Design of Fuzzy Systems for the Modelling of Explosive Cutting Process of Plates Using Singular Value Decomposition

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Abstract: Fuzzy systems are used for modelling of explosive cutting process of plates by shaped charges. The aim of such modelling is to show how the depth of penetration varies with the variation of important parameters. It is also demonstrated that singular value decomposition (SVD) can be effectively used to fine tuning of such fuzzy models in a noniterative process. Such application of SVD will highly improve the performance of fuzzy systems to model the very complex process of explosive cutting of plates by shaped charges.

Key Words: Fuzzy systems, Explosive Cutting Process, SVD

1 Introduction

Modelling of processes and system identification using input-output data have always attracted many research efforts. In fact, system identification techniques are applied in many fields in order to model and predict the behaviours of unknown and/or very complex systems based on given input-output data [1]. Theoretically, in order to model a system, it is required to understand the explicit mathematical input-output relationship precisely. Such explicit mathematical modelling is, however, very difficult and is not readily tractable in poorly understood systems. Alternatively, soft-computing methods [2], which concern computation in imprecise environment, have gained significant attention. The main components of soft computing, namely, fuzzy-logic, neural network, and genetic algorithm have shown great ability in solving complex non-linear system identification and control problems. Several research efforts have been expended to use evolutionary methods as effective tools for system identification [3-6]. Moreover, these methodologies, fuzzy rule-based systems have been an active research field for their unique ability to build models based on experimental data. The concept of fuzzy sets which deal with uncertain or vague information, paved the way for applying them to real and complex tasks [7]. Indeed, fuzzy-logic, coupled with rule-based systems, has the ability of modelling the approximate and imprecise reasoning processes which is common in human thinking or human problem solving. This results a policy which can be accordingly evaluated mathematically by using fuzzy set theory. Therefore, fuzzy systems as universal approximator [8-10] can be effectively employed to perform input-output mapping. Such fuzzy systems can be iteratively designed using different evolutionary search methods [11-13] or using hybrid learning rule in ANFIS [14]. In fact, these fuzzy systems are trained by examples \((X_i, y_i) \ (i=1, 2, \ldots, M)\) in terms of input-output pairs. Recently, a combination of orthogonal transformation and backpropagation methods has been proposed to train a candidate fuzzy model and to remove its unnecessary fuzzy rules [15].

Explosive cutting of plates using shaped charges is one of the processes in mechanical engineering in which the physical interactions of various involved parameters are rather complex [16]. In fact, during the last few decades the use of explosives as a source of energy has found many applications in engineering. The main difference between explosives, magnetomotive forces, impact and any other source of energy is that a very large amount of energy is made available to do work in a very short period of time. Explosives are now used in such diverse fields as welding, bulk cladding of plates, forming, sizing, powder compaction, hardening, and cutting. The use of explosives is not due to solely to their speed, but also that sometimes there may be no other way of achieving the same results as in explosive welding of dissimilar metals.
In cutting metals using linear shaped charge, an explosive charge with a metallic liner is placed at a certain distance from the metal part. The cutting action is the consequence of the development of a very high-speed jet of molten metal produced by the collapse of the liner. The explosive is usually detonated from one end. As the detonation proceeds down the length of the charge, the metallic liner collapses inwards and is projected as a high velocity linear jet of metallic particles. A standoff between the cutter and the target is essential to the proper formation of the jet and problems occur in the use of linear shaped charges underwater as the standoff must be filled with air. The normal parameters of interest for any shaped charge that affect its performance as reported in the literature, are the liner material, liner thickness, the type of explosive, the explosive weight, the liner shape, and the standoff distance.

In this paper, it is shown that optimally-designed fuzzy system can effectively model the depth of penetration as a function of important input parameters in explosive cutting process, namely, the apex angle, the standoff, the liner thickness, and the mass of charge. In this way, a simple heuristic method for designing fuzzy systems from input-output data pairs is enhanced using singular value decomposition (SVD). Consequently, fuzzy systems for modelling data which obtained from high-energy rate explosive cutting process can be effectively constructed and tuned in a noniterative procedure. The number of rules derived from all input-output data pairs has less impact on the computation cost and is thus possible to approximate the training data more precisely.

2 Experimental Procedure and Results

For the first set of tests, the standoff distance was varied whilst the liner thickness, the apex angle defined as the perpendicular distance from the end of the charge to the target, and the weight of the charge were maintained constant. In the second set of data, the standoff distance was varied whilst the apex angle, the liner thickness, and the weight of the charge were maintained constant. In the third set of data, the liner thickness was varied whilst the apex angle, the standoff, and the liner thickness were kept constant. It should be noted that this experiment was repeated for three different value of charge weight. Finally, in the fourth set of data, the charge weight together with the liner thickness was varied whilst the apex angle and the standoff were kept constant.

Experimental results obtained [17][18] indicate that the standoff distance has an appreciable effect on penetration. Increasing the standoff allows the jet to elongate before it runs into the target. An optimum standoff is found to exist after which the penetration is shallower. The depth of penetration was also found to be affected by the apex angle.

3 Modelling Using Fuzzy Systems

Fuzzy systems which consist of set of fuzzy IF-THEN rules can be used in modelling in order to map inputs to outputs. The formal definition of the identification problem is to find a function \( \hat{f} \) so that can be approximately used instead of actual one, \( f \), in order to predict output \( \hat{y} \) for a given input vector \( X = (x_1, x_2, x_3, \ldots, x_n) \) as close as possible to its actual output \( y \). Therefore, given \( M \) observation of multi-input-single-output data pairs so that

\[
y_i = f(x_{i1}, x_{i2}, \ldots, x_{in}) \quad (i=1,2,\ldots,M) \quad (1)
\]

It is now possible to build a look-up table to be used to train a fuzzy system to predict the output values \( \hat{y}_i \) for any given input vector \( X = (x_{i1}, x_{i2}, x_{i3}, \ldots, x_{in}) \), that is

\[
\hat{y}_i = \hat{f}(x_{i1}, x_{i2}, x_{i3}, \ldots, x_{in}) \quad (i=1,2,\ldots,M). \quad (2)
\]

The problem is now to determine a fuzzy system so that the difference between the actual output and the predicted one is minimised, that is

\[
\sum_{i=1}^{M} [\hat{f}(x_{i1}, x_{i2}, x_{i3}, \ldots, x_{in}) - y_i] \rightarrow \min . \quad (3)
\]

In this way, a set of linguistic fuzzy IF-THEN rules is designed to approximate \( f \) by \( \hat{f} \) using \( M \) observation of n-input-single-output data pairs \( (X_i, y_i) \) \( (i=1, 2, \ldots, M) \). The fuzzy rules embodied in such fuzzy models can be conveniently expressed using the following generic form
Rule \(_l\) : IF \(x_1\) is \(A^{(j_1)}_l\) AND \(x_2\) is \(A^{(j_2)}_l\) AND, 
\(...\), \(x_n\) is \(A^{(j_n)}_l\) THEN \(y\) is \(B^{(k)}_l\) 
(4)
in which \(j_i \in \{1, 2, ..., r\}\) and \(k \in \{1, 2, ..., s\}\). The entire fuzzy sets in \(x_i\) and \(y\) spaces are given as

\[A^{(i)} = \{A^{(1)}, A^{(2)}, A^{(3)}, ..., A^{(r)}\} \quad (5a)\]

and

\[B^{(i)} = \{B^{(1)}, B^{(2)}, B^{(3)}, ..., B^{(s)}\}. \quad (5b)\]

These entire fuzzy sets are assumed symmetric triangular defined on the domains \([-\alpha_i, +\beta_i]\) and \([-\gamma, +\theta]\), respectively. These domains are selected so that all the fuzzy sets to be complete; that is for any \(x_i \in [-\alpha_i, +\beta_i]\) there exist \(A^{(j_i)}_l\) in equation (5a) such that the degree of membership function in non-zero, \(\mu_{A^{(j_i)}_l}(x_i) \neq 0\).

After defining fuzzy sets to completely cover the input and output spaces, it is now possible to generate one rule from each input-output data pairs [19]. However, it can be readily observed that more than one rule is possible for each input-output data pair. It is evident that rules with the same antecedents but different consequents are reduced to the one having the highest degree. The fuzzy rule expressed in equation (4) is a fuzzy relation in \(U \times V\) in which \(A^{(i)}\) and \(B\) are fuzzy sets in \(U_i\) and \(V\) so that \(U = U_1 \times U_2 \times U_3 \times \ldots \times U_n\) and \(Rule = A^{(j_1)}_1 \times A^{(j_2)}_2 \times A^{(j_3)}_3 \times \ldots \times A^{(j_n)}_n \rightarrow B\). It is evident that the input vector \(X = (x_1, x_2, x_3, ..., x_n)^T \in U\) and \(y \in V\). Using Mamdani algebraic product implication, the degree of such local fuzzy IF-THEN rule can be evaluated in the form

\[
\mu_{Rule}^{(i)} = \mu_{A^{(j_1)}_l \times A^{(j_2)}_2 \times \ldots \times A^{(j_n)}_n}(x_1, x_2, ..., x_n) \times \mu_{B^{(k)}}(y)
\]

(6)

where

\[
\mu_{A^{(j_i)}_l} = \prod_{i=1}^{n} \mu_{A^{(j_i)}_l}(x_i)
\]

(7)

In these equations \(\mu_{A^{(j_i)}_l}\) and \(\mu_{B^{(k)}}\) represent the degree of membership of input \(x_i\) and output \(y\) regarding their \(lth\) fuzzy rule’s linguistic value, \(A^{(j_i)}_l\) and \(B^{(k)}_l\), respectively. If the fuzzy set \(B^{(k)}_l\) is normal with centre \(\hat{y}_I\), then using singleton fuzzifier, product inference engine, and centre average defuzzifier leads to the fuzzy system in the form

\[
f(X) = \frac{\sum_{l=1}^{N} \bar{y}_{Rule}^{(l)} (\prod_{i=1}^{n} \mu_{A^{(j_i)}_l}(x_i))}{\sum_{l=1}^{N} (\prod_{i=1}^{n} \mu_{A^{(j_i)}_l}(x_i))}
\]

(8)

when a certain set containing \(N\) fuzzy rules in the form of equation(4) is available. It is then clear that in equation(8) \(\bar{y}_{Rule}^{(l)} \in \{\bar{y}_1, \bar{y}_2, ..., \bar{y}_s\}\) where \(s\) is the total number of fuzzy linguistic terms of the consequence. Since some of rules’ consequences may be the same, equation(8) can be alternatively represented in the following linear regression form

\[
f(X) = \sum_{k=1}^{s} p_k(X) \bar{y}_k + D
\]

(9)

where \(D\) is the difference between \(f(X)\) and corresponding actual output, \(y\), and

\[
p_k(X) = \frac{\sum_{t=1}^{n_k} (\prod_{i=1}^{n} \mu_{A^{(j_i)}_l}(x_i))}{\sum_{l=1}^{N} (\prod_{i=1}^{n} \mu_{A^{(j_i)}_l}(x_i))} \quad \text{if } n_k \geq 1
\]

(10)

\[
p_k(X) = 0. \quad \text{if } n_k = 0.
\]
In equation (10), \( n_k \) is the number of the similar \( y_k \in \{ y_1, y_2, \ldots, y_s \} \) which appear when the numerator of equation (8) is expanded for \( \bar{y}_{Rule} \).

It is therefore evident that equation (9) can be readily expressed in a matrix form for a given \( M \) input-output data pairs \((x_i, y_i)\) \((i=1, 2, \ldots, M)\) in the form

\[
Y = P \bar{y}_{Rule} + D
\]  

(11)

where \( Y = [y_1, y_2, \ldots, y_M]^T \in \mathbb{R}^M \) and \( P = [p_1, p_2, \ldots, p_s]^T \in \mathbb{R}^{M \times S} \) with \( p_i = [p_{i1}, p_{i2}, \ldots, p_{im}]^T \in \mathbb{R}^M \). Such firing strength matrix \( P \) and associated fuzzy consequences are obtained when input and output spaces are partitioned into certain number of fuzzy sets and, in addition, the related fuzzy rules are extracted based on direct matching [20]. However, such procedure has been modified in this paper to including all possibilities of fuzzy rules of given input-output data pair \((x_i, y_i)\) unless a stronger rule with the same antecedents yet different consequence is generated according to the other given data pairs. It is evident that the number of available training data pairs is usually larger than the fuzzy linguistic terms of the consequence, that is \( M \geq S \). This situation turns the equation (11) into a least squares estimation process in terms of unknowns, \( \bar{y}_{Rule} \), so that the difference \( D \) is minimized. The governing normal equations can be expressed in the form

\[
\bar{y}_{Rule} = (P^T P)^{-1} P^T Y
\]  

(12)

Such modification of output fuzzy partitioning leads to better approximation of given data pairs in terms of minimization of difference vector \( D \). However, such solution directly from normal equations is rather susceptible to roundoff error and, more importantly, to the singularity of these equations.

4 Application of Singular Value Decomposition to the Design of Fuzzy Models

Singular value Decomposition (SVD) is the method for solving most linear least squares problems that some singularities may exist in the normal equations. The SVD of a matrix, \( P \in \mathbb{R}^{M \times S} \), is a factorisation of the matrix into the product of three matrices, column-orthogonal matrix \( U \in \mathbb{R}^{M \times M} \), diagonal matrix \( W \in \mathbb{R}^{S \times S} \) with non-negative elements (singular values), and orthogonal matrix \( V \in \mathbb{R}^{S \times S} \) such that

\[
P = U W V^T
\]  

(13)

The most popular technique for computing the SVD was originally proposed in [21]. The problem of optimal selection of \( \bar{y}_{Rule} \) in equations (8)(11) is firstly reduced to finding the modified inversion of diagonal matrix \( W \) [22] in which the reciprocals of zero or near zero singulars (according to a threshold) are set to zero. Then, such optimal \( \bar{y}_{Rule} \) are obtained using the following relation

\[
\bar{y}_{Rule} = V \{ diag(1/w_j) \} U^T Y
\]  

(14)

Such procedure which involves the direct matching extraction fuzzy rules together with SVD approach of finding the optimal fuzzy consequences, \( \bar{y}_{Rule} \), is noniterative and, therefore, can be effectively employed to build fuzzy models based on input-output observation data pairs.

5 Fuzzy Modelling of Explosive Cutting Process of Plates by Shaped Charges

Direct matching rule extraction and SVD are used to design a fuzzy system for a set of given experimental input-output data in a series of explosive cutting tests of plates by shaped charges. The parameters of interest in this multi-input single-output system which affect the performance of the shaped charge and, in turn, the depth of penetration are the liner material, the explosive material, the liner shape, the apex angle, the liner...
thickness, the explosive weight, and distribution, and the standoff distance. Among these parameters, the liner and explosive material together with the liner shape have been kept fixed. The liner material, explosive material, and the liner shape have been selected as Copper/Polymer, SX2, and ‘V’ shape, respectively. Accordingly, there have been a total number of 43 input-output experimental data considering 4 input parameters, namely, the apex angle, the standoff, the liner thickness, and the explosive weight, in four different groups. In the first set of data, the apex angle varied from 45 degree to 135 degree whilst other parameters were fixed as the standoff=0., the explosive weight=50g, and the liner thickness=0.9mm. In the second set of data, the standoff varied from -0.4 to 1 (or, equivalently, non-effective standoff from 4.3mm to 14.5 mm) whilst other parameters were fixed as the apex angle=100 degree, the explosive weight=50g, and the liner thickness=0.9mm. In the third set of data, the liner thickness varied whilst other parameters were fixed as the apex angle=100 degree, the standoff=0., and the explosive weight=50g, 150g, and 250g. Finally, in the fourth set of data, the explosive weight and the liner thickness varied simultaneously, whilst other parameters were fixed as the apex angle=100 degree and the standoff=0. In order to model the 4-input-single-output set of data, a fuzzy system with 5 linguistic terms in each antecedent and 35 linguistic terms in consequence was considered, that is, r=5 and s=35. Then applying the modified direct matching fuzzy rule extraction method with the modification of including all possibilities of fuzzy rules for each pair of input-output data, a set consisting of 132 rules was constructed. In order to tune the obtained fuzzy system, SVD is then applied to identify some singular values in firing strength matrix. The result of these singular values is depicted in Fig. 1. It is evident that the dimension of firing strength matrix, as well as the number of fuzzy linguistic terms in the consequence, can now be appropriately reduced. The experimental data obtained from the above-mentioned procedure constitute a 43 4-input-single-output data used independently by the two direct matching extraction fuzzy rule and the tuned-fuzzy model methods. Fig.2 shows the behaviour of the fuzzy model when its rules were extracted using direct matching. Such behaviour has been improved significantly, as shown in Fig.3, when the obtained model was further tuned using singular value decomposition of the constructed firing strength matrix. It should be
further tuned using singular value decomposition of the constructed firing strength matrix. It should be noted that each data index corresponds to one 4-input-single output data obtained experimentally as discussed before.

6 Conclusion

Fuzzy systems have been successfully used for the modelling of very complex process of explosive cutting of plates by shaped charges. In this way, it has been shown that fuzzy systems provide effective means to model and predict the depth of penetration according to different input. Moreover, it has been shown that singular value decomposition can significantly improve the performance of such fuzzy systems which can be constructed by direct matching using a look-up table. Such efficient and noniterative procedure will also reduce the dimension of the firing strength matrix as the unnecessary values of output fuzzy partitions are removed.

References