 0.9$
- Rule 8: $U_{\text{medium}}(\text{yellow time}) = 0.7$
- Rule 23: $U_{\text{long}}(\text{yellow time}) = 0.5$

Let the output variable (yellow time) and weights based on the degree-of-membership values for each of the three rules be:

- Rule 2: Yellow time = 3.5 sec, Weight = 0.9
- Rule 8: Yellow time = 4 sec, Weight = 0.7
- Rule 23: Yellow time = 4.5 sec, Weight = 0.5

Then by the weighted average method the resultant is:

$$\frac{3.5 \times 0.9 + 4 \times 0.7 + 4.5 \times 0.5}{0.9 + 0.7 + 0.5} = 3.9 \text{ (sec)}$$

The weighted average has some shortcomings. Although very strong conceptually, it suffers from ambiguity as a result of an output membership function specifying more than one output values.

Another defuzzification method is the centroid technique. In this technique the center of gravity of the area bounded by the membership function curve is computed as the most typical crisp value of the fuzzy quantity; i.e.,

$$Y = \frac{\int U_i(y) \cdot y \cdot dy}{\int U_i(y) \cdot dy}$$

The center-of-gravity method is one of the most common defuzzification methods, though it is computationally intensive and suffers from additional shortcomings.

In this study the centroid and the singleton methods will be used to combine and defuzzify the outputs into
3.4 The System Output

3.4.1 Definition of Output

The system has three outputs:

- **YELLOW-TIME**, the duration of the change interval.
- **ALL-RED**, the duration of the clearance interval.
- **GREEN-EXTENSION**, the duration of the green extension.

For **YELLOW-TIME** we have the following fuzzy values:
- SHORT-for short change interval
- MEDIUM-for an average change interval
- LONG-for long intervals

For **ALL-RED** we have created the following fuzzy values:
- SHORT-for provision of no all-red or short all-red time duration.
- MEDIUM-for an average clearance interval
- LONG-for long clearance intervals

Similarly, for **GREEN-EXTENSION** we have created the following fuzzy values:
- SHORT-for provision of no green-extension or short green-extension time duration
- MEDIUM-LONG-for an average green-extension interval
- LONG-for long green-extension intervals

3.4.2 The Combination of Four Scenarios

By combining the WIDTH and the GRADE we have four scenarios, which modify the system. They are illustrated in Figure 2.

4. Experiment Design

4.1 Purpose of Experiment

The main purpose of the experiment is to test and illustrate various fundamental features of the rule-based fuzzy logic system.

Two scenarios were tested in the experiment. For each scenario all the outputs from the 27 different combinations formed the crossing of the primary variables (speed (SP), congestion factor (CF), and driver location (DL)) were included in the experiment. The two scenarios are, namely:

1. **Scenario 1** – Medium intersection width and flat grade, and
2. **Scenario 2** – Wide intersection width and flat grade.

Descriptive statistics are also used to describe the results of the rule-based fuzzy logic system. It also provides important characteristics of the system and its outputs for further examination such as the operating ranges and variances. Typical modification at this stage is altering the membership functions of the appropriate primary variables and running the system again until the system reflects field conditions or recommended operating ranges.

4.2 Method of Analysis and Inference

4.2.1 The Inference of the Yellow Interval

From the analysis of variance (see Table 2), it is shown that only two of the primary variables speed (SP) and driver location (DL) significantly affect the yellow interval timings. The congestion factor was not significant in the yellow interval timings. As expected, this is consistent with driver behavior and reasoning. Drivers based their reaction on how far they are from the intersection and the speed at which they are traveling in responding to the yellow signal.

Table 2: Analysis of Variance and Regression for Yellow Interval Timing

<table>
<thead>
<tr>
<th>Analysis of Variance (ANOVA) Table</th>
</tr>
</thead>
<tbody>
<tr>
<td>df</td>
</tr>
<tr>
<td>----</td>
</tr>
<tr>
<td>REGRESSION</td>
</tr>
<tr>
<td>RESIDUAL</td>
</tr>
<tr>
<td>TOTAL</td>
</tr>
</tbody>
</table>

**Regression Statistics**

<table>
<thead>
<tr>
<th>Coeff.</th>
<th>SE</th>
<th>t</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>INTERCEPT</td>
<td>2.810</td>
<td>0.134</td>
<td>21.004</td>
</tr>
<tr>
<td>SP(Speed)</td>
<td>0.027</td>
<td>0.003</td>
<td>9.660</td>
</tr>
<tr>
<td>CF(Congestion)</td>
<td>0.152</td>
<td>0.092</td>
<td>1.653</td>
</tr>
<tr>
<td>DL(Location)</td>
<td>0.001</td>
<td>0.000</td>
<td>7.171</td>
</tr>
</tbody>
</table>

*significant at 0.05 level

4.2.2 The Inference of the All-Red Interval

The analysis of variance (see Table 3) shows that the primary variable speed (SP) is not a determining factor or has no significant effect on the duration of the all-red time. The results show that driver location (DL) and congestion factor (CF) significantly affects the
all-red interval timings. Field observations indicate that the higher the congestion factor the more likely drivers are to overuse the yellow time and cross the intersection on red indication. As such, providing an all-red indication for congested intersections does provide adequate safety and also accommodate driver behavior.

### Table 3: Analysis of Variance and Regression for All-Red Interval Timing

<table>
<thead>
<tr>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>REGRESSION</td>
<td>3</td>
<td>0.971</td>
<td>0.324</td>
<td>21.12</td>
</tr>
<tr>
<td>RESIDUAL</td>
<td>23</td>
<td>0.353</td>
<td>0.015</td>
<td></td>
</tr>
<tr>
<td>TOTAL</td>
<td>26</td>
<td>1.324</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Coeff.</th>
<th>SE</th>
<th>t</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>INTERCEPT</td>
<td>0.633</td>
<td>0.141</td>
<td>4.481</td>
</tr>
<tr>
<td>SP(Speed)</td>
<td>0.000</td>
<td>0.003</td>
<td>0.000</td>
</tr>
<tr>
<td>CF(Congestion)</td>
<td>0.692</td>
<td>0.097</td>
<td>7.110</td>
</tr>
<tr>
<td>DL(Location)</td>
<td>0.001</td>
<td>0.000</td>
<td>3.580</td>
</tr>
</tbody>
</table>

*significant at 0.05 level

Regression Statistics

| Multiple R | 0.857 |
| R Square   | 0.734 |
| Standard Error | 0.124 |
| Observation | 27 |

Besides congestion factor and driver location, intersection width plays a significant role in all-red interval timing. Although the all-red interval is not provided at all intersections, it is highly recommended for wide intersections to prevent collisions (ITE, May 1985).

### 4.2.3 The Inference of Green Extension

The analysis of variance (see Table 4) shows that only one of the primary variable congestion factor (CF) significantly affects the green extension timing. The results show that the driver’s location (DL) and speed of approaching vehicle (SP) have little or no significant effect on green extension timing. Field observations and expert advice indicate that green extensions are provided at high-speed intersections with high truck volumes and at congested intersections to improve safety and throughput of intersections. However, in this rule-based fuzzy logic system no provision was made at high-speed intersections with a high percentage of truck volume. Needless to say, the model can be modified easily to include the speed and truck percentages as a control factor for the green extension.

### Table 4: Analysis of Variance and Regression for All-Red Interval Timing

<table>
<thead>
<tr>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>REGRESSION</td>
<td>3</td>
<td>9.417</td>
<td>3.139</td>
<td>107.01</td>
</tr>
<tr>
<td>RESIDUAL</td>
<td>23</td>
<td>0.675</td>
<td>0.029</td>
<td></td>
</tr>
<tr>
<td>TOTAL</td>
<td>26</td>
<td>10.091</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Coeff.</th>
<th>SE</th>
<th>t</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>INTERCEPT</td>
<td>-0.749</td>
<td>0.195</td>
<td>-3.832</td>
</tr>
<tr>
<td>SP(Speed)</td>
<td>0.000</td>
<td>0.004</td>
<td>0.000</td>
</tr>
<tr>
<td>CF(Congestion)</td>
<td>2.410</td>
<td>0.135</td>
<td>17.908</td>
</tr>
<tr>
<td>DL(Location)</td>
<td>0.000</td>
<td>0.000</td>
<td>0.598</td>
</tr>
</tbody>
</table>

*significant at 0.05 level

Regression Statistics

| Multiple R | 0.966 |
| R Square   | 0.933 |
| Standard Error | 0.171 |
| Observation | 27 |

The study has shown that rule-based fuzzy logic systems can be used to set up the clearance and change intervals for traffic signals. The model parameters evaluated included speed of vehicle, driver location, and congestion factor. The study shows that one or more factors affect the change interval. The results of the rule-based fuzzy logic system is also consistent with field observations and expert advice.

### 5. References