A Novel Approach for Chattering Minimization in Sliding Mode Control

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Abstract: The purpose of this paper is to design a high performance sliding mode controller through the use of a new switching function. This method uses the idea of boundary layer sliding mode while taking the boundary layer width as a function of the angle between the state trajectory and the sliding surface which we call approach angle. By incorporating the approach angle into the switching function, the overall sliding mode controller guarantees the asymptotical stability of the system while having only a slight amount of chattering. This method overcomes the shortcomings of a pure discontinuous switching such as excessive chattering, while maintaining its benefit that is the asymptotical stability is guaranteed. The proposed method is used to control an inverted pendulum. It is also compared with a pure switching function. Simulation results show that the new sliding mode controller has good control performance with negligible chattering.

Keywords: Sliding mode controller, chattering reduction, switching function, boundary layer.

1 Introduction

Sliding mode control [1] is a particular type of VSCS. The most distinguishing property of VSCS is that the closed loop system is completely insensitive to system uncertainties and external disturbances. In sliding mode control, VSCS are designed to drive and then constrain the system to lie within a neighborhood of a suitable switching surface in the state space [3]. As long as the system trajectory stays on this surface, the closed loop dynamics are completely governed by equations that define the surface. Some advantages of this approach are flexible design (any control is suitable so long as the resulting trajectories are directed towards the switching surface), robustness (since the parameters defining the switching surface are chosen by the designer, the dynamic behavior of the closed loop system are independent of changes in the plant parameters as long as the system stays on the switching surface, and invariance (motion on the sliding surface is invariant with respect to bounded disturbances that are in the range space of the input vector).

A disadvantage of the sliding mode is that the discontinuous control signal may excite high frequency dynamics of the system neglected in the course of modeling such as unmodeled structural modes, time delays and so on [2]. This causes fast, finite-amplitude oscillations known as “chattering”, which would result in loud noise, high wear of moving mechanical parts and thus should be definitely eliminated.

Different schemes have been proposed in the research literature to eliminate the chattering. The most commonly cited approach to reduce the effects of chattering has been the so called piecewise linear or smooth approximation of the switching element in a boundary layer in the vicinity of the sliding surface [6], [7]. Inside the boundary layer, the switching function is approximated by a linear feedback gain. This proposed method has wide acceptance by many sliding mode researchers, but unfortunately it does not resolve the core problem of the robustness of sliding mode as exhibited in chattering.

Various complex hybrid sliding mode controller structures also have been proposed. These hybrid controllers try to ensure the asymptotical stability...
and to reduce the chattering by combining sliding mode control with other techniques, such as adaptive control techniques [13] and fuzzy control techniques [14]. Most of these hybrid controllers require complex implementation algorithms. In the sliding mode controller considered in this work, a changing boundary layer width is used to adjust for the trade off between robustness and chatter elimination. It is based on the idea that the amount of chattering depends on the angle under which the state trajectory approaches the sliding surface. The larger the angle is the more chattering is caused. By incorporating the approach angle into the switching function the boundary layer width and consequently the feedback gain within boundary layer can be adjusted according to the state approach angle in order to achieve minimum chattering with maximum robustness. A new switching function based on this idea is presented to design a sliding mode controller and its performance is investigated in this work.

The organization of the paper is as follows: in Section 2 the design procedure of sliding mode control is briefly reviewed. Then, in Section 3 the use of the new switching function for sliding mode design is introduced. In Section 4 two indices are introduced for evaluation of the controller performance. In section 5 the control of a nonlinear inverted pendulum using the proposed control is presented. Section 6 contains the simulation results. Finally, Section 7 concludes the paper.

2 Sliding Mode Control Design

A Sliding Mode Controller is a Variable Structure Controller (VSC). Without loss of generality, consider the design of a sliding mode controller for the following second order system:

\[ x = f(x, x, t) + bu(t) \]  

(1)

Here we assume \( b > 0 \). \( u(t) \) is the input to the system. The following is a possible choice of the structure of a sliding mode controller [10]:

\[ u = -k \operatorname{sgn}(s) + u_{eq} \]  

(2)

where \( u_{eq} \) is called equivalent control which dictates the motion of the state trajectory along the sliding surface. \( k \) is a constant, representing the maximum controller output. \( s \) is called switching function because the control action switches its sign on the two sides of the switching surface \( s = 0 \). For a second order system \( s \) is defined as:

\[ s = \ddot{x} + \lambda \dot{x} \]  

(3)

where \( \ddot{x} = x - x_d \) is the tracking error vector and \( x_d \) is the desired state and \( \lambda \) is a constant. The definition of \( \ddot{x} \) here requires that \( k \) in (2) be positive. \( \operatorname{sgn}(s) \) is the sign function:

\[ \operatorname{sgn}(s) = \begin{cases} -1 & \text{if } s < 0 \\ 1 & \text{if } s > 0 \end{cases} \]  

(4)

The control strategy presented by (2) is called the discontinuous sliding mode. It will guarantee that the system trajectory moves toward and stays on the sliding surface \( s = 0 \) from any initial condition if the following condition meets:

\[ ss \leq -\eta |s| \]  

(5)

where \( \eta \) is a positive constant that guarantees the system trajectories hits the sliding surface in a finite time.

As explained before, using a sign function often causes chattering in practice. One solution is to introduce a boundary layer around the switch surface [2] and use a linear approximation of the sign function within the boundary layer:

\[ u = -k \operatorname{sat}(s/\phi) + u_{eq} \]  

(6)

where the constant factor \( \phi \) defines the thickness of the boundary layer. \( \operatorname{sat}(s/\phi) \) is a saturation function defined as:

\[ \operatorname{sat}(s/\phi) = \begin{cases} \frac{s}{\phi} & \text{if } |s| \leq 1 \\ \operatorname{sgn}(s) & \text{if } |s| > 1 \end{cases} \]  

(7)

It is proven that if \( k \) is large enough, the sliding mode controllers of (2) and (6) are guaranteed to be asymptotically stable [10].

The consequence of this control scheme is that invariance property of sliding mode control is lost. The system robustness is a function of the width of the boundary layer. The thinner the boundary layer the more robust will be the controller.

3 The New Switching Function

The slope of the saturation function and thus the control gain applied to the system within the boundary layer is inversely proportional to the width of the boundary layer. On the other hand,
large control gain is equivalent to large system chattering. Thus system chattering can be decreased by extending the boundary layer width. Of course, a wide boundary layer may cause the system to lose its asymptotical stability and also decreases the system’s robustness.

Examining the chattering behavior of a system, one can easily see that excessive chattering occurs when the state trajectory approaches the sliding surface under a large approach angle. Thus, by taking the boundary layer width as a function of the approach angle we can adjust the boundary layer to be wide when the approach angle is large, to reduce the chattering, and to make it thin when the approach angle is small to increase the robustness and satisfy the asymptotical stability of the system.

Thus we add the state approach angle, $\theta$, to the boundary layer of the continuous witching function.

So the control, $u$, takes the following form:

$$u = u_{eq} - k \cdot \text{sat}(\frac{s}{|\theta|})$$

It can be shown that by adding the continuous switching term of the form: $-k \cdot \text{sat}(s/|\theta|)$ to equivalent control, $u_{eq}$, the asymptotical stability of the overall system is guaranteed.

Using this perception it can be shown that the overall control system is asymptotically stable.

4 Control of an Inverted Pendulum

In order to investigate the effectiveness of the new switching control, it was applied to an inverted pendulum regulation problem using computer simulation. Here are the equations for the state space model of an inverted pendulum:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = g \sin x_1 - \frac{mLx_2^2 \cos x_1 \sin x_1}{(m+m_c)}$$

$$\dot{y} = x_1 + b(x_1)u$$

where $x_1$ is the angle between the pendulum and vertical direction, $u$, is the control force applied to the cart and $y$ is the system output. Other parameters can be seen in figure 2.

In order to verify the robustness of the proposed controller, the system function, $f$, and the control gain, $b$, are assumed to be unknown but bounded functions which (using the parameter values given in table 1) can be estimated as:

$$\hat{f}(x_1, x_2) = 2.63x_1 - 0.073x_2^2$$

and

$$\hat{b} = \sqrt{b_{\text{min}} b_{\text{max}}}$$

that is the geometric mean of $b_{\text{min}}$ and $b_{\text{max}}$, the
lower bound and upper bound of $b$.
The control gain margin, $\beta$, is defined as:
$$\beta = \sqrt{\frac{b_{\text{max}}}{b_{\text{min}}}}.$$  
(13)

The equivalent control $u_{eq}$ then can be approximated using the estimation of $f$. To achieve $\dot{s} = 0$ it becomes:
$$\dot{u}_{eq} = -\hat{f} + \ddot{x}_d - \dot{\lambda}\dot{x}$$
(14)

So the overall control will be:
$$u = \dot{\beta}^{-1}\left[\dot{u}_{eq} - K_{\text{sat}}(s \Phi)\right]$$
(15)

In order for the sliding condition of (5) to hold, it is sufficient that the switching gain, $k$, be chosen large enough to satisfy the following condition:
$$k \geq \beta(F + \eta) + (\beta - 1)|u_0|$$
(16)

where the function $F$ is the upper bound for the estimation error of $f$.
$$|f - \hat{f}| < F$$
(17)

In many practical cases where it is difficult to calculate the proper control gain each time, a large enough constant can be used instead.

5 Definition of Performance Measures

The purpose of control is to regulate the states to zero with minimum system chattering. Thus it is appropriate to evaluate the sliding mode control performance from the viewpoint of both control error and actuator chattering. For this, two indices indicating the extents of error and chattering are respectively introduced as follows:

$$E = \frac{c_e \int_{t_0}^{t_f} \|x\| \, dt}{t_f - t_0}$$
(18)

$$C = \frac{c_c \int_{t_0}^{t_f} \|\dot{x}\| \, dt}{t_f - t_r}$$
(19)

where $c_e$, $c_c$ are proportional constants, $t_r$ is the reaching time that is the time by which the states hit the sliding surface for the first time and $t_f$ is the final time. The index $E$ is for the evaluation of control error and shows the average distance of the states from their desired value that is zero. The other index, $C$, is for evaluation of chattering and is equal to the average change rate of the slope of actuator output during the time interval between when the states reach the sliding surface and the final time. As a result smaller values of $E$, $C$ are preferred for better regulation and less chattering, respectively.

6 Simulation Results

The controller proposed in this work was used to regulate the inverted pendulum described in section 3 with the parameter values listed in table 1 from the initial state $x(0) = 1$, $x_2(0) = 0$ to the origin.

**Table 1. Inverted pendulum parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>0.1 kg</td>
</tr>
<tr>
<td>$l$</td>
<td>3 m</td>
</tr>
<tr>
<td>$m_c$</td>
<td>1 kg</td>
</tr>
<tr>
<td>$g$</td>
<td>9.8 m/sec²</td>
</tr>
</tbody>
</table>

In this simulation the new switching function incorporating the state approach angle, $\theta$, was applied to the inverted pendulum. In order to compare the new controller with ordinary sliding mode controllers a discontinuous switching sliding mode controller was also used. Figures 3-7 show the simulation results for the two mentioned methods.

**Fig. 3.** Position and Speed vs. Time. (a) for the proposed controller. (b) for discrete switching controller.
In fig. 3 the states are displayed versus time for the two controllers. The states show little difference in their overall behavior and are regulated to zero in both cases. In fig. 4 the control action, \( u \), is shown and compared for the proposed controller and the classic sliding mode controller.

![Fig. 4. Control action, \( u \), vs. Time (a) for proposed controller. (b) for discrete switching controller.](image)

It is obvious that the proposed controller takes considerably less control action to regulate the system and chattering is negligible in the control signal produced by the proposed controller. The chattering suppression is the result of attenuating the state approach angle, \( \theta \), after the state trajectory enters the original boundary layer. As the state trajectory nears the sliding surface it takes on the behavior of a discontinuous switching because of the fact that the boundary layer is contracting to the sliding surface. Figure 5 shows the state approach angle, \( \theta \), versus time which is also a measure of the boundary layer width for the new sliding mode controller. The boundary layer width, \( \Phi|\theta| \), does not truly settle onto the sliding surface because of the uncertainty in the system.

![Fig. 5. Approach angle (boundary layer width) vs. time for proposed controller](image)

In figure 6 the state trajectory and the sliding surface are shown for the system controlled by the two methods. The state trajectory achieves asymptotical stability by reaching, and maintaining the sliding surface.

![Fig. 6. Phase trajectory and sliding surface. (a) for proposed controller. (b) for discrete switching controller](image)

In order to have a quantitative evaluation of the new
controller’s performance the two performance measuring indices defined in section 5 were calculated for the new controller and the ordinary discontinuous switching controller the results of which can be seen in table 2.

| Method | Index | sat(s/Φ|θ|) sign(s) |
|--------|-------|-------------|
| tᵣ    | 1.8   | 0.88        |
| E      | 1.823 | 1.814       |
| C      | 1.073 | 814.5       |

As it was expectable the reaching time is smaller for the discontinuous sliding mode controller. The error index, E, has had little increase in the proposed controller in comparison with the discontinuous switching controller while the chattering index, C, has had great reduction. So the use of the new controller can well be justified.

7 Conclusion

The use of a new switching function incorporating the state approach angle was considered. It was shown that sliding mode controller using this switching function can assure the asymptotical stability of the system while producing negligible chattering. The advantage of the proposed method to ordinary sliding mode methods was illustrated using the simulation study of an inverted pendulum. Two quantitative indices were also introduced and calculated to verify the good performance of the new controller.

References: